Logic for computer scientists

Tarski’s world and AnaCon

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Tarski’s world and AnaCon

- How can we understand Fitch’s AnaCon procedure?
- Can we reconstruct it (on a computer)?
We have encountered arguments that are valid in Tarski’s World but not FO valid.

\[ \forall x (\text{Cube}(x) \leftrightarrow \text{SameShape}(x, c)) \]

\[ \text{Cube}(c) \]

The replacement method yields an invalid argument:

\[ \forall x (P(x) \leftrightarrow Q(x, c)) \]

\[ P(c) \]
Axiomatic method: bridge the gap between Tarski’s World validity and FO validity by systematically expressing facts about the meanings of the predicates, and introduce them as axioms. Axioms restrict the possible interpretation of predicates. Axioms may be used as premises within arguments/proofs.
The argument revisited

\[\forall x (\text{Cube}(x) \leftrightarrow \text{SameShape}(x, c))\]
\[\forall x \text{SameShape}(x, x)\]

\[\text{Cube}(c)\]

The replacement method yields a valid argument:

\[\forall x (\text{P}(x) \leftrightarrow \text{Q}(x, c))\]
\[\forall x \text{Q}(x, x)\]

\[\text{P}(c)\]
The basic shape axioms

1. $\neg \exists x (\text{Cube}(x) \land \text{Tet}(x))$
2. $\neg \exists x (\text{Tet}(x) \land \text{Dodec}(x))$
3. $\neg \exists x (\text{Dodec}(x) \land \text{Cube}(x))$
4. $\forall x (\text{Tet}(x) \lor \text{Dodec}(x) \lor \text{Cube}(x))$
An argument using the shape axioms

\begin{align*}
\neg \exists x (\text{Dodec}(x) \land \text{Cube}(x)) \\
\forall x (\text{Tet}(x) \lor \text{Dodec}(x) \lor \text{Cube}(x)) \\
\neg \exists x \ \text{Tet}(x) \\
\forall x (\text{Cube}(x) \iff \neg \text{Dodec}(x)) \\
\neg \exists x (\text{P}(x) \land \text{Q}(x)) \\
\forall x (\text{R}(x) \lor \text{P}(x) \lor \text{Q}(x)) \\
\neg \exists x \ \text{R}(x) \\
\forall x (\text{Q}(x) \iff \neg \text{P}(x))
\end{align*}
SameShape introduction and elimination axioms

1. \( \forall x \forall y ((\text{Cube}(x) \land \text{Cube}(y)) \rightarrow \text{SameShape}(x, y)) \)
2. \( \forall x \forall y ((\text{Dodec}(x) \land \text{Dodec}(y)) \rightarrow \text{SameShape}(x, y)) \)
3. \( \forall x \forall y ((\text{Tet}(x) \land \text{Tet}(y)) \rightarrow \text{SameShape}(x, y)) \)
4. \( \forall x \forall y ((\text{SameShape}(x, y) \land \text{Cube}(x)) \rightarrow \text{Cube}(y)) \)
5. \( \forall x \forall y ((\text{SameShape}(x, y) \land \text{Dodec}(x)) \rightarrow \text{Dodec}(y)) \)
6. \( \forall x \forall y ((\text{SameShape}(x, y) \land \text{Tet}(x)) \rightarrow \text{Tet}(y)) \)
How can we understand Fitch’s AnaCon procedure?
Can we reconstruct it (on a computer)?

Answer: use an axiomatization of Tarski’s world plus a first-order theorem prover (e.g. resolution-based)
This method also works for other domains