Timed CSP

Syntax and semantics of Timed CSP according to Steve Schneider, Concurrent and Real–time Systems, The CSP Approach, John Wiley & Sons, Ltd, 2000.

1 Syntax

P	$::=$	STOP	deadlock
	$\mid$	SKIP	successful termination
	$\mid$	$c!e \rightarrow Q$	communication (output)
	$\mid$	$c?e \rightarrow Q$	communication (input)
	$\mid$	$a \rightarrow Q$	event prefix
	$\mid$	$a : A \rightarrow Q$	prefix choice
	$\mid$	$a \xrightarrow{t} Q$	timed prefix
	$\mid$	$Q \overset{t}{\triangleright} R$	timeout
	$\mid$	$Q \Box R$	external choice
	$\mid$	$Q \sqcap R$	internal choice
	$\mid$	$\mu X \bullet P$	recursion
	$\mid$	$Q \mathbin{{}_{A}\|_{B}} R$	alphabetized parallel
	$\mid$	$Q \mid\mid\mid R$	interleaving
	$\mid$	$Q \underset{A}{\|} R$	synchronized parallel
	$\mid$	$Q \setminus A$	hiding
	$\mid$	$f(Q)$	renaming
	$\mid$	$f^{-1}(Q)$	backward renaming
	$\mid$	$Q ; R$	sequential composition
	$\mid$	$Q \triangle R$	interrupt
	$\mid$	$Q \triangle_t R$	timed interrupt
	$\mid$	$Wait\ t$	delay
	$\mid$	X	process reference
	$\mid$	(Q)	brackets

2 Operational Semantics

Deadlock (DL)

(DL1)

$$\frac{}{\text{STOP} \overset{d}{\rightsquigarrow} \text{STOP}}$$

Successful Termination (T)

(T1)

$$\frac{}{\text{SKIP} \overset{d}{\rightsquigarrow} \text{SKIP}}$$

(T2)

$$\frac{}{\text{SKIP} \xrightarrow{\checkmark} \text{STOP}}$$

Event Prefixing (PR)

(PR1)

$$\frac{}{(a \rightarrow P) \overset{d}{\rightsquigarrow} (a \rightarrow P)}$$

(PR2)

$$\frac{}{(a \rightarrow P) \xrightarrow{a} P}$$

Prefix Choice (PC)

(PC1)

$$\frac{}{x : A \rightarrow P(x) \overset{d}{\rightsquigarrow} x : A \rightarrow P(x)}$$

(PC2)

$$\frac{}{x : A \rightarrow P(x) \xrightarrow{a} P(a)} \quad [\, a \in A \,]$$

Timed Event Prefix (TPR)

(TPR1)

$$\frac{}{(a@u \rightarrow Q) \overset{d}{\rightsquigarrow} (a@u \rightarrow Q[(u+d)/u])}$$

(TPR2)

$$\frac{}{(a@u \rightarrow Q) \xrightarrow{a} Q[0/u]}$$

Timeout (TO)

(TO1)

$$\frac{P \overset{d'}{\rightsquigarrow} P'}{P \overset{d}{\triangleright} Q \overset{d'}{\rightsquigarrow} P' \overset{d-d'}{\triangleright} Q} \quad [\, 0 < d' \leq d \,]$$

(TO2)

$$\frac{P \xrightarrow{\mu} P'}{P \overset{d}{\triangleright} Q \xrightarrow{\mu} P'} \quad [\, \mu \neq \tau \,]$$

(TO3)

$$\frac{P \xrightarrow{\tau} P'}{P \overset{d}{\triangleright} Q \xrightarrow{\tau} P' \overset{d}{\triangleright} Q}$$

(TO4)

$$\frac{}{P \overset{0}{\triangleright} Q \xrightarrow{\tau} Q}$$

External Choice (EC)

(EC1)

$$\frac{P \overset{d}{\rightsquigarrow} P' \quad Q \overset{d}{\rightsquigarrow} Q'}{P \;\Box\; Q \overset{d}{\rightsquigarrow} P' \;\Box\; Q'}$$

