Universität Bremen

Fachbereich 03 - Informatik

Logic Synthesis

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Due Date: Nov. 14, 2006

WORK SHEET 1

Problem 1. Let A, B, and C be sets. Does each of the following holds? If it holds, then prove it, otherwise show a counterexample.

(a) $A \cup B = A \cup C \Rightarrow B = C$

(b) $A \cap B = A \cap C \Rightarrow B = C$

(c) $A \oplus B = A \oplus C \Rightarrow B = C$, where $A \oplus B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$.

(d) $(A \cap \overline{B}) \cup (A \cap \overline{C}) = A \cap (\overline{B \cap C})$.

Problem 2. Consider the equivalence relation on $A = \{a_1, a_2, a_3, a_4\}$. How many equivalence relations are there?

Problem 3. Let R and S be equivalent relations on the set A. Is each of the following and equivalence relation? If it is an equivalence relation, then prove it, otherwise show a counter example.

(a) $R \cup S$,

(b) $R \cap S$.

Problem 4. How many unary operations on $L = \{a, b, c\}$? How many unary operations on $A = \{0, 1, 2, 3\}$?

Problem 5. Obtain all possible partitions of $T = \{0, 1, 2\}$.

Problem 6. Given the partial order $\langle A, \leq \rangle$, where $x, y, z \in A$, $\cdot$ denotes the GLB, and $\vee$ denotes the LUB. Prove or disprove the following:

(a) $x \leq (y \vee z) \Rightarrow (x \leq y \text{ or } x \leq z)$.

(b) $x \leq (y \cdot z) \Rightarrow (x \leq y \text{ and } x \leq z)$.

Problem 7. Let $R = \{(a, a), (b, b), (a, c), (c, a), (c, b)\}$ be a binary relation on $S = \{a, b, c\}$. Does R satisfy each of the following. If not show a counterexample.

(a) reflective,

(b) symmetric,

(c) antisymmetric,

(d) transitive,

(e) partial order relation,

(f) equivalence relation,

(g) function.