RevKit — User Manual

Contents

1. Introduction

RevKit is an open source toolkit aimed to make recent developments in the domain of reversible circuit design accessible to other researchers. Therefore, RevKit provides core functionality (like parsers, export functions, cost calculations, etc.), but also elaborated methods for synthesis, optimization, and verification of reversible (and quantum) circuits. More precisely, the following approaches are available in RevKit.

Synthesis

- A transformation-based method inspired by the concepts of [6]

- The BDD-based synthesis method as introduced in [9]

- The KFDD-based synthesis method as introduced in [7]

- The heuristic synthesis with output permutation method as introduced in [10]

- The ESOP-based synthesis method inspired by the concepts of [2]

- The exact synthesis method as introduced in [3]

Optimization

- The window optimization method as introduced in [8]

- The circuit line reduction method as introduced in [13]

Verification

- The SAT-based equivalence checker as introduced in [11]

Further Methods

- A naïve method to embed irreversible functions into reversible ones (needed e.g. to synthesize irreversible functions using the transformation-based method)

- A simple simulation engine (for reversible circuits working on Boolean values)

- A simple decomposition method that maps a given reversible circuit (composed of Toffoli, Fredkin, and Peres gates) to its equivalent quantum circuit (composed of NOT, CNOT, V, and V+ gates) inspired by the concepts of [1] and [4].

This document provides a manual describing how to apply the provided approaches and core functionalities of RevKit[1]. The main aspects are thereby kept brief, but are illustrated by means of examples.

In order to invoke approaches, RevKit uses a command line interface enabling an easy and flexible access to the functions and algorithms in the framework. This interface allows to create, modify, and display circuits as well as to call the above mentioned approaches. Additionally, to a certain degree it is also possible to extend the framework with new functionality (even if this is not the main intention of the interface). The interface itself is based on the Python programming language. As a result, commands (or sequences of commands) can easily be specified and processed using a common Python interpreter.

In the following, the usage of RevKit using its command interpreter is described as follows: First, how to get and how to install RevKit is shown, respectively. Section 3 provides a brief introduction into the usage of the basic functionality of RevKit followed by some more advanced examples in Section 4. Finally, the last sections provide the documentation of all data-structures and functions supported by the interpreter so far (including synopsis, examples, etc.).

2. Download and Installation

RevKit can be downloaded from the *www.revkit.org* website. Opening a Bash shell and assuming that the file *revkit-1.0.tar.gz* is in the current working directory, RevKit can be compiled using the following command.

```
>    tar  xvfz revkit−1.0.tar .gz
>    cd revkit−1.0
>    sudo apt−get install  build−essential cmake python−dev ipython python−qt4
>    ./build.sh −DBUILD_BINDINGS=ON
```

The *build.sh* script will download necessary libraries and compile the source code. Enabling the BUILD_BINDINGS option ensures, that the Python interface is compiled. Furthermore options that concern the developers' perspective are described in the developers' documentation and in the README file, respectively. Note that the third command installs required dependencies using the distribution's package manager. This command used in the example will only work with Ubuntu-based systems. Requirement-installation commands for other distributions can be taken from the README file.

After building RevKit, it can be started by invoking the scripts in the *tools* directory or by entering *ipython* followed by

```
from revkit import ∗
```

---

[1]For a description of how to extend RevKit with own approaches or how to integrate RevKit in own C++-projects, respectively, we refer the reader to the developers' documentation. The developers' documentation (including an API) is provided by means of doxygen in the sources of RevKit as well as on http://www.revlib.org.

How to enter the commands is in the scope of this document. To get an overview of all available commands, type

```
revkit_commands()
```

into the python shell.

3. Getting Started

After the installation, RevKit is fully functional and ready to use. In this section, how to apply the most important functions is briefly described. This should provide a starting point to become familiar with the framework and the syntax of the command line interface. Using this as a basis, all remaining functionality can be similarly applied. The last sections of this documentation provide the respective documentation on that. Besides that, Section 4 gives some further examples on more advanced application scenarios.

3.1. Using Approaches

When using a standard approach as for example the exact synthesis [3], it is not necessary to start the Python shell and enter the respective commands manually. For these purposes ready-to-use scripts, so called tools, are provided in the homonymous folder *tools*. Given a RevLib specification file called *function.spec*, the circuit can be obtained and written into the file *circuit.real* using the following command

> ./tools/exact_synthesis.py −−filename **function**.spec −−realname circuit.real

There are further options which can be passed to the *exact_synthesis.py* tool. They can be listed using the *help* option:

> ./tools/exact_synthesis.py −−help

For each approach implemented in RevKit a corresponding script is available in the *tools* folder. Call them using the *help* option to learn more about their usage.

In the tools folder, there is also a script called *viewer.py*, which displays circuits and its truth table. It is possible to open a *RevLib* realization file by adding it as the argument or with the corresponding menu entry in the application.

3.2. Creating a circuit

How to create a circuit using RevKit is described by means of *Multiple Control Toffoli gates* (MCT) in the following. Therefore, four steps are performed:

1. Importing the the revkit module,

2. declaring a circuit including 3 lines,

3. adding the respective Toffoli gates to the circuit, and

4. printing the circuit in ASCII format to the standard output[2].

This can be performed using the following Python code:

---

[2]Note that there is a special function print_function, which can be used to print the circuit to the standard output accepting more options to change the appearance. More information can be get from the reference in the remainder of this document

```
1  #!/usr/bin/python
2  from revkit import *
3
4  circ = circuit ( 3 )
5
6  append_toffoli( circ , [2], 1 )
7  append_toffoli( circ , [0, 1], 2 )
8  append_toffoli( circ , [1, 2], 0 )
9  append_toffoli( circ , [0, 1], 2 )
10 append_toffoli( circ , [2], 1 )
11
12 print circ
```
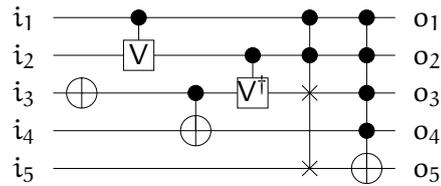
The second parameter of the append_toffoli function gives thereby a list of indices denoting the control line locations, while the last parameter gives the index of the target line. All lines are thereby counted starting with 0, whereby 0 denotes the top line.

