

Logik für Informatiker  
Logic for computer scientists  
Proof rules for quantifiers

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WiSe 2009/10

## Universal Elimination ( $\forall$ Elim)

$$\triangleright \left| \begin{array}{l} \forall x S(x) \\ \vdots \\ S(c) \end{array} \right.$$

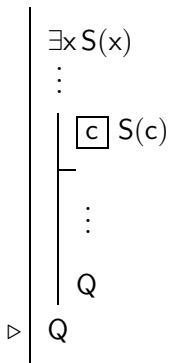
## Existential Introduction ( $\exists$ Intro)

$$\triangleright \left| \begin{array}{l} S(c) \\ \vdots \\ \exists x S(x) \end{array} \right.$$

## Example: $\forall$ -Elim and $\exists$ -Intro

$$\begin{array}{|l} \forall x[\text{Cube}(x) \rightarrow \text{Large}(x)] \\ \forall x[\text{Large}(x) \rightarrow \text{LeftOf}(x, b)] \\ \text{Cube}(d) \\ \hline \exists x[\text{Large}(x) \wedge \text{LeftOf}(x, b)] \end{array}$$

## Existential Elimination ( $\exists$ Elim):



Where  $c$  does not occur outside the subproof where it is introduced.

## Example: $\exists$ -Elim

$$\begin{array}{|l} \forall x[\text{Cube}(x) \rightarrow \text{Large}(x)] \\ \forall x[\text{Large}(x) \rightarrow \text{LeftOf}(x, b)] \\ \exists x \text{Cube}(x) \\ \hline \exists x[\text{Large}(x) \wedge \text{LeftOf}(x, b)] \end{array}$$

## General Conditional Proof ( $\forall$ Intro):

$$\begin{array}{l} \vdash \left| \begin{array}{l} \boxed{c} P(c) \\ \hline \vdots \\ Q(c) \end{array} \right. \\ \forall x (P(x) \rightarrow Q(x)) \end{array}$$

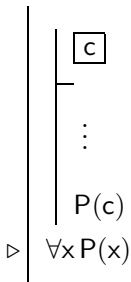
Where  $c$  does not occur outside the subproof where it is introduced.

# Example: General Conditional Proof

$$\left| \begin{array}{l} \forall x[\text{Cube}(x) \rightarrow \text{Large}(x)] \\ \forall x[\text{Large}(x) \rightarrow \text{LeftOf}(x, b)] \\ \hline \forall x[\text{Cube}(x) \rightarrow \text{LeftOf}(x, b)] \end{array} \right.$$



## Universal Introduction ( $\forall$ Intro):



Where  $c$  does not occur outside the subproof where it is introduced.

# Prenex normal form (reminder)

$$\left\{ \begin{array}{l} \exists x \text{Cube}(x) \rightarrow \forall y \text{Small}(y) \\ \forall x \forall y (\text{Cube}(x) \rightarrow \text{Small}(y)) \end{array} \right.$$

# Example with multiple quantifiers

$$\left\{ \begin{array}{l} \exists y[\text{Girl}(y) \wedge \forall x(\text{Boy}(x) \rightarrow \text{Likes}(x, y))] \\ \forall x[\text{Boy}(x) \rightarrow \exists y(\text{Girl}(y) \wedge \text{Likes}(x, y))] \end{array} \right.$$

# Example: de Morgan's Law

$$\left\{ \begin{array}{l} \neg \forall x P(x) \\ \exists x \neg P(x) \end{array} \right.$$

(is not valid in intuitionistic logic, only in classical logic)

# Example: The Barber Paradox

$$\exists z \exists x [ManOf(x, z) \wedge \forall y (ManOf(y, z) \rightarrow (Shave(x, y) \leftrightarrow \neg Shave(y, y)))]$$

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