Definition 3.12 We define CTL formulas inductively via a Backus Naur form as done for LTL:

$$\begin{aligned}\phi ::= \ & \bot \mid \top \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \rightarrow \phi) \mid \mathrm{AX}\,\phi \mid \mathrm{EX}\,\phi \mid \\ & \mathrm{AF}\,\phi \mid \mathrm{EF}\,\phi \mid \mathrm{AG}\,\phi \mid \mathrm{EG}\,\phi \mid \mathrm{A}[\phi \ \mathrm{U}\ \phi] \mid \mathrm{E}[\phi \ \mathrm{U}\ \phi]\end{aligned}$$

where p ranges over a set of atomic formulas.

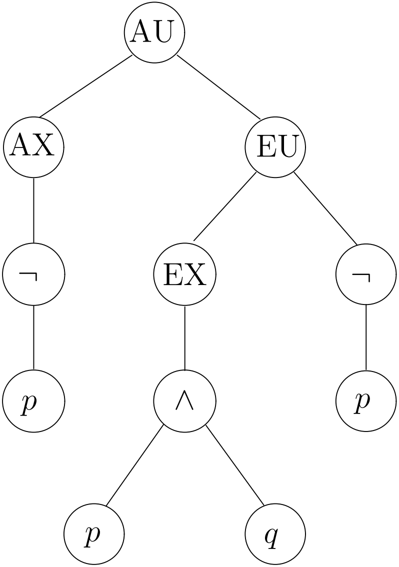

Figure 3.18. The parse tree of a CTL formula without infix notation.

Definition 3.15 Let $\mathcal{M} = (S, \rightarrow, L)$ be a model for CTL, s in S, ϕ a CTL formula. The relation $\mathcal{M}, s \vDash \phi$ is defined by structural induction on ϕ:

1. $\mathcal{M}, s \vDash \top$ and $\mathcal{M}, s \nvDash \bot$
2. $\mathcal{M}, s \vDash p$ iff $p \in L(s)$
3. $\mathcal{M}, s \vDash \neg\phi$ iff $\mathcal{M}, s \nvDash \phi$
4. $\mathcal{M}, s \vDash \phi_1 \wedge \phi_2$ iff $\mathcal{M}, s \vDash \phi_1$ and $\mathcal{M}, s \vDash \phi_2$
5. $\mathcal{M}, s \vDash \phi_1 \vee \phi_2$ iff $\mathcal{M}, s \vDash \phi_1$ or $\mathcal{M}, s \vDash \phi_2$
6. $\mathcal{M}, s \vDash \phi_1 \rightarrow \phi_2$ iff $\mathcal{M}, s \nvDash \phi_1$ or $\mathcal{M}, s \vDash \phi_2$.
7. $\mathcal{M}, s \vDash \mathrm{AX}\,\phi$ iff for all s_1 such that $s \rightarrow s_1$ we have $\mathcal{M}, s_1 \vDash \phi$. Thus, AX says: 'in every next state.'
8. $\mathcal{M}, s \vDash \mathrm{EX}\,\phi$ iff for some s_1 such that $s \rightarrow s_1$ we have $\mathcal{M}, s_1 \vDash \phi$. Thus, EX says: 'in some next state.' E is dual to A – in exactly the same way that $\exists$ is dual to $\forall$ in predicate logic.
9. $\mathcal{M}, s \vDash \mathrm{AG}\,\phi$ holds iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and all s_i along the path, we have $\mathcal{M}, s_i \vDash \phi$. Mnemonically: for All computation paths beginning in s the property ϕ holds Globally. Note that 'along the path' includes the path's initial state s.
10. $\mathcal{M}, s \vDash \mathrm{EG}\,\phi$ holds iff there is a path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and for all s_i along the path, we have $\mathcal{M}, s_i \vDash \phi$. Mnemonically: there Exists a path beginning in s such that ϕ holds Globally along the path.

11. $\mathcal{M}, s \vDash \mathrm{AF}\,\phi$ holds iff for all paths $s_1 \rightarrow s_2 \rightarrow \ldots$, where s_1 equals s, there is some s_i such that $\mathcal{M}, s_i \vDash \phi$. Mnemonically: for All computation paths beginning in s there will be some Future state where ϕ holds.
12. $\mathcal{M}, s \vDash \mathrm{EF}\,\phi$ holds iff there is a path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and for some s_i along the path, we have $\mathcal{M}, s_i \vDash \phi$. Mnemonically: there Exists a computation path beginning in s such that ϕ holds in some Future state;
13. $\mathcal{M}, s \vDash \mathrm{A}[\phi_1 \mathrm{~U~} \phi_2]$ holds iff for all paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, that path satisfies $\phi_1 \mathrm{~U~} \phi_2$, i.e., there is some s_i along the path, such that $\mathcal{M}, s_i \vDash \phi_2$, and, for each $j < i$, we have $\mathcal{M}, s_j \vDash \phi_1$. Mnemonically: All computation paths beginning in s satisfy that ϕ_1 Until ϕ_2 holds on it.
14. $\mathcal{M}, s \vDash \mathrm{E}[\phi_1 \mathrm{~U~} \phi_2]$ holds iff there is a path $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and that path satisfies $\phi_1 \mathrm{~U~} \phi_2$ as specified in 13. Mnemonically: there Exists a computation path beginning in s such that ϕ_1 Until ϕ_2 holds on it.