3.3. Adding Gates

After getting to know about adding gates in general, in the following example, a circuit is created with different methods, i.e. using different gate types and positions where to insert the corresponding gate. Therefore,

1. An empty circuit with 5 lines is created,

2. Names for the input and output signals of the circuit are set.

3. A CNOT Gate with control at line 2 (counted from 0) and target at line 3 is added,

4. A V Gate (control on 0, target on 1) is prepended (added in the front of the circuit),

5. A Fredkin Gate with controls on 0 and 1 and targets 2 and 4 is appended at the end of the circuit,

6. A V+ Gate is inserted before the second gate (second parameter) with control on 1 and target on 2,

7. A NOT Gate is prepended at the beginning of the circuit,

8. A Toffoli gate with controls on 0, 1, 2, and 3 and target on 4 is added at the end of the circuit,

9. The LaTeX 2_ε code for drawing the circuit (using Ti*k*Z) is printed. Thereby the width between gates is adjusted.

This leads to the following circuit:

```
1   #!/usr/bin/python
2   from revkit import *
3
4   circ = circuit( 5 )
5   circ.inputs = [ "i_1", "i_2", "i_3", "i_4", "i_5" ]
6   circ.outputs = [ "o_1", "o_2", "o_3", "o_4", "o_5" ]
7
8   append_cnot( circ, 2, 3 )
9   prepend_v( circ, 0, 1 )
10  append_fredkin( circ, [0, 1], 2, 4 )
11  insert_vplus( circ, 2, 1, 2 )
12  prepend_not( circ, 2 )
13  append_toffoli( circ, [0, 1, 2, 3], 4 )
14
15  print create_image( circ, elem_width = 0.75 )
```

3.4. Reading and Writing a Circuit from a File

Instead of manually creating circuits, RevKit also supports circuit descriptions given in the RevLib format (see [12] for more information on RevLib and the supported formats). The following example demonstrates how a circuit given in this format can be imported, modified, and finally re-stored in a file. More precisely, the following code shows how

1. an empty circuit is created,

2. the *RevLib* realization file is parsed,

3. the circuit is modified (here, the gates are simply reversed), and

4. the circuit is re-stored to another file.

```
1   #!/usr/bin/python
2   from revkit import *
3
4   circ = circuit();
5   read_realization( circ, "circuit.real")
6   reverse_circuit( circ )
7   write_realization( circ, "circuit-copy.real")
```

3.5. Iterating through the Gates of a Circuit

Having a circuit available, RevKit provides functions in order to work with it. As an example, the following code shows how to iterate through the gates of circuit using the Python for ... in loop. For each gate the number of its control lines is printed to the standard output. Therefore,

1. an empty circuit is created,

2. a *RevLib* file is parsed,

3. every gate is traversed from left to right, and

4. for each gate the number of its control lines is printed.

```python
#!/usr/bin/python
from revkit import *

circ = circuit ()
read_realization ( circ , "circuit . real")

for g in circ .gates:
    print "Gate_has", g.num_controls, "controls."
```

3.6. Calling an approach

Having the basic functionality introduced so far, the main purpose of RevKit is to use the approaches e.g. for synthesis, optimization, verification, etc. This is exemplarily described in the following by means of the transformation based synthesis method (originally introduced in [6]).

In general, all approaches can be invoked using a generic signature of the respective functions. Usually, the first parameter denotes thereby the variable to which the result should be assigned (e.g. in the case of a synthesis approach, a variable representing the generated circuit). The following parameters denote all data, which might be required by the respective approach. In the case of the transformation based synthesis, this is a truth table description of the function to be synthesized. Finally, optional parameters can be delivered. If not, these parameters are initialized with default values. In case of a successful run, the return value of the respective functions is a *dictionary* (Python type dict) containing statistical data collected by the algorithm. Otherwise, the return value is a string containing an error message.

The following code shown the call of the transformation based synthesis with default parameters only.

```python
#!/usr/bin/python
from revkit import *

```

```
4   circ = circuit ()
5   spec = binary_truth_table ()
6   read_specification ( spec, "function.spec")
7
8   transformation_based_synthesis( circ, spec )
```

The function has an optional parameter bidirectional, which enables a special configuration of the approach (see Section C.3 for more details). By default, this option is enabled (i.e. the respective parameter is set to True). However, as the following code shows, this configuration can be easily modified.

```
1   #!/usr/bin/python
2   from revkit import *
3
4   circ = circuit ()
5   spec = binary_truth_table ()
6   read_specification ( spec, 'function.spec' )
7
8   transformation_based_synthesis( circ, spec, bidirectional = False )
```

A more detailed documentation of all parameters (also denoted by *settings*) can be found in the last sections of this manual (e.g. in case of the transformation based approach in Section C.3). These parameter always can be applied in every order after the mandatory parameters.

Besides the settings, there are also statistical variables, denoted by *statistics* in the following. In the case of the transformation based algorithm, the only statistical information is the run-time. In the following example, this statistic should be printed after the execution of the approach. The value is thereby assigned to a Python dict variable. It can be accessed by the name of the statistical parameter which are specified in the documentaion for each approach as well. However, in the case the execution of the approach fails, a string containing an error message is returned and, thus, the statistical values cannot be accessed. Thus, we have to check first whether the algorithm succeeded.

```
1   #!/usr/bin/python
2   from revkit import *
3
4   circ = circuit ()
5   spec = binary_truth_table ()
6   read_specification ( spec, "function.spec" )
7
8   r = transformation_based_synthesis( circ, spec, bidirectional = False )
9
10  if type(r) == dict:      # Success
11      print "Runtime", r["runtime"], "seconds."
12  else:                    # Fail
```

```
13    print r
```

3.7. Displaying a Circuit

RevKit contains some basic GUI functionality. As described above, a circuit can be printed to the standard output using Python's print or the print_circuit command. Additionally, RevKit provides functions to visualize a circuit. The following example demonstrates how to use the GUI functions. Therefore,

1. a circuit is read from a RevLib realization file,

2. the GUI is initialized (needs to be done only once per session),

3. a window (represented by the variable w) displaying the circuit is created and shown, and

4. an input pattern is assigned to the circuit in order to simulate it (in this example, it is assumed that the circuit has three lines).

```python
#!/usr/bin/python
from revkit import *

circ = circuit ()
read_realization ( circ , "circuit . real" )

init_gui ()

w = display_circuit ( circ )
w.simulate( [1,0,0] )
```

It is possible to zoom in and out into the circuit using the mouse wheel.

