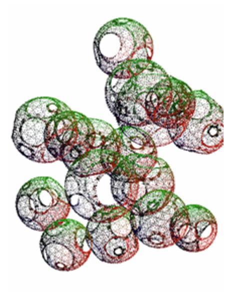
Point Cloud Collision Detection

- Modern acquisition methods lead to modern object representations.
- Efficient rendering (splatting & ray-tracing).
- Only little work on interaction.



Goals

- Fast collision detection between point clouds.
- No polygonal reconstruction.

Surface Definition

- Approximate surface by implicit function

  $$S = \{\mathbf{x} : f(\mathbf{x}) = 0, \mathbf{x} \in \mathbb{R}^3\}$$

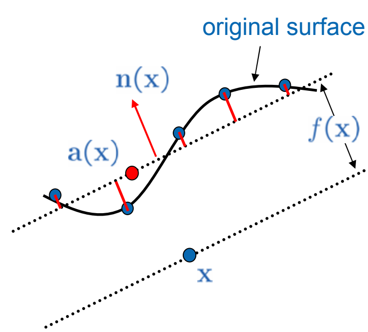
- Define $\mathbf{n}(\mathbf{x})$ by weighted least squares.

- Weight

  $$\theta(\mathbf{x}, \mathbf{p}) = e^{-\frac{d(x,p)^2}{h^2}}$$
  $$d(\mathbf{x}, \mathbf{p}) = \text{some distance measure}$$

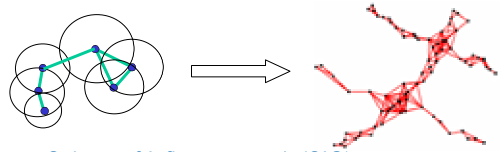
- $f(\mathbf{x}) = \mathbf{n}(\mathbf{x}) \cdot (\mathbf{a}(\mathbf{x}) - \mathbf{x})$

- Which distance measure to use?



[Klein & Zachmann, 2004]

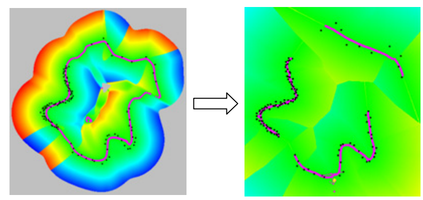
- Geometric proximity graph:
  - nodes = points
  - edges = "neighboring" points



Sphere-of-Influence graph (SIG)

- Approximate geodesic distance by shortest path.

- Properties:
  - Nice surface
  - Efficient evaluation
  - Implicit function throughout space
  - Surface with boundaries allowed



Contributions

- Novel, fast intersection computation for point clouds
- Utilizes proximity graph
- Runtime O(log log N), if constant number of intersection points is sufficient.
- Quality/resolution of output is adjustable.

Problem Statement

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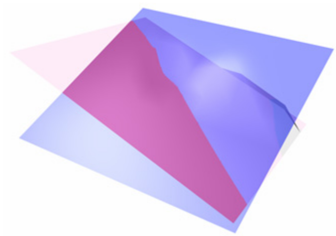
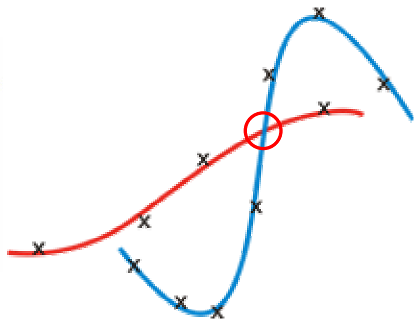
- Given two point clouds A and B (or subsets thereof),
  - decide if there is an intersection
  - construct a sampling of

$$\mathcal{Z} = \{x \mid f_A(x) = f_B(x) = 0\}.$$





Overview

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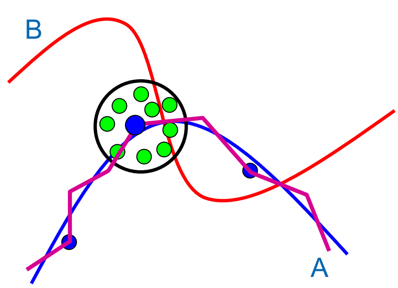
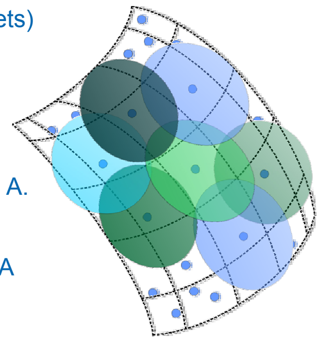
1. Bracket intersections by pairs of points.
2. Find approximate intersection point (AIP) by interpolation search.
3. Refine AIP by (randomized) sampling.