(EC2)

$$\frac{P \xrightarrow{\tau} P'}{\begin{array}{l} P \;\Box\; Q \xrightarrow{\tau} P' \;\Box\; Q \\ Q \;\Box\; P \xrightarrow{\tau} Q \;\Box\; P' \end{array}}$$

(EC3)

$$\frac{P \xrightarrow{\mu} P'}{\begin{array}{l} P \;\Box\; Q \xrightarrow{\mu} P' \\ Q \;\Box\; P \xrightarrow{\mu} P' \end{array}} \quad [\, \mu \neq \tau \,]$$

Internal Choice (IC)

(IC1)

$$\frac{}{\begin{array}{l} P \sqcap Q \xrightarrow{\tau} P \\ P \sqcap Q \xrightarrow{\tau} Q \end{array}}$$

(Mutual) Recursion (REC)

In these rules, σ represents the syntactic environment of the process N (which is called "surrounding context of process definitions" by Schneider). In contrast to earlier operational semantics definitions for Timed CSP, recursion unfolding does not produce an internal event in Schneider's definition.

(REC1)

$$\frac{P \overset{d}{\rightsquigarrow} P'}{N \overset{d}{\rightsquigarrow} P'} \quad [\, (N = P) \in \sigma \,]$$

(REC2)

$$\frac{P \xrightarrow{\mu} P'}{N \xrightarrow{\mu} P'} \quad [\, (N = P) \in \sigma \,]$$

Alphabetized Parallel (AP)

(AP1)

$$\frac{P \overset{t}{\rightsquigarrow} P' \quad Q \overset{t}{\rightsquigarrow} Q'}{P \;{}_{A}\|_{B}\; Q \overset{t}{\rightsquigarrow} P' \;{}_{A}\|_{B}\; Q'}$$

(AP2)

$$\frac{P \xrightarrow{\mu} P'}{P \;{}_{A}\|_{B}\; Q \xrightarrow{\mu} P' \;{}_{A}\|_{B}\; Q} \quad [\, \mu \in (A \cup \{\tau\}) \setminus B \,]$$

(AP3)

$$\frac{Q \xrightarrow{\mu} Q'}{P \;{}_{A}\|_{B}\; Q \xrightarrow{\mu} P \;{}_{A}\|_{B}\; Q'} \quad [\, \mu \in (B \cup \{\tau\}) \setminus A \,]$$

(AP4)

$$\frac{P \xrightarrow{\mu} P' \quad Q \xrightarrow{\mu} Q'}{P \;{}_{A}\|_{B}\; Q \xrightarrow{\mu} P' \;{}_{A}\|_{B}\; Q'} \quad [\, \mu \in (A \cap B)^{\checkmark} \,]$$

Interleaving (IL)

(IL1)

$$\frac{P \overset{t}{\leadsto} P' \quad Q \overset{t}{\leadsto} Q'}{P \mid\mid\mid Q \overset{t}{\leadsto} P' \mid\mid\mid Q'}$$

(IL2)

$$\frac{P \xrightarrow{\mu} P'}{\begin{array}{c} P \mid\mid\mid Q \xrightarrow{\mu} P' \mid\mid\mid Q \\ Q \mid\mid\mid P \xrightarrow{\mu} Q \mid\mid\mid P' \end{array}} \quad [\, \mu \neq \checkmark \,]$$

(IL3)

$$\frac{P \xrightarrow{\checkmark} P' \quad Q \xrightarrow{\checkmark} Q'}{P \mid\mid\mid Q \xrightarrow{\checkmark} P' \mid\mid\mid Q'}$$

Interface Parallel (IP)

(IP1)

$$\frac{P \overset{t}{\leadsto} P' \quad Q \overset{t}{\leadsto} Q'}{P \underset{A}{\parallel} Q \overset{t}{\leadsto} P' \underset{A}{\parallel} Q'}$$