In the tools folder, there is a script called *viewer.py*, which encapsulates the *display_circuit* behavior in an application. It is possible to open a *RevLib* realization file by adding it as the argument or with the corresponding menu entry in the application.

3.8. Miscellaneous

In this section, some tips in the usage of RevKit with the *IPython* interpreter are given. As already mentioned above, all available data structures and commands can be listed by entering

```
revkit_commands()
```

into the Python interpreter. To get the synopsis and some documentation of a command, the name of the command followed by a question mark can be entered, e.g.

swop?

Entering two question marks will print out the Python source code implementation of the command:

swop??

4. Advanced Examples

4.1. A Stand-alone Program

This section illustrates how to build a stand-alone program (also denoted as *tool*) using RevKit in combination with Python. As an example, the transformation-based synthesis approach is used.

First, the Python header is set up, and the revkit as well as the sys libraries are loaded. The latter is used for accessing the command line parameters.

```python
1  #!/usr/bin/python
2  from revkit import *
3  import sys
```

Note that the sys library is not loaded into the global namespace. Now, the program options are set up. Therefore, different methods are available (for a comprehensive overview, see the reference in the remainder of this user guide). In the following, we need a parameter for providing a specification file to read from and a parameter for providing a realization file to write to. Furthermore, two user defined options are given. The first one is used to enable an ASCII print out and the second one to choose if the bidirectional approach for the transformation based synthesis should be used or not.

```python
4  opts = program_options()
5  opts.add_read_specification_option() \
6      .add_write_realization_option() \
7      .add_option( "print", "prints_the_circuit" ) \
8      .add_option( "bidirectional", True, "Bidirectional_approach" )
```

The methods for adding program options can be added successively, but note that in the end of each line a backslash has to be written, since there is no end of statement character in Python. The parameter for controlling the bidirectional flag comes with a default value, i.e. True.

After all parameters are set up, they have to be parsed and checked if they are entered correctly. This is done with the methods parse and good, respectively.

```python
9   opts.parse( sys.argv )
10
11  if not opts.good():
```

11

```
12        print opts
13        exit ()
```

The parameter for parse is sys.argv, i.e. the argument values from the command line. It checks, if the names of the parameters have been correctly entered and if values for all mandatory parameters have been provided. If the method good fails, a string for the usage of the program options is printed to the standard output and the program quits.

For the synthesis function, an empty circuit and binary truth table is required. The truth table should be parsed from the given program option.

```
14  circuit  circ ;
15  binary_truth_table  spec;
16
17  read_specification ( spec, opts. read_specification_filename () )
```

After that, we are ready to call the synthesis function. The parameter to enable or disable the bidirectional approach is thereby directly taken from the program options using the [] operator. Since we defined a default value for this parameter, it is assured that it yields a valid value. The key is the same string which was given by the call of add_option in line 8.

```
18  res = transformation_based_synthesis( circ,  spec,  \
19                                         bidirectional  = opts["bidirectional"] )
```

The result of the algorithm is saved in the variable res. As mentioned above, if the algorithm failed, res is a string. Otherwise, it is a Python dictionary (dict) with statistical information. Thus, first it is checked, whether the algorithm succeeded. If not, the error message is printed and the program quits.

```
20  if type(r)  == str :
21        print r
22        exit ()
```

However, if the functional call succeeded, first it should be checked, whether the circuit should be printed to the standard output. This can be controlled using the program option print. Whether a program option is set or not can be checked with the method is_set.

```
23  if opts. is_set ( "print" ):
24        print circ
```

Then, it should be checked whether the circuit realization should be dumped to a RevLib file. Since a predefined method of program_options was used to add this option, there exists a predefined option for checking and reading the value of that option as well.

```
25  if opts. is_write_realization_filename_set ():
26        write_realization ( circ,  opts. write_realization_filename () )
```

Finally, statistical information of the circuit as well as of the synthesis process (e.g. the run-time)is printed to the standard output.

```
27   print_statistics ( circ , res["runtime"] )
```

A. Core Data Structures

A.1. Gate

The class gate represents a gate in a circuit. It is a collection of control and target lines. Furthermore, a distinct type is set to each gate. Usually, gates are added using helper functions, e.g. append_toffoli.

Constructors

> **gate**() Initializes an empty gate

Properties

> **controls** Line iterator, allows the use of a for . . . in loop (*read-only*)
>
> **targets** Line iterator, allows the use of a for . . . in loop (*read-only*)
>
> **size** Size of the gate, which is the sum of number of control lines and target lines (*read-only*)
>
> **num_controls** Number of control lines (*read-only*)
>
> **num_targets** Number of target lines (*read-only*)
>
> **type** Type of the gate, which can be gate_type.toffoli, gate_type.peres, gate_type.fredkin, gate_type.peres, gate_type.v, and gate_type.vplus

Methods

> **add_control**(*l*) Adds a control at line *l*
>
> **remove_control**(*l*) Removes the control at line *l*
>
> **add_target**(*l*) Adds a target at line *l*
>
> **remove_target**(*l*) Removes the control at line *l*

A.2. Circuit

A circuit is the central data structure in the RevKit framework. It can be seen as a container of lines. Furthermore, it has properties for meta-data information such as names of the inputs, declaration of constant inputs, etc. A sub-circuit is also a circuit, shares the same data structure, and, thus, the same properties and operations. It is created with the subcircuit constructor.

