

# Logics and categories for software engineering and artificial intelligence

Till Mossakowski, Lutz Schröder  
Summer Semester 2009

University of Bremen  
Department of Computer Science

## Exercise Sheet 2

Due: April 28, 2009

**Exercise 2.1** (Logical consequence or not?)

Evaluate the validity of the following argument. If it is a logical consequence, use the programs *SPASS*, *Fitch* and *Jitpro* to construct formal (resolution, natural deduction, tableau) proofs to show this. Otherwise, use *Tarski's World* to construct a counterexample.<sup>1</sup>

1		$\text{Cube}(a) \vee (\text{Cube}(b) \rightarrow \text{Tet}(c))$
2		$\text{Tet}(c) \rightarrow \text{Small}(c)$
3		$(\text{Cube}(b) \rightarrow \text{Small}(c)) \rightarrow \text{Small}(b)$
4		$\neg\text{Cube}(a) \rightarrow \text{Small}(b)$

**Exercise 2.2** (Inconsistency)

Consider the set  $\mathcal{T} = \{(A \wedge B) \rightarrow \neg A, C \vee A, \neg A \rightarrow A, B\}$ . Use *SPASS*, *Fitch* and *Jitpro* to construct formal proofs showing that  $\mathcal{T} \vdash \perp$ .

**Exercise 2.3** (New connectives)

Consider the following truth table for the ternary connective  $\diamond$ .

$P$	$Q$	$R$	$\diamond(P, Q, R)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Express  $\diamond$  using only the connectives  $\vee$ ,  $\wedge$ , and  $\neg$ . Can you simplify the result such that the simplified sentence has no more than two occurrences each of  $P$ ,  $Q$ , and  $R$ , and no more than six occurrences of the Boolean connectives  $\vee$ ,  $\wedge$ , and  $\neg$ ?

<sup>1</sup>SPASS is available within Hets, see <http://www.dfki.de/sks/hets>. Fitch and Tarski's World can be downloaded from an internal web page shown in the lecture. Jitpro is available under <http://ps.uni-sb.de/jitpro/prover.php>.