Logik für Informatiker
Logic for computer scientists

Till Mossakowski

WiSe 2009/10

Rooms

- Monday 12:00 - 14:00 MZH 1400
- Thursday 14:00 - 16:00 MZH 5210
- Exercises (bring your Laptops with you!)
  - Wednesday 8:00 - 10:00 Sportturm C 5130
  - or within the course
- Web: `www.informatik.uni-bremen.de/agbkb/lehre/ws09-10/Logik/`

The formal language PL1

PL1 is the formal language of first-order predicate logic

Why do we need a formal language?
$\Rightarrow$ Slides from Prof. Barbara König, Universität Duisburg-Essen
`http://www.ti.inf.uni-due.de/teaching/ws0607/logik/folien/einfuehrung.pdf`

The language of PL1: individual constants

- Individual constants are symbols that denote a person, thing, object
- Examples:
  - Numbers: 0, 1, 2, 3, . . .
  - Names: Max, Claire
  - Formal constants: a, b, c, d, e, f, n1, n2
- Each individual constant must denote an existing object
- No individual constant can denote more than one object
- An object can have 0, 1, 2, 3 . . . names

The language of PL1: predicate symbols

- Predicate symbols denote a property of objects, or a relation between objects
- Each predicate symbol has an arity that tell us how many objects are related
- Examples:
  - Arity 0: Gate0_is_low, A, B, ...
  - Arity 1: Cube, Tet, Dodec, Small, Medium, Large
  - Arity 2: Smaller, Larger, LeftOf, BackOf, SameSize, Adjoins ...
  - Arity 3: Between

The interpretation of predicate symbols

- In Tarski's world, predicate symbols have a fixed interpretation, that not always completely coindices with the natural language interpretation
- In other PL1 languages, the interpretation of predicate symbols may vary. For example, $\leq$ may be an ordering of numbers, strings, trees etc.
- Usually, the binary symbol $=$ has a fixed interpretation: equality

Atomic sentences

- in propositional logic (Boole):
  - propositional symbols: a, b, c, ...
- in PL1 (Tarski's world):
  - application of predicate symbols to constants: Larger(a,b)
  - the order of arguments matters: Larger(a,b) vs. Larger(b,a)
  - Atomic sentences denote truth values (true, false)

Function symbols

- Function symbols lead to more complex terms that denote objects. Examples:
  - father, mother
  - +, -, *, /
- This leads to new terms denoting objects:
  - father(max)   mother(father(max))
  - 3*(4+2)
- This also leads to new atomic sentences:
  - Larger(father(max),max)
  - 2<3*(4+2)

Logical validity; satisfiability

A sentence A is a logically valid, if it is true in all circumstances.
A sentence A is a satisfiable, if it is true in at least one circumstance.
A circumstance is

- in propositional logic: a valuation of the atomic formulas in the set { true, false }
- in Tarski's world: a block world

Consequences . . .

Logical consequence

A sentence B is a logical consequence of A_1, ..., A_n, if all circumstances that make A_1, ..., A_n true also make B true.
In symbols: $A_1, \ldots, A_n \models B$.
$A_1, \ldots, A_n$ are called premises, B is called conclusion.
In this case, it is a valid argument to infer B from $A_1, \ldots A_n$. If also $A_1, \ldots A_n$ are true, then the valid argument is sound.

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Logical consequence — examples

- All men are mortal. Socrates is a man. So, Socrates is mortal. (valid, sound)
- All rich actors are good actors. Brad Pitt is a rich actor. So he must be a good actor. (valid, but not sound)
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Fitch notation

All men are mortal
Socrates is a man

So, Socrates is mortal

A_1
$\ldots$
A_n

B

Premise_1
$\ldots$
Premise_n

Conclusion

Methods for showing (in)validity of arguments

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Validity To show that an argument is valid, we must provide a proof. A proof consists of a sequence of proof steps, each of which must be valid.

- In propositional logic, we also can use truth tables to show validity. This it not possible in first-order logic.

Invalidity An argument can shown to be invalid by finding a counterexample (model), i.e. a circumstance where the premises are true, but the conclusion is false.