Constructors

> **circuit**() Initializes an empty circuit with 0 lines
>
> **circuit**(*n*) Initializes an empty circuit with *n* lines
>
> **subcircuit**(*base*, *from*, *to*) Initializes a sub-circuit with *base* as circuit basis, including the gates *from* to *to*, where *to* is excluded

subcircuit(*base*, *from*, *to*, *filter*) Initializes a sub-circuit with *base* as circuit basis, including the gates *from* to *to*, where *to* is excluded. Furthermore, the lines are restricted to the indices in the list *filter*

Properties

lines Number of lines

num_gates Number of gates (*read-only*)

gates Gate iterator, allows the use of a for . . . in loop (*read-only*)

inputs Input labels

outputs Output labels

constants Determines constant inputs, i.e. a list which assigns the values True, False, or None to each input

garbage Determines garbage outputs, i.e. a list which assigns the values True or False to each output

circuit_name Name of the circuit

filter The filter is a list [*s*,*f*]. In the case the circuit is a sub-circuit and restricted by its lines, then s is the number of lines of the base circuit and f is the set of lines present in the sub-circuit. Otherwise, s is 0 and f is empty

Methods

append_gate(*g*) Appends gate g

prepend_gate(*g*) Prepends gate g

insert_gate(*n*, *g*) Inserts gate g in front of the gate at position n

remove_gate_at(*n*) Removes the gate at position n

is_subcircuit() Returns whether circuit is a sub-circuit or not

[*n*] **Accessor** Gets the *n*-th gate, counting from 0 (*read-only*)

Example

Two different methods for iterating through the gates.

```python
#!/usr/bin/python

circ = circuit ()
read_realization( circ , 'circuit.real' )

for g in circ :
  print g.num_controls()

for i in range(0, circ .num_gates):
  print circ [i].num_controls()
```

A.3. Truth Table

As truth table, the user interface of RevKit provides a binary_truth_table containing Boolean values only.

Constructors

> **binary_truth_table**() Initializes an empty binary truth table, working on the values True, False, and None (*don't care*)

Properties

> **entries** Entry iterator, allows the use of a for . . . in loop (*read-only*)
>
> **num_inputs** Number of input variables. The value initially is 0 and is determined after the first call of add_entry() (*read-only*)
>
> **num_outputs** Number of output variables. The value initially is 0 and is determined after the first call of add_entry() (*read-only*)
>
> **permutation** Current output permutation, i.e. a list with the numbers from 0 to $n - 1$, where n is the number of primary outputs. The permutation can also be changed with permute()
>
> **inputs** Input labels
>
> **outputs** Output labels
>
> **constants** Determines constant inputs, i.e. a list which assigns the values True, False, or None to each input
>
> **garbage** Determines garbage outputs, i.e. a list which assigns the values True or False to each output

Methods

> **add_entry**(*in*, *out*) Adds an entry with inputs *in* and outputs *out*. The first call determines the number of input and output variables. Afterwards, the size of *in* and *out* must be conform to them
>
> **clear**() Clears the truth table, including all meta-data and number of inputs and outputs
>
> **permute**() Permutes the output variables. Returns False when no more new permutation can be set

Example

Iterating through the entries of a specification.

```
1  #!/usr/bin/python
2
3  spec = binary_truth_table ()
4  read_specification ( spec, "function.spec" )
```

```
5
6  for entry in spec.entries :
7      print entry [0],  "maps_to", entry[1]
```

B. Core Functions

B.1. Basic Functions

Adding Circuits

append_circuit(*circ*, *src*)

> Appends the circuit *src* to the circuit *circ*.

prepend_circuit(*circ*, *src*)

> Inserts the circuit *src* at the beginning of circuit *circ*.

insert_circuit(*circ*, *pos*, *src*)

> Inserts the circuit *src* before gate with index *pos* of the circuit *circ*. The index is counted from 0.

Adding Gates

append_toffoli(*circ*, *controls*, *target*)

> Appends the Toffoli gate with control lines in the list *controls* and target line on *target* to the circuit *circ*.

append_fredkin(*circ*, *controls*, *target1*, *target2*)

> Appends the Fredkin gate with control lines *controls* and target lines *target1*, *target2* to the circuit *circ*.

append_peres(*circ*, *control*, *target1*, *target2*)

> Appends the Peres gate with control line *control* and target lines *target1*, *target2* to the circuit *circ*.

append_cnot(*circ*, *control*, *target*)

> Appends the CNOT gate with control line *control* and target line *target* to the circuit *circ*.

append_not(*circ*, *target*)

> Appends the NOT gate with target line *target* to the circuit *circ*.

append_v(*circ*, *control*, *target*)

> Appends the V gate with control line *control* and target line *target* to the circuit *circ*.

append_vplus(*circ*, *control*, *target*)

> Appends the V+ gate with control line *control* and target line *target* to the circuit *circ*.

prepend_toffoli(*circ*, *controls*, *t*

> Prepends the Toffoli gate with control lines *controls* and target line *target* to the circuit *circ*.

prepend_fredkin(*circ*, *controls*, *target1*, *target2*)

> Prepends the Fredkin gate with control lines *controls* and target lines *target1*, *target2* to the circuit *circ*.

prepend_peres(*circ*, *control*, *target1*, *target2*)

> Prepends the Peres gate with control line *control* and target lines *target1*, *target2* to the circuit *circ*.

prepend_cnot(*circ*, *control*, *target*)

> Prepends the CNOT gate with control line *control* and target line *target* to the circuit *circ*.

prepend_not(*circ*, *target*)

> Prepends the NOT gate with target line *target* to the circuit *circ*.

prepend_v(*circ*, *control*, *target*)

> Prepends the V gate with control line *control* and target line *target* to the circuit *circ*.

prepend_vplus(*circ*, *control*, *target*)

> Prepends the V+ gate with control line *control* and target line *target* to the circuit *circ*.

insert_toffoli(*circ*, *n*, *controls*, *t*

> Inserts the Toffoli gate with control lines *controls* and target line *target* to the circuit *circ*.

insert_fredkin(*circ*, *pos*, *controls*, *target1*, *target2*)

> Inserts the Fredkin gate with control lines *controls* and target lines *target1*, *target2* to the circuit *circ* at position *pos*.

insert_peres(*circ*, *pos*, *control*, *target1*, *target2*)

> Inserts the Peres gate with control line *control* and target lines *target1*, *target2* to the circuit *circ* at position *pos*.

insert_cnot(*circ*, *pos*, *control*, *target*)

> Inserts the CNOT gate with control line *control* and target line *target* to the circuit *circ* at position *pos*.

insert_not(*circ*, *pos*, *target*)

> Inserts the NOT gate with target line *target* to the circuit *circ* at position *pos*.

insert_v(*circ*, *pos*, *control*, *target*)

> Inserts the V gate with control line *control* and target line *target* to the circuit *circ* at position *pos*.

insert_vplus(*circ*, *pos*, *control*, *target*)

> Inserts the V+ gate with control line *control* and target line *target* to the circuit *circ* at position *pos*.

