Homework 1

Algorithmic Game Theory — Summer semester 2010

Exercise 1 (15 points)**.** Consider the following 3-player strategic game:

$3:E$		
1 \ 2	C	D
A	(0, 0, 0)	(0, 1, -1)
B	(2, 2, 2)	(-1, 3, 4)

$3:F$		
1 \ 2	C	D
A	(8, 4, 2)	(7, 7, -2)
B	(9, 2, 5)	(-10, 3, 0)

Find all Pareto-optimal strategy profiles, dominant strategies, strongly and weakly dominated strategies and pure Nash equilibria.

Exercise 2 (25 points)**.** Consider the following strategic game.

1 \ 2	C	D	E
A	(a, b)	(c, d)	(e, f)
B	(g, h)	(i, j)	(k, l)

Determine the conditions under which:

1. the strategy profile AC Pareto-dominates BD
2. the strategy profile BE is Pareto-optimal
3. the strategy A strictly dominates B for player 1
4. the strategy D is a dominant strategy for player 2
5. the strategy profile AD is a Nash equilibrium

Exercise 3 (15 points)**.** Suppose customers are uniformly distributed along a beach that is 1 kilometer long. Ice-cream prices on the beach are fixed, so customers always choose the nearest vendor. If more than one vendor is at the same location, they split the business evenly.

Consider a game in which there are two ice-cream vendors, and each must select a location in $[0, 1]$ for his shop. Show that the strategy profile $(0.5, 0.5)$, in which both vendors place their ice cream stands at the middle of the beach, is the unique pure Nash equilibrium in this game.

Exercise 4 (15 points)**.** In this exercise, we consider *iterated elimination of* ***weakly*** *dominated stategies* (IEWDS), which is defined analogously to IESDS seen in class.

- Show using the following game that IEWDS can lead to different results depending on the order in which strategies are eliminated.

1 \ 2	C	D	E
A	(1, 1)	(1, 3)	(2, 4)
B	(2, 3)	(1, 3)	(2, 2)

- Show using the following game that some pure Nash equilibria might be eliminated by IEWDS.

1 \ 2	D	E	F
A	(1, -1)	(-1, 1)	(-1, -1)
B	(-1, 1)	(1, -1)	(-1, -1)
C	(-1, -1)	(-1, -1)	(-1, -1)

Exercise 5 (15 points)**.** Prove that a pure Nash equilibrium can never be removed by IESDS. Hint: use proof by contradiction.

Exercise 6 (15 points)**.** Show that a mixed strategy profile is a Nash equilibrium if and only if the following holds: if the mixed strategy for a player attaches a positive probability to a pure strategy a then a is a best response to the mixed strategies of the other players.