

# Logics and categories for software engineering and artificial intelligence

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## Exercise Sheet 1

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### Exercise 1.1 (Propositional Logic I)

- (a) Use the equivalence rules introduced in the lecture to push all occurrences of the negation symbol “ $\neg$ ” next to the atoms in the following expressions:
- $\neg((A \rightarrow B) \vee ((A \rightarrow C) \wedge \neg A))$
  - $\neg(A \wedge \neg B) \rightarrow A$
- (b) Below are two arguments in English. Translate each argument into logic using an appropriate dictionary, and check whether the argument is logically valid.
- If the king is in the room, then the courtiers laugh only if he laughs. The courtiers always laugh when the jester is in the room. The king never laughs when the jester is in the room. *Therefore*, either the king or the jester is not in the room.
  - If Jones did not meet Smith last night, then either Smith was a murderer, or Jones is telling a lie. If Smith was not a murderer, then Jones did not meet Smith last night, and the murder happened after midnight. If the murder happened after midnight, then either Smith was a murderer, or Jones is telling a lie, but not both. *Therefore*, Smith was a murderer.

### Exercise 1.2 (Propositional Logic II)

- (a) (i) Find formulae  $A$ ,  $B$ , and  $C$  such that  $\{A, B\}$ ,  $\{A, C\}$ , and  $\{B, C\}$  are consistent, while  $\{A, B, C\}$  is not.  
(ii) For any  $n$ , find an inconsistent set of  $n$  formulae, of which every  $n - 1$  formulae are consistent.
- (b) (i) Find four pairwise inconsistent non-contradictory formulae.  
(ii) State the maximal number of pairwise inconsistent non-contradictory formulae with two atomic propositions  $p$  and  $q$ .
- (c) Check the validity of the following rules:

$$\frac{A \wedge B}{A} \text{ (\wedge elimination)} \qquad \frac{A \quad B}{A \wedge B} \text{ (\wedge introduction)}$$

$$\frac{\begin{array}{c} [A] \quad [B] \\ \vdots \quad \vdots \\ C \quad C \end{array}}{C} \text{ (\vee elimination)} \qquad \frac{A \quad \neg A}{B} \text{ (\neg elimination)}$$

**Exercise 1.3** (Propositional Logic III)

Suppose the engine of a car does not perform properly. We want to decide whether we should replace the engine, repair the engine, or replace auxiliary equipment. For the diagnosis, the following symptoms, intermediate conclusions and final decisions or diagnoses should be considered.

Variable	Meaning
<i>black_exhaust</i>	Engine fumes are black
<i>blue_exhaust</i>	Engine fumes are blue
<i>low_power</i>	Engine has low power
<i>overheat</i>	Engine overheats
<i>ping</i>	Engine emits a pinging sound under load
<i>incorrect_timing</i>	Ignition timing is incorrect
<i>low_compression</i>	Compression of engine is low
<i>carbon_deposits</i>	Cylinders have carbon deposits
<i>clogged_filter</i>	Air filter is clogged
<i>clogged_radiator</i>	Radiator is clogged
<i>defective_carburetor</i>	Carburetor is defective
<i>worn_rings</i>	Piston rings are worn
<i>worn_seals</i>	Valve seals are worn
<i>replace_auxiliary</i>	Replace auxiliary equipment
<i>repair_engine</i>	Repair engine
<i>replace_engine</i>	Replace engine

The following facts relate symptoms to intermediate conclusions (facts (i) through (vi)) and intermediate conclusions to final decisions (facts (vii) through (ix)).

- (i) If the engine overheats and the ignition is correct, then the radiator is clogged.
- (ii) If the engine emits a pinging sound under load and the ignition timing is correct, then the cylinders have carbon deposits.
- (iii) If power output is low and the ignition timing is correct, then the piston rings are worn, or the carburetor is defective, or the air filter is clogged.
- (iv) If the exhaust fumes are black, then the carburetor is defective, or the air filter is clogged.
- (v) If the exhaust fumes are blue, then the piston rings are worn, or the valve seals are worn.
- (vi) The compression is low if and only if the piston rings are worn.
- (vii) If the piston rings are worn, then the engine should be replaced.
- (viii) If carbon deposits are present in the cylinders or the carburetor is defective or valve seals are worn, then the engine should be repaired.
- (ix) If the air filter or radiator is clogged, then that auxiliary equipment should be replaced.

Suppose the car owner complains that the engine overheats. Due to a recent engine check, it is known that the ignition timing is correct. What should be done to eliminate the problem?

Answer this question by translating the given information into a propositional CASL specification and checking with HETS which of the final decisions (diagnoses) follow from the symptoms.