Circuit Lines

add_line_to_circuit(*circ*, *input*, *output*, *is_control* = None, *is_garbage* = False)

> Adds a line to the circuit.

control_lines(*g*)

> Returns a list of the control lines of *g*.

target_lines(*g*)

> Returns a list of the target lines of *g*.

find_non_empty_lines(*circ_or_gate*, *begin* = None, *end* = None)

> Returns the non empty lines in a circuit(range) or gate. The first parameter can be a circuit or a gate. If the first paramater is a circuit, then the gates can be selected by a range from *begin* to *end* (exclusive).

find_empty_lines(*circ_or_gate*, *begin_or_line_size* = None, *end* = None)

> Returns the empty lines in a circuit, a circuit range, or a gate. The first parameter can be a circuit or a gate. If the first paramater is a circuit, then the gates can be selected by a range from begin (*begin_or_line_size* parameter) to *end* (exclusive). If the first parameter is a gate then the second parameter is used to specify the number of lines in the gate.

Copying, Modifying and clearing circuits

clear_circuit(*circ*)

> Clears the circuit *circ* completely, i.e. gates, lines, and meta-data are deleted. The object is still valid.

circuit_to_truth_table(*circ*, *spec*)

> Generates the truth table for the circuit *circ*.

copy_circuit(*src*, *dest*)

> Copies all relevant data including lines, gates, and meta-data from *src* to *dest*.

copy_metadata(*base*, *circ*)

> Copies data from a specification or circuit *base* including inputs, outputs, garbage lines and constant lines to the circuit *circ*.

reverse_circuit(*src* [, *dest*])

> Reverses the circuit *src* and write the result into *dest*, if given. Otherwise the circuit is reversed in-place.

expand_circuit(*sub*, *circ*)

> Expands the sub-circuit *sub* by the lines of its base circuit and copies the result to *circ*.

Truth Table Information and Modification

fully_specified(*spec*)

> Returns True, if *spec* is a fully specified truth table. Otherwise False.

extend_truth_table(*spec*)

> Removes the Don't Cares Values of a binary truth table *spec*.

B.2. Input/Output

Creating Images

create_image(*circ*, *generator* = create_tikz_settings(), *elem_width* = 0.5, *elem_height* = 0.5, *line_width* = 0.3, *control_radius* = 0.1, *target_radius* = 0.2)

> Creates an image from *circ* and prints out the code to generate the image, e.g. LATEX. The target code can be specified using the *generator* parameter. In the default case, the output is TikZ code for LATEX. Another possible generator is create_pstricks_settings. Furthermore layout options can be specified with the remaining parameters.

Printing a circuit to console

print_circuit(*circ*, *print_inputs_and_outputs* = False, *print_gate_index* = False, *control_char* = '*', *line_char* = '-', *gate_spacing* = 0, *line_spacing* = 0)

> Prints the circuit *circ* as an ASCII representation to the console. The remaining parameters can adjust the appearance.

Printing statistics

print_statistics(*circ*, *runtime* = -1.0, *main_template* = '...', *runtime_template* = '...')

> Prints statistics of *circ* to the console. If *runtime* is not -1.0 it is printed as well. For more information about the templates, we refer to the corresponding entry in the API of the developers' documentation.

Reading and writing circuits and specifications

read_realization(*circ*, *filename*)

> Read-in routine for *RevLib* realization files. The circuit *circ* has to be empty.

write_realization(*circ*, *filename*, *version* = '1.0', *header* = 'This file has been generated using RevKit (www.revkit.org)')

> Dumps the circuit *circ* as *RevLib* realization file called *filename*.

read_specification(*spec*, *filename*)

> Read-in routine for *RevLib* specification files. The binary truth table *spec* has to be empty.

write_specification(*spec*, *filename*, *version* = '1.0', *header* = 'This file has been generated using RevKit (www.revkit.org)', *output_order* = [])

> Dumps the binary truth table *spec* as *RevLib* specification file called *filename*. Using *output_order* the order of the outputs can be changed. If specified, the length of the list has to match the number of outputs and all indices must be contained in the list.

read_pla(*spec*, *filename*, *extend* = True) Read-in routine for *PLA* specification files. The binary truth table *spec* has to be empty. The PLA gets extended using extend_truth_table automatically. This behavior can be disabled by setting extend to False.

write_blif(*circ*, *filename*, *tmp_signal_name* = 'tmp')

> Dumps the circuit *circ* as *BLIF* circuit to a file called *filename*.

B.3. Utilities

Cost Functions

costs(*circ*, *cost_function*)

> Returns the costs for the circuit *circ* base on the costs function *cost_function*, which can be either gate_costs(), quantum_costs(), transistor_costs(), or line_costs().

Program Options

Constructors

> **program_options**() Initializes an instance of type program_options which has initially only the *help* option.

Methods

> **add_costs_option**() Adds an option *costs* to specify a cost function.

> **add_read_specification_option**() Adds a mandatory option *filename* to specify a *RevLib* specification to read from. If this method was called, add_read_realization_option cannot be called anymore.

> **add_read_realization_option**() Adds a mandatory option *filename* to specify a *RevLib* realization to read from. If this method was called, add_read_specification_option cannot be called anymore.

> **add_write_realization_option**() Adds an option *realname* to specify a *RevLib* realization to read to.

> **add_numeric_option**(*name*, *description*) Adds an option getting a numeric value without a default value having the name *name* and a description *description*.

> **add_double_option**(*name*, *description*) Adds an option getting a floating number value without a default value having the name *name* and a description *description*.

> **add_option**(*name*, *description*) Adds an option getting a string value without a default value having the name *name* and a description *description*.

> **add_option**(*name*, *default_value*, *description*) Adds an option with a default value. The corresponding type can be determined from the default value, which can be either numeric or a string.

> **costs**() Returns the selected costs function, if a respective option was added.

> **good**() Evaluates to True, iff all mandatory options were specified and the *help* option was not requested.

> **is_set**(*name*) Returns True, if the option with name *name* was set as argument.

> **parse**(*arguments*) Parses the program arguments, usually in sys.argv.

> **read_realization_filename**() Value of the *filename* (as realization) option, if specified.

> **read_specification_filename**() Value of the *filename* (as specification) option, if specified.

> **write_realization_filename**() Value of the *realname* option, if specified.

> **is_write_realization_filename_set**() Returns True, iff the *realname* option is set as an argument.

