WORK SHEET 4

Problem 1. Prove or show a counterexample for each of the following statements.

(a) If the MSOP for the function $f$ is unique, then all prime implicants for $f$ are essential prime implicants.
(b) If the MSOP for the function $f$ is unique, then the minimum POS of $f$ is unique.
(c) Let $p$ be an arbitrary prime implicant of $f$. There is an MSOP that has $p$.
(d) The MSOP for a completely specified function $f$ is unique.
(e) The number of product terms in an MSOP for a completely specified function $f$ is equal to the number of sums in an MPOS for $f$.

Problem 2. Using Karnaugh maps find all prime implicants, all essential prime implicants, and a MSOP for the following functions:

(a) $f = \sum m(0, 2, 4, 5, 8, 9, 10, 13, 14, 15)$
(b) $g = \sum m(0, 1, 4, 5, 11, 15) + d(6, 7)$
(c) $h = \sum m(0, 4, 8, 10, 11, 15) + d(5, 7, 13)$

Problem 3. Show that an MSOP for $f = (x_1 \lor x_2 \lor \cdots \lor x_n)(\overline{x_1} \lor \overline{x_2} \lor \cdots \lor \overline{x_n})$ has at least $n$ products.

Problem 4. Using Quine-McCluskey’s method find a MSOP for $f = \sum m(0, 4, 8, 10, 11, 15, 21, 23, 29, 31) + d(5, 7, 13)$.

Problem 5. Which of the following are self-dual functions:

(a) $f = xy$
(b) $g = \overline{y}$
(c) $h = xyz \lor \overline{xy} \lor \overline{yz}$

Problem 6. Let $f$ be a monotone increasing function of $n$ variables. Prove the following:

(a) All prime implicants of $f$ are essential prime implicants.
(b) There is a unique MSOP for $f$

Problem 7. Prove that the number of the monotone increasing functions of $n$ variables is at least $2^N$, where $N = \binom{n}{[n/2]}$, and $[k]$ denotes the maximum integer not greater than $k$.

Problem 8. Let $f(x_1, x_2, \ldots, x_n) = f(x_2, x_1, x_3, \ldots, x_n) = f(x_2, x_3, \ldots, x_n, x_1)$. Prove that $f$ is a totally symmetric function.