Universität Bremen
Fachbereich 03 - Informatik

Logic Synthesis
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WORK SHEET 1

Problem 1. Let $A$, $B$, and $C$ be sets. Does each of the following holds? If it holds, then prove it, otherwise show a counterexample.

(a) $A \cup B = A \cup C \Rightarrow B = C$
(b) $A \cap B = A \cap C \Rightarrow B = C$
(c) $A \oplus B = A \oplus C \Rightarrow B = C$, where $A \oplus B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$.
(d) $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$.

Problem 2. Consider the equivalence relation on $A = \{a_1, a_2, a_3, a_4\}$. How many equivalence relations are there?

Problem 3. Let $R$ and $S$ be equivalent relations on the set $A$. Is each of the following and equivalence relation? If it is an equivalence relation, then prove it, otherwise show a counter example.

(a) $R \cup S$,
(b) $R \cap S$.

Problem 4. How many unary operations on $L = \{a, b, c\}$? How many unary operations on $A = \{0, 1, 2, 3\}$?

Problem 5. Obtain all possible partitions of $T = \{0, 1, 2\}$.

Problem 6. Given the partial order $\langle A, \leq \rangle$, where $x, y, z \in A$, $\cdot$ denotes the GLB, and $\vee$ denotes the LUB. Prove or disprove the following:

(a) $x \leq (y \vee z) \Rightarrow (x \leq y \lor x \leq z)$,
(b) $x \leq (y \cdot z) \Rightarrow (x \leq y \land x \leq z)$.

Problem 7. Let $R = \{(a, a), (b, b), (a, c), (c, a), (c, b)\}$ be a binary relation on $S = \{a, b, c\}$. Does $R$ satisfy each of the following. If not show a counterexample.

(a) reflexive,
(b) symmetric,
(c) antisymmetric,
(d) transitive,
(e) partial order relation,
(f) equivalence relation,
(g) function.