[*name*] **Accessor** Returns the value of the option with name *name*, if specified (*read-only*)

C. Synthesis

C.1. Synthesis with Boolean Decision Diagrams

This algorithm implements the BDD based synthesis approach based on [9]. It supports complemented edges, different re-ordering strategies and the generation of both, Toffoli and elementary quantum gates.

The function representation can be read from a BLIF or PLA file-name. Thereby the extension is used to determine the file type, so it has to be ensured that a BLIF file has the extension *.blif and a PLA file has the extension *.pla, respectively.

Synopsis

```
bdd_synthesis(circ, filename[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm

filename A file which contains a function described as PLA or BLIF

Settings for the algorithm:

complemented_edges Specifies whether complemented edges should be used by the BDD. The default value is True.

reordering The reordering strategy for choosing the variable ordering. The default value is 4.

dotfilename If this string is specified, i.e. if it is not empty, then a graph representation of the BDD in DOT format is written to that file-name.

infofilename If this string is specified, i.e. if it is not empty, then information about the generated BDD are dumped to that file-name.

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

node_count Number of nodes of the generated BDD

Example

The following example creates a circuit using the BDD synthesis approach and dumps the BDD as a graph. Using dot, the graph can be displayed with the command

```
cat /tmp/test.dot | dot -Tpng | display
```

```
1  #!/usr/bin/python
2  from revkit import *
3
4  circ = circuit ()
```

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```
5
6  bdd_synthesis(circ, 'function.pla', dotfilename = '/tmp/test.dot')
```

C.2. Synthesis with Kronecker Functional Decision Diagrams

This synthesis approach constructs KFDDs from a given functional representation in PLA or BLIF and constructs a reversible circuit by constructing cascades for every node type as proposed in [7]. Thereby, re-ordering strategies as well as different decomposition types can be used.

Synopsis

```
kfdd_synthesis(circ, filename[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm

filename A file which contains a function described as PLA or BLIF

Settings for the algorithm:

default_decomposition The default decomposition type (Shannon = 0, positive Davio = 1, negative Davio = 2) used when initially constructing the KFDD. The default value is 0.

reordering The reordering strategy for choosing the variable ordering. The default value is 0.

sift_factor Sets a factorial limit for the growth during a siftprocess cause although the outcome will be improved, during sifting the KFDD might explode if not kept at bay. Suggested values are in between 2 and 3. The default value is 2.5

sifting_growth_limit This parameter (possible values are 'r' for relative and 'a' for absolute) determines whether the given sift-factor should be treated as relative or absolute growth limit. In the case of a relative treatment, after each repositioning of a sifting variable the comparison size for the growing is actualized. In the case of an absolute treatment, the comparison size is the intial size of the KFDD for the complete sifting process. The default value is 'a'.

sifting_method Sets the kind of choice for the next sifting candidate. Possible values and their meaning are listed in the following table:

Method	Description
'r' (Random)	The random selection was introduced for comparison reasons.
'i' (Initial)	The sifting variables are chosen in the order given before the sifting procedure starts.
'g' (Greatest)	Chooses the variable in the level with the largest number of nodes.

'l' (Loser first)	Although the complete sifting process will reduce the number of DD-Nodes (or at least keep the same size if no improvement can be done) after each repositioning of a sifting variable there will occasionally be some levels that grow. The loser first strategy chooses the next sifting candidate as the variable in the level with the least increase in size.
'v' (Verify)	Calculates the number of node eliminations due to the reduction rules of OKFDDs if a variable is repositioned in a specific level. It then chooses the best position according to the highest count result.

The default value is 'v'.

dotfilename If this string is specified, i.e. if it is not empty, then a graph representation of the KFDD in DOT format is written to that file-name.

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

node␣count Number of nodes of the generated BDD

Example

The following example synthesizes a circuit using the KFDD synthesis approach. The negative Davio decomposition is used as the default in the construction process.

```python
#!/usr/bin/python
from revkit import *

circ = circuit ()
kfdd_synthesis( circ , 'function.pla', default_decomposition = 2 )
```

C.3. Transformation-based Synthesis

This transformation based synthesis algorithm is based on [5]. The idea is to traverse the truth table rows from top to bottom and add gates to the circuit to obtain the identity. In the paper, two strategies were proposed, a basic approach adding gates in the end of the circuit and a bidirectional approach also adding gates in the beginning which can lead to fewer costs. Both approaches are implemented in this algorithm.

Synopsis

```
transformation_based_synthesis(circ, spec[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm

spec A fully specified binary truth-table which is basis for the synthesis algorithm

Settings for the algorithm:

bidirectional Determines whether the bidirectional approach should be used or not. The default value is True

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

Example

The following example synthesizes two reversible circuits using the function described in the file *function.spec*. First, the basic approach is applied and afterwards the bidirectional extension is enabled.

```
1   circ1 = circuit ()
2   circ2 = circuit ()
3
4   spec = binary_truth_table ()
5
6   read_specification (spec, 'function.spec')
7
8   # bidirectional approach
9   transformation_based_synthesis(circ1, spec)
10
11  # no bidirectional approach
12  transformation_based_synthesis(circ2, spec, bidirectional = False)
```

C.4. Exact Synthesis using Boolean Satisfiability

Synthesizes a minimal circuit (with respect to the number of gates) using the SAT-based exact synthesis approach as presented in [3].

Synopsis

```
exact_synthesis(circ, spec[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm

spec A fully specified binary truth-table which is basis for the synthesis algorithm

Settings for the algorithm:

solver The solver to be used in the approach. The default (and currently only available) value is MiniSAT.

max_depth The maximal considered circuit depth. The default value is 20.

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

Example

In the following example, a circuit is synthesized using Boolean Satisfiability.

```
1  #!/usr/bin/python
2  from revkit import *
3
4  spec = binary_truth_table ()
5  circ = circuit ()
6
7  read_specification ( spec, "function.spec" )
8  exact_synthesis( circ , spec )
```

C.5. ESOP-based Synthesis

This algorithm implements the ESOP based synthesis approach as introduced in [2]. The basic approach, where each input signal requires to line for its positive and negative polarity version, can be enabled by setting the setting separate_polarities to True. If one line is used for both polarities, which is the default case, a functor can be specified to reorder the cubes in order to minimize inverter gates. Two functors are provided which are, no_reordering which keeps the initial order from the truth table, and weighted_reordering which is proposed in [2] as reordering strategy.

Synopsis

```
esop_synthesis(circ, filename[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm

filename A file which contains a function described as ESOP cubes

Settings for the algorithm:

separate_polarities If True, the basic approach using two circuit lines for each input variable is used. Furthermore, in that case, no reordering functor has to be specified. The default value is False.

reordering Function for reordering the cubes to obtain a better result by using less NOT gates. The default value is weighted_reordering with default values.

garbage_name A string for the name of the garbage outputs which are possible created during embedding. The default value is 'g'.