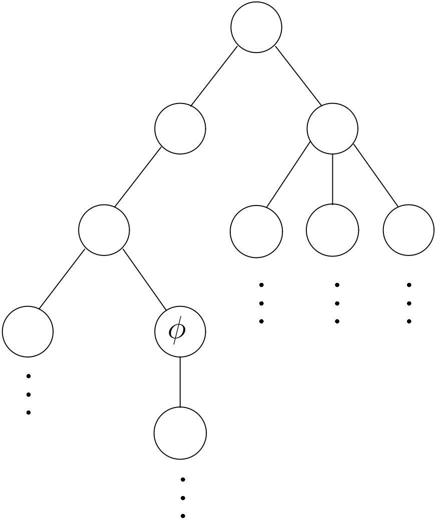

Figure 3.19. A system whose starting state satisfies EF ϕ.

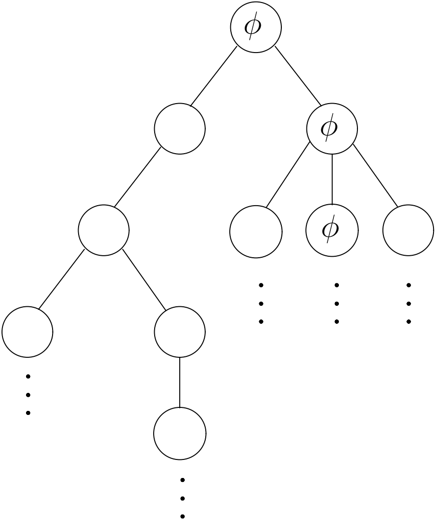

Figure 3.20. A system whose starting state satisfies $\mathrm{EG}\ \phi$.

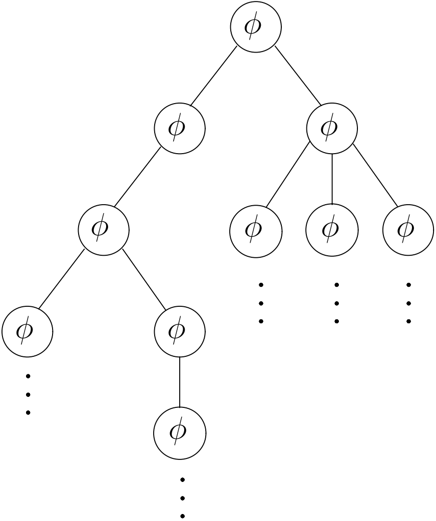

Figure 3.21. A system whose starting state satisfies $\mathrm{AG}\ \phi$.

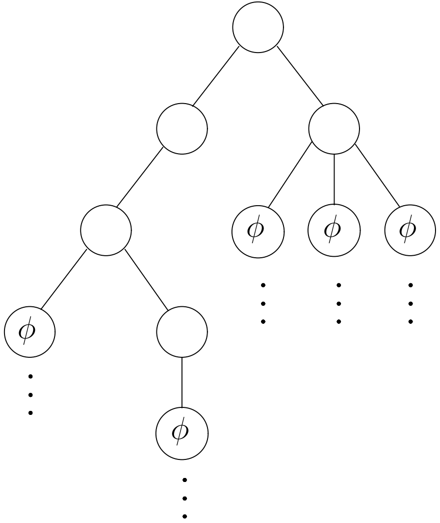

Figure 3.22. A system whose starting state satisfies $\mathrm{AF}\ \phi$.

The syntax of CTL* involves two classes of formulas:

- *state formulas*, which are evaluated in states:

  $$\phi ::= \top \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid \mathrm{A}[\alpha] \mid \mathrm{E}[\alpha]$$

  where p is any atomic formula and α any path formula; and
- *path formulas*, which are evaluated along paths:

  $$\alpha ::= \phi \mid (\neg\alpha) \mid (\alpha \wedge \alpha) \mid (\alpha \;\mathrm{U}\; \alpha) \mid (\mathrm{G}\,\alpha) \mid (\mathrm{F}\,\alpha) \mid (\mathrm{X}\,\alpha)$$

where ϕ is any state formula. This is an example of an inductive definition which is *mutually recursive*: the definition of each class depends upon the definition of the other, with base cases p and $\top$.

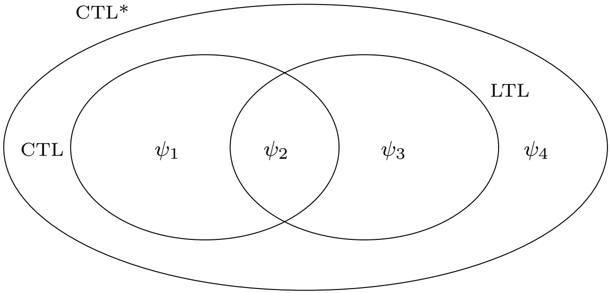

Figure 3.23. The expressive powers of CTL, LTL and CTL*.