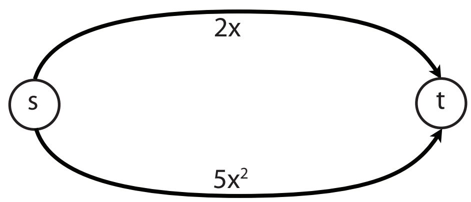
# Homework 4

Algorithmic Game Theory — Summer semester 2010

To get full marks, please make sure to justify your answers.

**Exercise 1** (40 points)**.** Consider the following selfish routing problem, where 1 unit of traffic is to be routed from $s$ to $t$.




(a) Consider a flow with $p$ on the top path and $1 - p$ on the bottom path. State the cost of both paths in this flow.

(b) Compute an equilibrium flow. What is its cost?

(c) Compute an optimal flow. What is its cost?

(d) Using parts (b) and (c), compute the price of anarchy of this game.

Note: for parts (b) and (c), you will likely need to solve some quadratic and/or cubic equations. You can do this with a calculator or an online tool.

**Exercise 2** (30 points)**.** Consider the following voter profile:

$$
\begin{array}{rl}
33: & a \succ b \succ c \succ d \succ e \\
16: & b \succ d \succ c \succ e \succ a \\
3: & c \succ d \succ b \succ a \succ e \\
8: & c \succ e \succ b \succ d \succ a \\
18: & d \succ e \succ c \succ b \succ a \\
22: & e \succ c \succ b \succ d \succ a
\end{array}
$$

Determine the winner for each of the following voting rules: plurality, plurality with runoff, Borda, veto, STV, Copeland, and pairwise elimination with ordering *dbcea*. Also determine whether this profile has a Condorcet winner.

**Exercise 3** (10 points)**.** Show that the Borda voting rule does not satisfy the Condorcet condition by finding a voter profile in which the Condorcet winner is not selected by the Borda rule.

**Exercise 4** (10 points)**.** A social choice function $f$ satisfies *unanimity* if whenever every agent $i$ has $o$ as its most preferred alternative, then $f$ must select $o$. It is called *onto* if for every outcome $o$, there exists some preference profile for which $f$ selects $o$. Prove that a social choice function that is onto and satisfies monotonicity also satisfies unanimity.