Universität Bremen
Fachbereich 03 - Informatik

Logic Synthesis
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WORK SHEET 2

Problem 1. Consider the algebra on the set \( A = \{0, 1, a\} \) that is defined by the following tables.

\[
\begin{array}{c|ccc}
\lor & 0 & 1 & a \\
\hline
0 & 0 & 1 & a \\
1 & 1 & 1 & 1 \\
a & a & a & a \\
\end{array}
\quad
\begin{array}{c|ccc}
\cdot & 0 & 1 & a \\
\hline
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
a & a & 1 & a \\
\end{array}
\quad
\begin{array}{c|c}
x & x \\
\hline
0 & 1 \\
1 & 0 \\
a & a \\
\end{array}
\]

Check whether each of the axioms in the Huntington’s postulates holds.

Problem 2. Show that the following two algebras are isomorphic to each other.

\textit{Algebra 1: } \( \langle A, \lor, \land \neg, O_A, I_A \rangle \).
Let \( A \) be the positive integers that are divisors of 120. For each \( x, y \in A \), define the following:
- \( y \cdot y = \text{GCD}(y, x) \): the greatest common divisor of \( x \) and \( y \).
- \( x \lor y = \text{LCM}(x, y) \): the least common multiple of \( x \) and \( y \).
- \( \overline{x} = 120/x \): the quotient of 120 divided by \( x \).
- \( I_A = 120, O_A = 1 \).

\textit{Algebra 2: } \( \langle B, \lor, \land \neg, O_B, I_B \rangle \).
Let \( B = \{0, 1, 2, 3\} \times \{0, 1\} \times \{0, 1\} \). Then, the vector \( x = (x_1, x_2, x_3) \) that satisfies \( x_1 \in \{0, 1, 2, 3\}, x_2 \in \{0, 1\}, \) and \( x_3 \in \{0, 1\} \) is an element of \( B \).
Let \( x = (x_1, x_2, x_3) \) and \( y = (y_1, y_2, y_3) \) be elements of \( B \), define the algebra as follows:
- \( y \cdot y = (\min(x_1, y_1), \min(x_2, y_2), \min(x_3, y_3)) \).
- \( x \lor y = (\max(x_1, y_1), \max(x_2, y_2), \max(x_3, y_3)) \).
- \( \overline{x} = (3 - x_1, 1 - x_2, 2 - x_3) \).
- \( I_A = (3, 1, 1), O_A = (0, 0, 0) \).

Problem 3. In a Boolean algebra, prove the following without using truth tables: If \( a \lor b = a \lor c \) and \( ab = ac \), then \( b = c \).

Problem 4. Prove that an arbitrary \( n \)-variable logic function has an SOP with no more than \( 2^{n-1} \) products.

Problem 5. Represent the logic function \( f = x(y \oplus z) \) by the canonical SOP, canonical POS, and the positive polarity Reed-Muller expression.

Problem 6. Prove or show a counterexample for the following proposition:
\[ x \oplus (y \lor z) = (x \oplus y) \lor (x \oplus z). \]

Problem 7. Calculate the maximum number of nodes in a ROBDD with 10 variables. Show your work!

Problem 8. Calculate the maximum number of nodes in a ROBDD with edge negations for 10 variables. Show your work!