1. Root Bracketing

- Goal:
  - The pairs should evenly sample the surface.
  - The two points should not be too far apart.
  - Do it without explicit spatial data structure!
- Task: construct a pairs of points (to be root brackets)



- Thought experiment:
  - Assume surface is covered by a surfels.
  - Cover each surfel with at least one point from A.
    (candidate points for root brackets)
  - For each point: try to find another point from A
    lying on *the other side* of B.
    (completing the brackets)

Covering the Surfels

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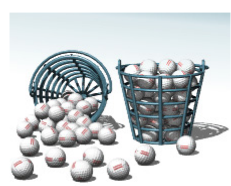
Avoid spatial data structure $\rightarrow$ pursue probabilistic approach: occupy all a surfels with high probability!



- Assumption: A is uniformly sampled.
- Lemma from paper $\rightarrow$
  draw $O(a \ln a)$ random and independent points
  from $A \cap Vol(A \cap B)$.
  Proof: see paper.

Premise: number of intersection points should be bounded by a constant.

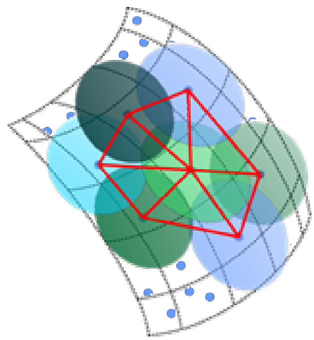
Consequence: choose a constant, or choose a depending on surfels size and surface area

Completing the Brackets

- Use $f_B(p_i) \cdot f_B(p_j) < 0$ as an indicator.
- Test only points p_j that
  - belong to the randomly chosen points
  - are close to each other

  Only a few points have to be tested!
- Solution: SIG

Finding brackets:
O(a ln a * d),
where d = max. out-degree;
average-ase: O(1)



2. Interpolation Search

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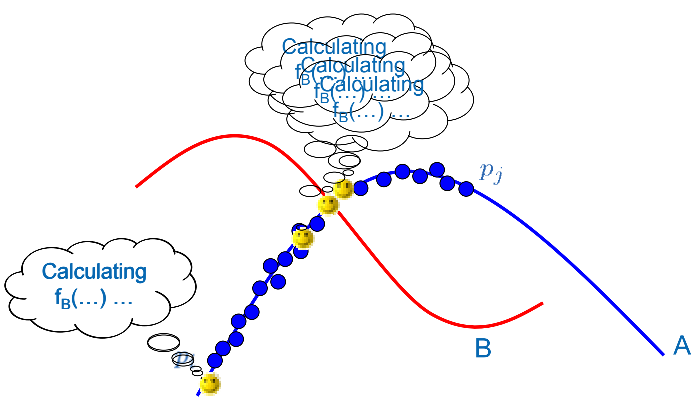
- Find $\hat{\mathbf{p}} \in A$ along shortest path $\overline{p_i p_j}$ in the geometric proximity graph, such that $|f_B(\hat{\mathbf{p}})|$ is minimal.
- Utilize *interpolation search!* → O(log log *m*), m = # elements



Interpolation Search

- Assumptions:
  - Shortest path are precomputed and stored in LUT.
  - f_B is monotone along shortest path.

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}

$P_i \in A$

$\uparrow l = 1$ $\qquad \uparrow r = 11$

- Interpolation parameter: $x = l + \lceil \frac{-f_B(P_l)}{f_B(P_r) - f_B(P_l)} (r - l) \rceil$

- Large point clouds:
  - Memory consumption could be too high.
    $\rightarrow$ compute paths on-the-fly.
  - In practice: runtime still behaves sublinear.

3. Precise Intersection Points

- Refine approximate intersection point.

