Logik für Informatiker
Logic for computer scientists

Proof rules for quantifiers

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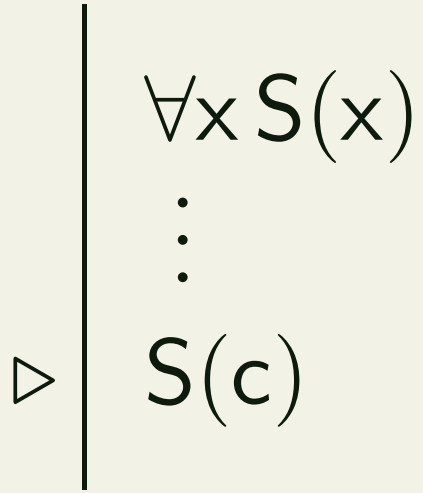
Universal Elimination ($\forall$ Elim)



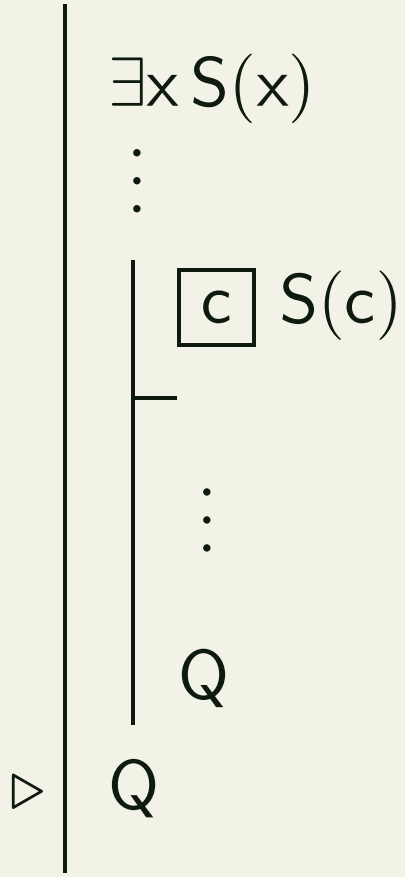
Existential Introduction ($\exists$ Intro)

$$\begin{array}{ll} & \mathsf{S}(\mathsf{c}) \\ & \vdots \\ \triangleright & \exists \mathsf{x}\, \mathsf{S}(\mathsf{x}) \end{array}$$

Example: $\forall$-Elim and $\exists$-Intro

$$\begin{array}{l} \forall x[\mathsf{Cube}(x) \rightarrow \mathsf{Large}(x)] \\ \forall x[\mathsf{Large}(x) \rightarrow \mathsf{LeftOf}(x, b)] \\ \mathsf{Cube}(d) \\ \hline \exists x[\mathsf{Large}(x) \wedge \mathsf{LeftOf}(x, b)] \end{array}$$

Existential Elimination ($\exists$ Elim):

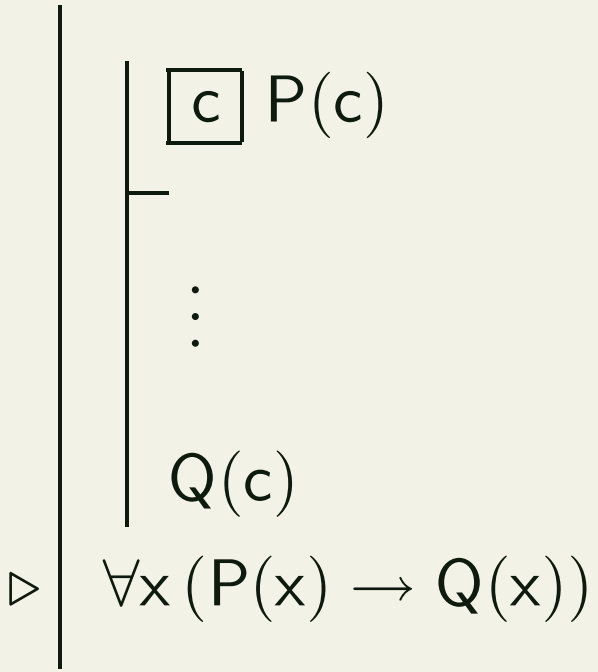


Where c does not occur outside the subproof where it is introduced.

Example: $\exists$-Elim

$$\begin{array}{l} \forall x[\mathsf{Cube}(x) \rightarrow \mathsf{Large}(x)] \\ \forall x[\mathsf{Large}(x) \rightarrow \mathsf{LeftOf}(x, b)] \\ \exists x\ \mathsf{Cube}(x) \\ \hline \exists x[\mathsf{Large}(x) \wedge \mathsf{LeftOf}(x, b)] \end{array}$$

General Conditional Proof ($\forall$ Intro):

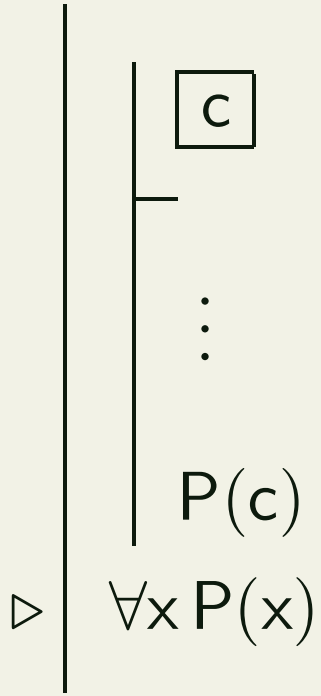


Where c does not occur outside the subproof where it is introduced.

Example: General Conditional Proof

$$\begin{array}{l} \forall x[\mathsf{Cube}(x) \rightarrow \mathsf{Large}(x)] \\ \forall x[\mathsf{Large}(x) \rightarrow \mathsf{LeftOf}(x, b)] \\ \hline \forall x[\mathsf{Cube}(x) \rightarrow \mathsf{LeftOf}(x, b) \end{array}$$

Universal Introduction ($\forall$ Intro):



Where c does not occur outside the subproof where it is introduced.

Identity Introduction (= Intro)

$$\triangleright \;\Big|\; n = n$$

Example: $\forall$-Intro

$$\vdash \forall x\ x = x$$

Example with multiple quantifiers

$$\exists y[\mathsf{Girl}(y) \wedge \forall x(\mathsf{Boy}(x) \rightarrow \mathsf{Likes}(x, y))]$$

$$\vdash$$

$$\forall x[\mathsf{Boy}(x) \rightarrow \exists y(\mathsf{Girl}(y) \wedge \mathsf{Likes}(x, y))]$$

Example: de Morgan's Law

$$\neg\forall x\, P(x) \vdash \exists x\, \neg P(x)$$

(is not valid in intuitionistic logic, only in classical logic)

Example: The Barber Paradox

$$\exists z\ \exists x\ [ManOf(x,z) \wedge \forall y\ (ManOf(y,z) \rightarrow (Shave(x,y) \leftrightarrow \neg Shave(y,y)))] \vdash \bot$$