Logic

Basic Question

- How to select the intended Σ-Algabras?

  $\rightsquigarrow$ Sets of Formulae

Equational Logic (Formulas)

Let $\Sigma = (S, \Omega)$ be a signature.

$$EL(\Sigma) := \{\forall X.t = u \mid X \text{ set of variables for } \Sigma, \; s \in S, \; t, u \in T_{\Sigma(X),s} \}$$

Equational Logic (Satisfaction Relation)

Let $\Sigma = (S, \Omega)$ be a signature.

We define

for each Σ-algebra A, and

for each equation $\forall X.t = u$:

$$A \models_\Sigma \forall X.t = u$$

iff (for all assignments $\alpha : X \to A$)

$$A(\alpha)(t) = A(\alpha)(u)$$

CASL-Syntax for EL

```
BASIC-ITEMS ::= SIG-ITEMS
              | forall VAR-DECL ;...; VAR-DECL
                  "." FORMULA "."..."." FORMULA ;/
              | ...

VAR-DECL    ::= VAR ,..., VAR : SORT

FORMULA     ::= TERM = TERM | ...
```

spec NAT =

sort Nat

ops $0 : Nat;$

$suc : Nat \to Nat;$

$\_\_ + \_\_, \_\_ * \_\_ : Nat \times Nat \to Nat$

%**prec** $\{\_\_ + \_\_\} < \{\_\_ * \_\_\}$

forall $m, n : Nat$

- %[addNat_0] $0 + n = n$
- %[addNat_suc] $suc(m) + n = suc(m + n)$
- %[multNat_0] $0 * n = 0$
- %[multNat_suc] $suc(m) * n = m * n + n$

end

spec $\textsc{List} = \textsc{Nat}$ **then**

sort $List$;

ops $nil : List$;

$\_\_ :: \_\_ : Nat \times List \to List$;

$\_\_ ++ \_\_ : List \times List \to List$;

$\_\_!\_\_ : List \times Nat \to Nat$;

forall $i, n : Nat; K, L : List$

- %[concat_nil] $nil ++ K = K$
- %[concat_NonEmpyList] $(n :: L) ++ K = n :: (L ++ K)$
- %[select_zero] $(n :: L)!0 = n$
- %[select_suc] $(n :: L)!(suc(i)) = L!i$

end