Description Logics: a Nice Family of Logics
— Modularity —

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Plan for today

1. What is modularity good for?
2. Modules for reuse
3. Summary and Outlook
And now . . .

1. What is modularity good for?

2. Modules for reuse

3. Summary and Outlook
What can I do with my ontology?

Ontology users and engineers want to use ontologies to

- represent and archive knowledge \( (M) \)
  in a structured way

- compute inferences from archived knowledge \( (M) \)
  e.g., classification, query answering

- explain inferences \( (M) \)
  justifications = pinpointing, abduction

- reuse (parts of) other ontologies to build their ontology \( (M) \)
  import

- expose the logical structure of the represented knowledge \( (M) \)
  comprehension

\( (M) = \text{modularity helps} \)
What can I do with my ontology?

Building and using an ontology often requires

- fast reasoning (M)
  
  expressivity ↔ complexity; optimisations, incremental reasoning

- collaborative development (M)

- version control (M)

- efficient reuse (M)

- an understanding of the ontology’s content and structure (M)
  
  comprehension

(M) = modularity helps
A priori vs. a posteriori modularisation

**A priori** (not covered today)

- At first, a modular structure is decided on.
- Then, the ontology is developed and used according to that structure.

**A posteriori**

- The ontology is regarded as a monolithic entity.
- At some point, a module is extracted or the ontology is decomposed into several modules.
And now . . .

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Comparing two ontologies

Assume that . . .

- you want to buy a medical ontology from me
- I offer two medical ontologies $\mathcal{O}_1$ and $\mathcal{O}_2$

Q: which one do you choose?

Possible A: the one that contains more knowledge.

Q: how do you measure the amount of knowledge in $\mathcal{O}_i$?

Possible A: Number of axioms?

- Well, compare $\{A \sqsubseteq B, B \sqsubseteq A\}$ vs. $\{A \equiv B\}$
- or $\{A \sqsubseteq B, B \sqsubseteq A \cup \neg A, A \cap \neg A \sqsubseteq B\}$ vs. $\{A \equiv B\}$

Possible A: Number of entailments? Number of models?
Think of axioms as generating entailments – e.g.:

\[
\begin{align*}
A &\sqsubseteq \exists r. B \\
\exists r. T &\sqsubseteq C \cap D
\end{align*}
\]

\[\models A \sqsubseteq D\]

Q: how many entailments can a TBox have?

A: \[\infty\]

\[A \sqsubseteq D \quad A \sqsubseteq D \cup A \quad A \sqsubseteq D \cup (A \cap D), \quad \ldots\]
Think of axioms as **restricting possible models**

Axioms “filter out” unwanted models – e.g.:

- $\text{Hand} \sqsubseteq \exists \text{hasPart}.\text{Finger}$
  $\Rightarrow$ models cannot have instances of Hand with no hasPart-edge to an instance of Finger

- $\text{Hand} \sqsubseteq = 5\text{hasPart}.\text{Finger}$
  $\Rightarrow$ models cannot have instances of Hand with $\neq 5$ hasPart-edges to instances of Finger

**Q:** how many models can a TBox have?

**A:** 0
Next attempt at “more” entailments/models

We cannot compare \textit{numbers} of entailments or models

But we can use set inclusion:

“\(O\) knows at most as much as \(O'\)” if

- every entailment of \(O\) is one of \(O'\):
  \[
  \{ \eta \mid O \models \eta \} \subseteq \{ \eta \mid O' \models \eta \}
  \]
  or

- every model of \(O'\) is one of \(O\):
  \[
  \{ \mathcal{I} \mid \mathcal{I} \models O' \} \subseteq \{ \mathcal{I} \mid \mathcal{I} \models O \}
  \]

\textbf{Problem:}

How do we test these conditions?
Knowledge w.r.t. a signature

Let’s reformulate the initial dialogue.
Assume that . . .

- you want to buy a subset of a medical ontology \( \mathcal{O} \) from me that covers the subdomain of, say, diseases
- I offer two subsets \( \mathcal{M}_1 \) and \( \mathcal{M}_2 \)

**Q:** which one do you choose?

**Possible A:** the one that “knows more” about diseases!

**Q:** which is the best subset I can offer?

**Possible A:** a module for diseases

- \( \mathcal{M} \subseteq \mathcal{O} \) that knows as much as \( \mathcal{O} \) about diseases:
  - \( \mathcal{M} \) indistinguishable from \( \mathcal{O} \) w.r.t. all terms relevant for diseases
- \( \mathcal{M} \) as small as possible
Inseparability w.r.t. a signature

