1 Glossary of Symbols

Symbol	Meaning	Example
$s = \langle s(1), s(2), s(3), \ldots \rangle$	trace notation	
$\langle\ \rangle$	empty trace	
$\#s$	length of s	
$head(s)$	head of s	$head(\langle a, b, c, b, b \rangle) = a$
$front(s)$	front of s	$front(\langle a, b, c, b, b \rangle) = \langle a, b, c, b \rangle$
$tail(s)$	tail of s	$tail(\langle a, b, c, b, b \rangle) = \langle b, c, b, b \rangle$
$last(s)$	last of s	$last(\langle a, b, c, b, b \rangle) = b$
$s \leq u$	s prefix of u,	$\langle a, b \rangle \leq \langle a, b, c \rangle$
s **in** u	s segment of u	$\langle b, c \rangle$ **in** $\langle a, b, c, d \rangle$
$s \frown u$	concatenation of s and u	$\langle b, c \rangle \frown \langle a, b, c, d \rangle = \langle b, c, a, b, c, d \rangle$
$s \upharpoonright X$	s projected on elements of set V	$\langle a, b, c, d, c \rangle \upharpoonright \{a, c\} = \langle a, c, c \rangle$
$f^*(\langle b, c \rangle) = \langle f(b), f(c) \rangle$		
$ch^*(\langle c.x, d.y \rangle) = \langle c, d \rangle$		
$val^*(\langle c.x, d.y \rangle) = \langle x, y \rangle$		
$[P]^0$	$\{e \in \alpha P \mid (\exists\, u \in Traces(P) \bullet head(u) = e)\}$	

2 A Quick-Reference Guide to CSP

The CSP semantics $\mathcal{I}[\![\ \bullet\ ]\!]$ for nondeterministic processes maps each syntactic CSP unit P onto a triple of sets

$$\mathcal{I}[\![\, P\,]\!] = (A, F, D)$$

subject to the following conditions:

D0 A, F, D have the following structure:

- A is a finite set, called the *alphabet* of P, denoted $A = \alpha P$.
- $F \subseteq A^* \times \mathbb{P}\, A$ is called the *failures* of P, denoted $F = Fail(P)$. For each element $(s, X) \in F$, s is called a *trace* of P and X a *refusal (set)* of P/s. The set of all traces is denoted by $Traces(P)$.
- $D \subseteq A^*$ is called the *divergences* of P, denoted $D = Div(P)$.

C0 $(\langle\ \rangle, \emptyset) \in F$

C1 $(s \frown t, X) \in F \Rightarrow (s, \emptyset) \in F$

C2 $(s, Y) \in F \wedge X \subseteq Y \Rightarrow (s, X) \in F$

C3 $(s, X) \in F \wedge x \in A \Rightarrow (s, X \cup \{x\}) \in F \vee (s \frown \langle x \rangle, \emptyset) \in F$

C4 $D \subseteq \operatorname{dom} F$

C5 $s \in D \wedge t \in A^* \Rightarrow s \frown t \in D$

C6 $s \in D \wedge X \subseteq A \Rightarrow (s, X) \in F$

We explain the meaning of CSP operators by giving their failure sets and their divergences.

Internal Choice Operator The internal choice operator operator $\sqcap$ selects one of its operands nondeterministically without giving the environment the possibility to influence this decision:

$$Fail(P \sqcap Q) = Fail(P) \cup Fail(Q)$$

$$Div(P \sqcap Q) = Div(P) \cup Div(Q)$$

Prefixing Operator The prefixing operator $\rightarrow$ executes $P(x)$ after occurrence of an initial event $x \in B$.

$$Fail(x : B \rightarrow P(x)) = \{(\langle\,\rangle, U) \mid U \subseteq (\alpha P - B)\} \cup \{(\langle x \rangle \frown s, U) \mid x \in B \wedge (s, U) \in Fail(P(x))\}$$

$$Div(x : B \rightarrow P(x)) = \{\langle x \rangle \frown s \mid s \in Div(P(x)) \wedge x \in B\}$$

Parallel Operator The parallel operator $\|$ defines joint execution of P and Q with events common to both their alphabets executed synchronously.

$$\begin{aligned} Fail(P \parallel Q) = \; & \{(s, U \cup V) \mid s \in (\alpha P \cup \alpha Q)^* \wedge (s \upharpoonright \alpha P, U) \in Fail(P) \wedge (s \upharpoonright \alpha Q, V) \in Fail(Q)\} \\ & \cup \{(t, U) \mid t \in Div(P \parallel C)\} \end{aligned}$$

$$\begin{aligned} Div(P \parallel Q) = \; & \{s \frown t \mid t \in (\alpha P \cup \alpha Q)^* \wedge (s \upharpoonright \alpha P \in Div(P) \wedge s \upharpoonright \alpha Q \in Traces(Q) \vee \\ & \quad s \upharpoonright \alpha Q \in Div(Q) \wedge s \upharpoonright \alpha P \in Traces(P))\} \end{aligned}$$

External Choice Operator The external choice operator $\square$ executes P if the initial event is from P's alphabet and not from Q's alphabet, executes Q if the initial event is from Q's alphabet and not from P's alphabet and choses nondeterministically if the initial event is in the intersection of their alphabets.

$$Fail(P \,\square\, Q) = \{(s, U) \mid (s, U) \in Fail(P) \cap Fail(Q) \vee (s \neq \langle\,\rangle \wedge (s, U) \in Fail(P) \cup Fail(Q))\}$$

$$Div(P \,\square\, Q) = Div(P) \cup Div(Q)$$

Hiding Operator The Hiding operator $\setminus$ defines a new process as the original P with all events from C hidden.

$$Fail(P \setminus C) = \{(t \upharpoonright (\alpha P - C), U) \mid (t, U \cup C) \in Fail(P)\} \cup \{(t, U) \mid t \in Div(P \setminus C)\}$$

$$\begin{aligned} Div(P \setminus C) = \; & \{(s \upharpoonright (\alpha P - C)) \frown t \mid t \in (\alpha P - C)^* \wedge \\ & \quad (s \in Div(P) \vee (\forall n \bullet (\exists u \in C^* \bullet (\#u > n \wedge (s \frown u) \in Traces(P)))))\} \end{aligned}$$

Interleaving Operator The interleaving operator $|||$ defines interleaved execution of P and Q with events common to both alphabets nondeterministically executed either by P or by Q.

$$\begin{aligned} Fail(P \,|||\, Q) = \; & \{(s, U) \mid (\exists t, u \bullet s \; interleaves(t, u) \wedge (t, U) \in Fail(P) \wedge (u, U) \in Fail(Q))\} \\ & \cup \{(s, U) \mid s \in Div(P \,|||\, Q)\} \end{aligned}$$

$$\begin{aligned} Div(P \,|||\, C) = \; & \{u \mid (\exists s, t \bullet u \; interleaves(s, t) \wedge \\ & \quad (s \in Div(P) \wedge t \in Traces(Q) \vee s \in Traces(P) \wedge t \in Div(Q)))\} \end{aligned}$$

Interrupt Operator. $(P\hat{\ }Q)$ is defined to be the process that acts like P, but may be interrupted at any time by the first event of Q. From this event on, Q continues to run and P is never resumed. We have

$$\alpha(P\hat{\ }Q) = \alpha P \cup \alpha Q$$

and

$$Traces(P\hat{\ }Q) = \{s \frown t \mid s \in TracesP \wedge t \in TracesQ\}$$

It is assumed that Q's initial events are not contained in αP.