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

C.6. Truth Table Embedding

This algorithm takes an irreversible (incompletely) specified truth table, for example using read_pla and embeds it into a reversible specification. Thereby necessary garbage and constant lines are added. The function is always embedded using the 0 values of the constant lines and the method which is used is the "Greedy Method" applying possible assignments by the minimal hamming distance.

Synopsis

```
embed_truth_table(spec, base[, ...])
```

spec A truth table which will be created by this algorithm. Can be the same as base.

base The base truth table which is irreversible.

Settings for the algorithm:

garbage_name A string for the name of the garbage outputs which are possible created during embedding. The default value is 'g'.

output_order The initial has a number of output variables, say n, the initial order of them is $[0, \ldots, n - 1]$, i.e. the i-th variable is initially in the i-th column. However, with embedding garbage lines are possibly added, say l garbage lines. Usually, the garbage lines are *appended* to the output columns, i.e. the initial order of the output variables will not change. To change this behavior a list of indices can be specified. The list must have n different elements with the values from 0 to $(n + g - 1)$ or the list is empty meaning that the output order will remain the same. The default value is the empty list.

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

Example

In this example the AND function is specified manually and then embedded to be reversible. Finally the reversible specification is synthesized using the transformation based synthesis.

```python
1  #!/usr/bin/python
2  from revkit import *
3
4  spec = binary_truth_table()
5  spec.add_entry( [False, False], [False] )
6  spec.add_entry( [False, True], [False] )
7  spec.add_entry( [True, False], [False] )
```

```
 8  spec.add_entry( [True, True], [True] )
 9
10  embed_truth_table( spec, spec )
11
12  circ = circuit ()
13
14  transformation_based_synthesis( circ, spec )
```

C.7. Synthesis with Output Permutation

This is an implementation of the SWOP (Synthesis with Output Permutation) synthesis approach as introduced in [10]. Thereby it is generic and can be used with every truth table based synthesis approach, which gets a circuit and a truth table as parameters.

Synopsis

```
swop(circ, spec[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm

spec A fully specified binary truth-table which is basis for the synthesis algorithm

Settings for the algorithm:

enable This parameter enables the output permutation. Thus, when this parameter is False, the algorithm behaves the same as calling the chosen synthesis algorithm once. Therewith, embedding a synthesis algorithm in the swop algorithm enables three configurations: no swop (enable is False), heuristic and exhaustive (enable is True in combination with the exhaustive parameter). The default value is True.

exhaustive If this parameter is True, then all permutations are checked, otherwise the a good permutations is heuristically determined by sifting the permutations. The complexity of the SWOP algorithm (not considering the used synthesis approach) is $\mathcal{O}(2^n)$ if this parameter is True, and $\mathcal{O}(n^2)$ if this parameter is set to False. The default value is False.

synthesis A functor to the default synthesis approach which is used. The functor is of type truth_table_synthesis_func. The default value is transformation_based_synthesis_func().

cost_function A pointer to a cost function, which is used is criteria to minimize the circuit. The default value is gate_costs().

stepfunc A function which gets called after each iteration of the SWOP algorithm. The functor is of type swop_step_func. The default value is an empty function.

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

Example

In the following example the SWOP synthesis is used with a modified transformation based synthesis (using the non bidirectional approach) and a step function, which counts the number of iterations in a global variable named counter.

```
1   #!/usr/bin/python
2   from revkit import *
3
4   circ  =  circuit ()
5   spec = binary_truth_table ()
6
7   counter = 0
8
9   read_specification (spec, 'function.spec')
10
11  tbs  = transformation_based_synthesis_func(bidirectional = False)
12
13  def step ():
14      global counter
15      counter += 1
16
17  swop(circ, spec, synthesis = tbs, stepfunc = swop_step_func.from_callable(step))
18  print counter, ' iterations were performed'
```

C.8. Quantum Decomposition

This algorithm decomposes a reversible circuit into a quantum circuit based on the work of [1] and [4]. The resulting circuits do not necessarily coincide with the quantum costs calculated by quantum_costs(), since some further optimizations are not considered yet.

Synopsis

```
quantum_decomposition(circ, base[, ...])
```

circ An empty circuit, which will be filled with quantum gates by the algorithm.

base The base circuit, containing reversible gates which needs to be decomposed. This circuit will not be changed by the algorithm.

Settings for the algorithm:

helper_line_input In some cases a helper line is introduced by the algorithm (see above). This string specifies the input name for the helper line. The default value is 'w'.

helper_line_output In some cases a helper line is introduced by the algorithm (see above). This string specifies the output name for the helper line. The default value is 'w'.

gate_decomposition This parameter is a gate_decomposition_functor which decomposes a single gate and adds it to the quantum circuit. This fuctor is called by the algorithm for every gate. The default value is standard_decomposition, which implements the above described decomposition techniques.

Statistical information for the algorithm:

runtime Run-time used by the synthesis algorithm

Example

The following example decomposes the Toffoli gate as its quantum cascade and writes it to another realization file.

```python
1  #!/usr/bin/python
2  from revkit import *
3
4  circ = circuit (3)
5  append_toffoli( circ , [0,1], 2)
6
7  quancirc = circuit ()
8  quantum_decomposition(quancirc, circ)
```

```
9
10  write_realization (quancirc, ' circuit . real ')
```

D. Optimization

D.1. Window Optimization

This algorithm implements the window optimization approach as presented in [8]. The implementation is very generic and depends heavily on the functors defined in settings.

In a loop, a new window is selected using the select_window setting, and in case a window was found, the optimization approach using the optimization setting is applied.

The resulting new window is compared to the extracted one using the cost metric defined in the cost_function setting.

Synopsis

```
window_optimization(circ, base[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm by optimizing base.

base The base circuit which should be optimized.

