Logik für Informatiker
Xmas special

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Existence of Santa Clause

Theorem. Santa Clause exists.

Proof.

Assume to the contrary, that Santa Clause does not exist.
By $\exists$-Intro, there exists something that does not exist.
This is a contradiction. Hence, the assumption that Santa Clause does not exist must be wrong.
Thus, Santa Clause exists. □

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All reindeers have the same color

Theorem. Any number of reindeers have the same color.

Proof. By induction.

Basis: one reindeer has the same color (obviously!).

Inductive step: suppose that any collection of n reindeers has the same color. We need to show that $n+1$ reindeers have the same color, too. By induction hypothesis, the first n reindeers have the same color. Take out the last reindeer of these and replace it with the $n+1$st. Again by induction hypothesis, these have the same color. Hence, all $n+1$ reindeers have the same color. □

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Why the date of XMas cannot be surprising

Son: It is boring that XMas always is on the 24th.
Father: OK. This year, we will celebrate XMas on a day in the week from 23th to 29th. You will not know the date beforehand.
Son: Good! Then it cannot be the 29th — if we hadn't celebrated it until the 28th, I would know beforehand that it must be the 29th, since this is the last day of the week!
Moreover, it cannot be the 28th — if we hadn't celebrated it until the 27th, I would know beforehand that it must be the 28th (the 29th already has been excluded above).
Son (cont'd): Similarly, it can be neither the 27th, nor the 26th, nor the 25th, nor the 24th, nor the 23th.
Hence, you cannot fulfill you promise that I won't know the date beforehand.
Father: You will see, you won't know the date beforehand.

Why the date of XMas can be surprising

After all, XMas was celebrated on the 27th.
The son was quite surprised.

A scheduling problem

A camel must travel 1000 miles across a desert to the nearest city. She has 3000 bananas but can only carry 1000 at a time. For every mile she walks, she needs to eat a banana. What is the maximum number of bananas she can transport to the city?