$\rightarrow$ Details: see paper…

Runtime: O(*a* ln *a*)

Complexity Considerations

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- constant number of brackets, as a is constant
- Interpolation search: O(a ln a log log *m*) = O (log log *N*)
  (m= length of paths, is not constant!)
- Precise intersection points: O(*a* ln *a*) = O(1).
- f(x) can be evaluated in O(1).

Overall runtime: O(log log *N*)

Benchmark Scenario

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- Objects are scaled uniformly → cube size 2^3
- Perform a full tumbling turn by a fixed, large number (5000) of small steps.
- Average collision detection time for a complete revolution.

Minimal Bracket Density

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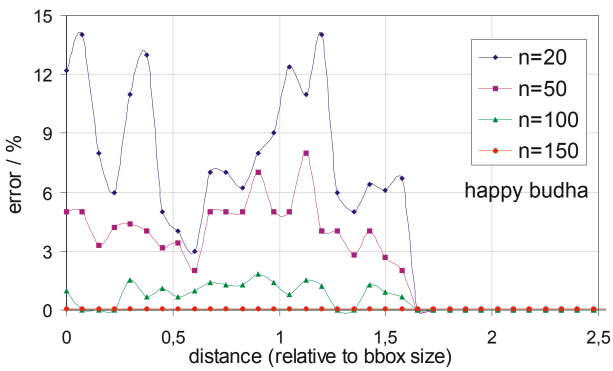
- If number of surfels is too small → influencing spheres in the graph are too large
  → likelihood increases that
  n(x) flips its sign without x changing sides.

- Use boolean collision queries to measure error.



60,000 points

Complexity

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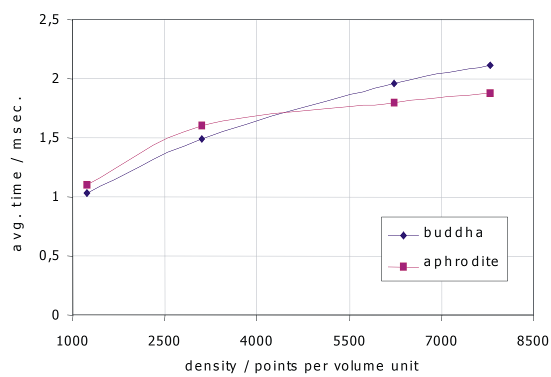
- Theoretical complexity: $O(\log \log N)$.

- Experimental complexity:



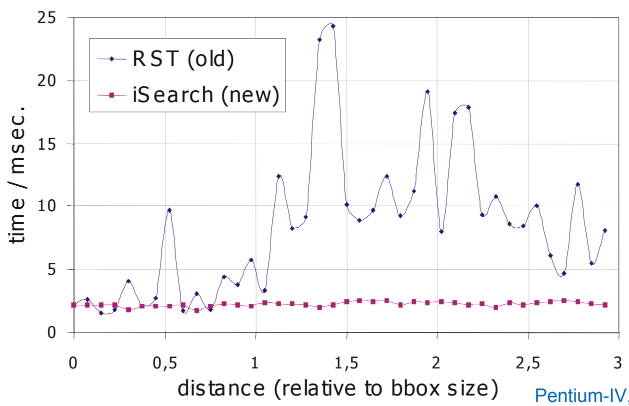
max. 60,000 points

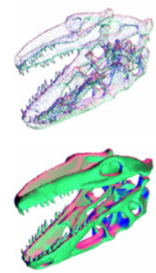
max. 90,000 points

Pentium-IV, 2.4GHz

Timings

- Benchmarking old vs. new method:
  – Old (RST) = brute-force sampling [EG'04]
  – iSearch = new





28,000 points

Pentium-IV, 2.4GHz

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Conclusion

- Technique:
  - utilizes a proximity graph for collision detection and surface definition.
  - needs no BV hierarchies and no spatial partitioning data structure.
  - any BV hierarchy can be augmented by new technique to increase performence.

- Runtime:
  - fast (approximate) collision detection
  - overall runtime: O(log log *N*) in average case.
  - speedup of factor 5–10 compared to "old" technique.

- Quality/resolution of output (intersection points) can be adjusted ($\rightarrow$ surfel radius)

Future Work

- Deformable point clouds, SIG can be updated in O($\log^3 N$).
- More rigorous estimation of minimal bracket density.
- Consistency of n(x).
- Out-of-core collision detection.

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Thank you!

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