Logik für Informatiker
Logic for computer scientists

Tarski's world and AnaCon

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Tarski's world and AnaCon

- How can we understand Fitch's AnaCon procedure?
- Can we reconstruct it (on a computer)?

TW consequence $\neq$ FO consequence

We have encountered arguments that are valid in Tarski's World but not FO valid.

$$\begin{array}{|l} \forall x(\mathrm{Cube}(x) \leftrightarrow \mathrm{SameShape}(x,c)) \\ \hline \mathrm{Cube}(c) \end{array}$$

The replacement method yields an invalid argument:

$$\begin{array}{|l} \forall x(P(x) \leftrightarrow Q(x,c)) \\ \hline P(c) \end{array}$$

The axiomatic method

Axiomatic method: bridge the gap between Tarski's World validity and FO validity by systematically expressing facts about the meanings of the predicates, and introduce them as *axioms*. Axioms restrict the possible interpretation of predicates.
Axioms may be used as premises within arguments/proofs.

The argument revisited

$$\begin{array}{|l} \forall x(\mathrm{Cube}(x) \leftrightarrow \mathrm{SameShape}(x, c)) \\ \forall x\,\mathrm{SameShape}(x, x) \\ \hline \mathrm{Cube}(c) \end{array}$$

The replacement method yields a valid argument:

$$\begin{array}{|l} \forall x(P(x) \leftrightarrow Q(x, c)) \\ \forall x\,Q(x, x) \\ \hline P(c) \end{array}$$

The basic shape axioms

1. $\neg\exists x(Cube(x) \land Tet(x))$
2. $\neg\exists x(Tet(x) \land Dodec(x))$
3. $\neg\exists x(Dodec(x) \land Cube(x))$
4. $\forall x(Tet(x) \lor Dodec(x) \lor Cube(x))$

An argument using the shape axioms

$$\begin{array}{|l}
\neg\exists x(\mathrm{Dodec}(x) \wedge \mathrm{Cube}(x)) \\
\forall x(\mathrm{Tet}(x) \vee \mathrm{Dodec}(x) \vee \mathrm{Cube}(x)) \\
\neg\exists x\ \mathrm{Tet}(x) \\
\hline
\forall x(\mathrm{Cube}(x) \leftrightarrow \neg\mathrm{Dodec}(x))
\end{array}$$

$$\begin{array}{|l}
\neg\exists x(P(x) \wedge Q(x)) \\
\forall x(R(x) \vee P(x) \vee Q(x)) \\
\neg\exists x\ R(x) \\
\hline
\forall x(Q(x) \leftrightarrow \neg P(x))
\end{array}$$

SameShape introduction and elimination axioms

1. $\forall x \forall y((Cube(x) \wedge Cube(y)) \rightarrow SameShape(x, y))$
2. $\forall x \forall y((Dodec(x) \wedge Dodec(y)) \rightarrow SameShape(x, y))$
3. $\forall x \forall y((Tet(x) \wedge Tet(y)) \rightarrow SameShape(x, y))$

4. $\forall x \forall y((SameShape(x, y) \wedge Cube(x)) \rightarrow Cube(y))$
5. $\forall x \forall y((SameShape(x, y) \wedge Dodec(x)) \rightarrow Dodec(y))$
6. $\forall x \forall y((SameShape(x, y) \wedge Tet(x)) \rightarrow Tet(y))$

- How can we understand Fitch's AnaCon procedure?
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Answer: use an axiomatization of Tarski's world plus a first-order theorem prover (e.g. resolution-based)
This method also works for other domains