(IP2)

$$\frac{P \xrightarrow{\mu} P'}{\begin{array}{c} P \underset{A}{\parallel} Q \xrightarrow{\mu} P' \underset{A}{\parallel} Q \\ Q \underset{A}{\parallel} P \xrightarrow{\mu} Q \underset{A}{\parallel} P' \end{array}} \quad [\, \mu \notin A^{\checkmark} \,]$$

(IP3)

$$\frac{P \xrightarrow{\mu} P' \quad Q \xrightarrow{\mu} Q'}{P \underset{A}{\parallel} Q \xrightarrow{\mu} P' \underset{A}{\parallel} Q'} \quad [\, \mu \in A^{\checkmark} \,]$$

Hiding (H)

(H1)

$$\frac{P \overset{d}{\leadsto} P' \quad \forall a \in A \bullet \neg(P \xrightarrow{a})}{P \setminus A \overset{d}{\leadsto} P' \setminus A}$$

(H2)

$$\frac{P \xrightarrow{\mu} P'}{P \setminus A \xrightarrow{\tau} P' \setminus A} \quad [\, \mu \in A \,]$$

(H3)

$$\frac{P \xrightarrow{\mu} P'}{P \setminus A \xrightarrow{\mu} P' \setminus A} \quad [\, \mu \notin A \,]$$

Event Renaming (ER)

(ER1)

$$\frac{P \overset{d}{\rightsquigarrow} P'}{f(P) \overset{d}{\rightsquigarrow} f(P')}$$

(ER2)

$$\frac{P \xrightarrow{\mu} P'}{f(P) \xrightarrow{f(\mu)} f(P')} \quad [\, \mu \neq \tau \,]$$

(ER3)

$$\frac{P \xrightarrow{\tau} P'}{f(P) \xrightarrow{\tau} f(P')}$$

Backward Renaming (BR)

(BR1)

$$\frac{P \overset{d}{\rightsquigarrow} P'}{f^{-1}(P) \overset{d}{\rightsquigarrow} f^{-1}(P')}$$

(BR2)

$$\frac{P \xrightarrow{f^{-1}(\mu)} P'}{f^{-1}(P) \xrightarrow{\mu} f^{-1}(P')} \quad [\, \mu \neq \tau \,]$$

(BR3)

$$\frac{P \xrightarrow{\tau} P'}{f^{-1}(P) \xrightarrow{\tau} f^{-1}(P')}$$

Sequential Composition (SC)

(SC1)

$$\frac{P \overset{d}{\rightsquigarrow} P' \quad \neg(P \xrightarrow{\checkmark})}{P \; ; \; Q \overset{d}{\rightsquigarrow} P' \; ; \; Q}$$

(SC2)

$$\frac{P \xrightarrow{\mu} P'}{P \; ; \; Q \xrightarrow{\mu} P' \; ; \; Q} \quad [\, \mu \neq \checkmark \,]$$

(SC3)

$$\frac{P \xrightarrow{\checkmark} P'}{P \; ; \; Q \xrightarrow{\tau} Q}$$

Interrupt (IR)

(IR1)

$$\frac{P \xrightarrow{\mu} P'}{P \triangle Q \xrightarrow{\mu} P' \triangle Q} \quad [\, \mu \neq \surd \,]$$

(IR2)

$$\frac{P \xrightarrow{\surd} P'}{P \triangle Q \xrightarrow{\surd} P'}$$

(IR3)

$$\frac{Q \xrightarrow{\tau} Q'}{P \triangle Q \xrightarrow{\tau} P \triangle Q'}$$

(IR4)

$$\frac{Q \xrightarrow{a} Q'}{P \triangle Q \xrightarrow{a} Q'}$$

(IR5)

$$\frac{P \overset{d}{\rightsquigarrow} P' \; Q \overset{d}{\rightsquigarrow} Q'}{P \triangle Q \overset{d}{\rightsquigarrow} P' \triangle Q'}$$