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Informal and formal proofs

- informal reasoning is used in everyday life
- semi-formal reasoning is used in mathematics and theoretical computer science
- balance between readability and precision
- formal proofs: follow some specific rule system,
- and are entirely rigorous
- and can be checked by a computer

An informal proof

- Since Socrates is a man and all men are mortal, it follows that Socrates is mortal.
- But all mortals will eventually die, since that is what it means to be mortal.
- So Socrates will eventually die.
- But we are given that everyone who will eventually die sometimes worries about it.
- Hence Socrates sometimes worries about dying.

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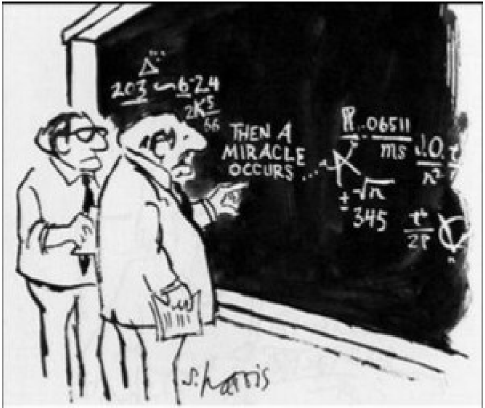
The need for formal proofs



"I think you should be more explicit here in step two."

A formal proof

1. Cube(c)
2. $c = b$

---

3. Cube(b) $=$**Elim:** 1,2

A formal proof

1.	Cube(c)	
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Four principles for the identity relation

1. $=$**Elim**: If $b = c$, then whatever holds of b holds of c (indiscernibility of identicals).
2. $=$**Intro**: $b = b$ is always true in FOL (reflexivity of identity).
3. Symmetry of Identity: If $b = c$, then $c = b$.
4. Transitivity of Identity: If $a = b$ and $b = c$, then $a = c$.

The latter two principles follow from the first two.

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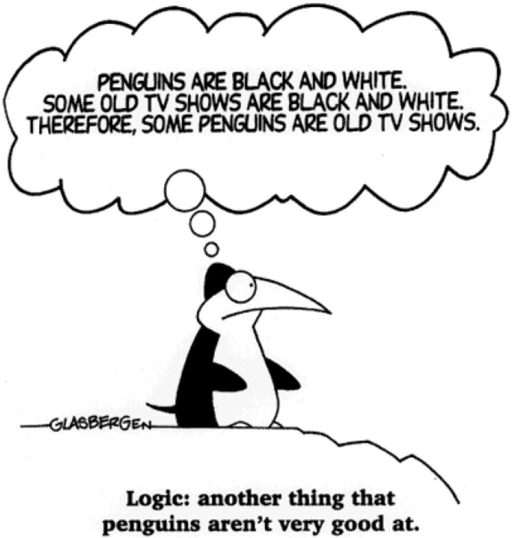
PENGUINS ARE BLACK AND WHITE.
SOME OLD TV SHOWS ARE BLACK AND WHITE.
THEREFORE, SOME PENGUINS ARE OLD TV SHOWS.
GLASBERGEN
Logic: another thing that penguins aren't very good at.

Informal proof of symmetry of identity

- Suppose that $a = b$.
- We know that $a = a$, by the reflexivity of identity.
- Now substitute the name b for the first use of the name a in $a = a$, using the indiscernibility of identicals.
- We come up with $b = a$, as desired.

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Formal proofs

P	
Q	
R	
S_1	Justification 1
...	
...	
S_n	Justification n
S	Justification n+1

Formal proof of symmetry of identity

1. $a = b$	
2. $a = a$	$=$**Intro:**
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Fitch rule: Identity introduction

Identity Introduction (= Intro):

$\triangleright \mid \mathsf{n} = \mathsf{n}$

Fitch rule: Identity elimination

Identity Elimination (= Elim):

$$
\begin{array}{l|l}
 & P(n) \\
 & \vdots \\
 & n = m \\
 & \vdots \\
\triangleright & P(m)
\end{array}
$$

Fitch rule: Reiteration

Reiteration (Reit):

$$\begin{array}{l|l} & P \\ & \vdots \\ \triangleright & P \end{array}$$