Logik für Informatiker
Logic for computer scientists

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LPL book	detailed introduction into first-order logic with many exercises
Boole	construct truth tables
Tarski's world	evaluate logical formulas within a blocks world
Fitch	construct proofs
Grinder	gives automatic feedback to your solutions $\rightarrow$ requires purchase of the CD (ca. 13 EUR) or the book (ca. 25 EUR, with CD)

Platform for exercises: `logic.informatik.uni-bremen.de`
also reachable via
`www.informatik.uni-bremen.de/agbkb/lehre/ws11-12/Logik/`

PL1 is the formal language of first-order predicate logic

Why do we need a formal language?
⇒ Slides from Prof. Barbara König, Universität Duisburg-Essen
`http://jordan.inf.uni-due.de/teaching/ss2010/logik/folien/folien.pdf`

The language of PL1: individual constants

- Individual constants are symbols that denote a person, thing, object
- Examples:
  - Numbers: 0, 1, 2, 3, ...
  - Names: Max, Claire
  - Formal constants: a, b, c, d, e, f, n1, n2
- Each individual constant must denote an existing object
- No individual constant can denote more than one object
- An object can have 0, 1, 2, 3 ... names

The language of PL1: predicate symbols

- Predicate symbols denote a property of objects, or a relation between objects
- Each predicate symbol has an arity that tell us how many objects are related
- Examples:
  - Arity 0: Gate0_is_low, A, B, ...
  - Arity 1: Cube, Tet, Dodec, Small, Medium, Large
  - Arity 2: Smaller, Larger, LeftOf, BackOf, SameSize, Adjoins ...
  - Arity 3: Between

The interpretation of predicate symbols

- In Tarski's world, predicate symbols have a fixed interpretation, that not always completely coindices with the natural language interpretation
- In other PL1 languages, the interpretation of predicate symbols may vary. For example, $\leq$ may be an ordering of numbers, strings, trees etc.
- Usually, the binary symbol $=$ has a fixed interpretation: equality

Atomic sentences

- in propositional logic (Boole):
  - propositional symbols: a, b, c, ...
- in PL1 (Tarski's world):
  - application of predicate symbols to constants: Larger(a,b)
  - the order of arguments matters: Larger(a,b) vs. Larger(b,a)
  - Atomic sentences denote truth values (true, false)

Function symbols

- Function symbols lead to more complex terms that denote objects. Examples:
  - father, mother
  - +, -, *, /
- This leads to new terms denoting objects:
  - father(max)   mother(father(max))
  - 3*(4+2)
- This also leads to new atomic sentences:
  - Larger(father(max),max)
  - 2<3*(4+2)

Logical validity; satisfiability

A sentence A is a logically valid, if it is true in all circumstances.

A sentence A is a satisfiable, if it is true in at least one circumstance.

A circumstance is

- in propositional logic: a valuation of the atomic formulas in the set { true, false }
- in Tarski's world: a block world

Consequences . . .

Logical consequence

A sentence B is a logical consequence of A_1, ..., A_n, if all circumstances that make A_1, ..., A_n true also make B true.
In symbols: $A_1, \ldots, A_n \models B$.
$A_1, \ldots, A_n$ are called premises, B is called conclusion.
In this case, it is a valid argument to infer B from $A_1, \ldots A_n$. If also $A_1, \ldots A_n$ are true, then the valid argument is sound.

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Logical consequence — examples

- All men are mortal. Socrates is a man. So, Socrates is mortal. (valid, sound)
- All rich actors are good actors. Brad Pitt is a rich actor. So he must be a good actor. (valid, but not sound)
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