Timed Interrupt (TIR)

(TIR1)

$$\frac{P \xrightarrow{\mu} P'}{P \triangle_d Q \xrightarrow{\mu} P' \triangle_d Q} \quad [\, \mu \neq \checkmark \,]$$

(TIR2)

$$\frac{P \xrightarrow{\checkmark} P'}{P \triangle_d Q \xrightarrow{\checkmark} P'}$$

(TIR3)

$$\frac{}{P \triangle_0 Q \xrightarrow{\tau} Q}$$

(TIR4)

$$\frac{P \overset{d'}{\rightsquigarrow} P'}{P \triangle_d Q \overset{d'}{\rightsquigarrow} P' \triangle_{d-d'} Q} \quad [\, d' \leqslant d \,]$$

Delay (D)

(D1)

$$\frac{}{\text{WAIT } u \overset{d}{\rightsquigarrow} \text{WAIT } (u - d)} \quad [\, d \leq u \,]$$

(D2)

$$\frac{}{\text{WAIT } 0 \xrightarrow{\tau} \text{SKIP}}$$

3 Labelled Transition Systems and Time

Recall the standard definition for *Labelled Transition Systems (LTS)*:

Definition 1 (Labelled Transition System (LTS)) *An LTS is a tuple* (S, E, T, s_0)*, where S is a set of* states*, E a set of* labels*,* $T \subseteq S \times E \times S$ *the* transition relation *and* s_0 *the* initial state*.* □

For describing the operational semantics of a TCSP process, a (timed) LTS (S, E, T, s_0) is used, where the state S contains process references or TCSP terms. The set of labels is defined as $E = \Sigma \cup \{\checkmark, \tau\} \cup \mathbb{R}_0^+$, with $(\Sigma \cup \{\checkmark, \tau\}) \cap \mathbb{R}_0^+ = \emptyset$. The subset $\Sigma \subset E$ is called the *alphabet* of the process. Transitions in T are defined by axioms which are written as inference rules in the usual SOS-style (structured operational semantics) of Plotkin.

Transitions carrying labels $\mu \in \Sigma \cup \{\checkmark, \tau\}$ are written $P \xrightarrow{\mu} P'$ and called *event transitions*. They are interpreted as instantaneous events performed by processes P and accompanied by a state change to a new process P'. If $\mu \in \Sigma$, it is interpreted as an observable event, while $\mu = \tau$ denotes an internal event which cannot be identified by a (testing) environment. Event $\checkmark$ signifies process termination. TCSP follows the principle of *maximal progress*: Events occur immediately if all participants are willing to engage in it. TCSP supports the specification of *non-deterministic* behaviour: If several transition may fire in a given state, one of them is selected in an arbitrary way. This transition may disable the other ones which have been simultaneously enabled. As a consequence, maximal progress does not guarantee that enabled events really occur in a situation where several transition rules apply. However, the principle of *maximal parallelism* guarantees that events which are simultaneously enabled by parallel processes all happen at the same time (via consecutive application of the associated transition rules and since event transitions are instantaneous), if previous transitions do not disable the following ones.

TCSP is based on *Newtonian time*, assuming the existence of a conceptual global clock. Time is dense; the possible values of time ticks are modelled by $\mathbb{R}_0^+$. Transitions with labels $d \in \mathbb{R}_0^+$ are called *evolution transitions* and written as $P \stackrel{d}{\rightsquigarrow} P'$. They specify how processes P evolve over time d to new process states P', without producing any visible events.

Since event transitions occur instantaneously, precautions are necessary to avoid the specification of unrealisitic processes producing infinitely many events within a bounded – or even zero-length – time interval. To this end, the concept of *time-guarded* TCSP terms has been introduced by Schneider, which basically requires that in every non-terminating recursive system of TCSP processes timing operators exist which guarantee that minimal waiting times occur between sequences of finitely many events. Time-guardedness ensures *finite variability*: In any bounded time interval, Processes can generate only finite numbers of events.