Settings for the algorithm:

> **select_window** A functor which selects the window which should be considered for local optimization. The default value is shift_window_selection_func with default parameters.

> **optimization** A functor which optimizes the window. The default value is resynthesis_optimization_func with default parameters.

> **cost_function** A pointer to a cost function, which is used is criteria to minimize the circuit. The default value is gate_costs().

Statistical information for the algorithm:

> **runtime** Run-time used by the synthesis algorithm

Example

In this circuit a circuit is read from a realization file and afterwards first optimized using shift window selection with a window length of 7 and an offset of 3. Finally, the circuit is again optimized using the line window selection scheme and quantum costs as cost criteria.

```python
1  #!/usr/bin/python
2  from revkit import *
3
4  circ = circuit ()
5  read_realization ( circ , ' circuit . real ')
6
```

```
7   opt_circ1  =  circuit ()
8   window_optimization(opt_circ1, circ, \
9       select_window = shift_window_selection_func(window_length = 7, offset = 3))
10
11  opt_circ2  =  circuit ()
12  window_optimization(opt_circ2, opt_circ1, \
13      select_window = line_window_selection_func(), cf = quantum_costs())
```

D.2. Line Reduction

This algorithm implements the approach presented in [13]. Windows are found and re-synthesized such that an output of that window is always returning a constant value, so that it can be used as replacement for another constant input line, often introduced by hierarchical synthesis methods.

Synopsis

```
line_reduction(circ, base[, ...])
```

circ An empty circuit, which is filled with gates by the algorithm by optimizing base.

base The base circuit which should be optimized.

  Settings for the algorithm:

>   **max_window_lines** Number of lines the selected windows can have initially. The default value is 6.

>   **max_grow_up_window_lines** When the truth table is not reversible, obtained by a window with max_window_lines lines, then the number of lines can be increased up at most this value. The default value is 9.

>   **window_variables_threshold** The possible window inputs are obtained by simulating its *cone of influence*. It is only simulated if the number of its primary inputs is less or equal to this value. The default value is 17.

>   **simulation** Simulation function used to simulate values inside the windows and inside the *cone of influence*. The default value is simple_simulation_func().

>   **window_synthesis** Functor used to re-synthesize the window. It only has to embed and synthesize the window. It is preferred to use embed_and_synthesize, whereby the parameters can be adjusted to use different synthesis algorithms. The default value is embed_and_synthesize() with default parameters.

  Statistical information for the algorithm:

>   **runtime** Run-time used by the synthesis algorithm

>   **num_considered_windows** Number of windows, which were considered in total.

>   **skipped_max_window_lines** Number of skipped windows due to maximum number of allowed primary inputs to be simulated, see window_variables_threshold.

>   **skipped_ambiguous_line** Number of skipped windows due to irreversible specification.

>   **skipped_no_constant_line** Number of skipped windows in the case that no constant line can be found for a garbage line.

>   **skipped_synthesis_failed** Number of skipped windows in the case that the synthesis of the window failed.

Example

First the line are reduced using the standard settings, meaning that the transformation based synthesis is exploited. Afterwards, line reduction is applied using the exact synthesis. To keep the number of window lines small when using the exact synthesis approach, the value for max_grow_up_window_lines is adjusted.

```python
#!/usr/bin/python
from revkit import *

circ = circuit ()
read_realization ( circ , 'circuit.real')

lr_circ1 = circuit ()
line_reduction( lr_circ1 , circ )

lr_circ2 = circuit ()
window_optimization(lr_circ2, circ, \
    max_grow_up_window_lines = 6, \
    window_synthesis = embed_and_synthesize( synthesis = exact_synthesis_func() ) )
```

E. Verification

References

[1] A. Barenco, C. H. Bennett, R. Cleve, D. DiVinchenzo, N. Margolus, P. Shor, T. Sleator, J. Smolin, and H. Weinfurter. Elementary gates for quantum computation. *The American Physical Society*, 52:3457–3467, 1995.

[2] K. Fazel, M. Thornton, and J. Rice. ESOP-based Toffoli gate cascade generation. In *Communications, Computers and Signal Processing, 2007. PacRim 2007. IEEE Pacific Rim Conference on*, pages 206 –209, 2007.

[3] D. Große, R. Wille, G. W. Dueck, and R. Drechsler. Exact multiple control Toffoli network synthesis with SAT techniques. *IEEE Trans. on CAD*, 28(5):703–715, 2009.

[4] D. Maslov and G. Dueck. Improved quantum cost for n-bit Toffoli gates. *Electronics Letters*, 39(25):1790 – 1791, 11 2003.

[5] D. Miller, D. Maslov, and G. Dueck. A transformation based algorithm for reversible logic synthesis. In *Design Automation Conference, 2003. Proceedings*, pages 318 – 323, 2-6 2003.

[6] D. M. Miller, D. Maslov, and G. W. Dueck. A transformation based algorithm for reversible logic synthesis. In *Design Automation Conf.*, pages 318–323, 2003.

[7] M. Soeken, R. Wille, and R. Drechsler. Hierarchical synthesis of reversible circuits using positive and negative Davio decomposition. In *Workshop on Reversible Computation*, 2010.

[8] M. Soeken, R. Wille, G. W. Dueck, and R. Drechsler. Window optimization of reversible and quantum circuits. In *IEEE Symposium on Design and Diagnostics of Electronic Circuits and Systems*, 2010.

[9] R. Wille and R. Drechsler. BDD-based synthesis of reversible logic for large functions. In *Design Automation Conf.*, pages 270–275, 2009.

[10] R. Wille, D. Große, G. Dueck, and R. Drechsler. Reversible logic synthesis with output permutation. In *VLSI Design*, pages 189–194, 2009.

[11] R. Wille, D. Große, D. M. Miller, and R. Drechsler. Equivalence checking of reversible circuits. In *Int'l Symp. on Multi-Valued Logic*, pages 324–330, 2009.

[12] R. Wille, D. Große, L. Teuber, G. W. Dueck, and R. Drechsler. RevLib: an online resource for reversible functions and reversible circuits. In *Int'l Symp. on Multi-Valued Logic*, pages 220–225, 2008. RevLib is available at http://www.revlib.org.

[13] R. Wille, M. Soeken, and R. Drechsler. Reducing the number of lines in reversible circuits. In *Design Automation Conf.*, 2010.