**Definition**

- **Signature** $\Sigma = \text{a set of concept/role names}
- The signature of axiom (ontology) $X$
  $= \text{all concept/role names in } X$
- $\mathcal{O}_1$ and $\mathcal{O}_2$ are $\Sigma$-inseparable w.r.t. a logic $\mathcal{L}$,
  written $\mathcal{O}_1 \equiv_\Sigma^\mathcal{L} \mathcal{O}_2$, if:
  for all $\eta \in \mathcal{L}$ with $\text{sig}(\eta) \subseteq \Sigma$,
  $$\mathcal{O}_1 \models \eta \iff \mathcal{O}_2 \models \eta$$
- $\mathcal{O}$ is a $\Sigma$-conservative extension (\(\Sigma\)-dCE) of $\mathcal{M}$ w.r.t. $\mathcal{L}$
  if $\mathcal{M} \subseteq \mathcal{O}$ and $\mathcal{M} \equiv_\Sigma^\mathcal{L} \mathcal{O}$

Alternative names:

- $\mathcal{M}$ **covers** $\mathcal{O}$ for $\Sigma$ w.r.t. $\mathcal{L}$
- $\mathcal{M}$ **is a module of** $\mathcal{O}$ for $\Sigma$ w.r.t. $\mathcal{L}$

[Konev et al. 2009]

[Ghilardi et al. 2006]
Choosing the signature $\Sigma$

Definition (repeated from previous slide)

$\mathcal{O}$ is a $\Sigma$-module of $\mathcal{M}$ w.r.t. $\mathcal{L}$ if $\mathcal{M} \subseteq \mathcal{O}$ and $\mathcal{M} \equiv_{\Sigma} \mathcal{O}$

The signature $\Sigma$ . . .

- can be seen as a “topic”
- that the module is required to cover
- is difficult to formulate:
  - $Q$: how many interesting entailments in $\Sigma = \{\text{Disease}\}$ can $\mathcal{O}$ possibly have?
Choosing the logic $\mathcal{L}$

Definition (repeated from previous slide)

$O$ is a $\Sigma$-module of $M$ w.r.t. $\mathcal{L}$

if $M \subseteq O$  and  $M \equiv^\Sigma_{\mathcal{L}} O$

Choice of $\mathcal{L}$ depends on your usage of the module:

- for ontology design: subsumptions betw. (complex?) concepts
- for ontology usage: your favourite query language
Modules for reuse

If we want to reuse module $\mathcal{M}$, we need a stronger guarantee:

$$\mathcal{M} \cup \mathcal{O}' \equiv \Sigma \mathcal{O} \cup \mathcal{O}'$$

for all $\mathcal{O}'$

i.e., we can safely import $\mathcal{M}$ into any $\mathcal{O}'$

Ensured by two additional requirements:

Lemma [Konev et al. 2009]

If $\mathcal{M} \subseteq \mathcal{O}$ and $\mathcal{M} \equiv \Sigma \mathcal{O}$, then $\mathcal{M} \cup \mathcal{O}' \equiv \Sigma \mathcal{O} \cup \mathcal{O}'$, for

1. every $\mathcal{O}'$ with $\text{sig}(\mathcal{O}) \cap \text{sig}(\mathcal{O}') \subseteq \Sigma$,

2. expressive enough $\mathcal{L}$, e.g. $\text{SROIQ}$ (OWL).

(1) means that $\mathcal{O}'$ may reuse only terms from $\Sigma$
How is a minimal $\Sigma$-module extracted?

Simple module extraction algorithm:

- $\mathcal{M} \leftarrow \emptyset$

- While $\mathcal{M} \setminus \{\alpha\} \equiv \Sigma \mathcal{O}$, for some $\alpha \in \mathcal{M}$,
  do $\mathcal{M} \leftarrow \mathcal{M} \setminus \{\alpha\}$

- Output $\mathcal{M}$

Observation:

Different orders of choosing $\alpha$
can lead to different minimal modules
Example

Let $\Sigma = \{\text{Knee}, \text{HingeJoint}\}$. Suppose Galen contains:

\begin{align*}
\text{Knee} \equiv & \text{Joint} \sqcap \exists \text{hasPart} . \text{Patella} \sqcap \exists \text{hasFunct} . \text{Hinge} \\
\text{Patella} \sqsubset & \text{Bone} \sqcap \text{Sesamoid} \\
\text{Ginglymus} \equiv & \text{Joint} \sqcap \exists \text{hasFunct} . \text{Hinge} \\
\text{Joint} \sqcap \exists \text{hasPart} . (\text{Bone} \sqcap \text{Sesamoid}) & \sqsubset \text{Ginglymus} \\
\text{Ginglymus} \equiv & \text{HingeJoint} \\
\text{Meniscus} \equiv & \text{FibroCartilage} \sqcap \exists \text{locatedIn} . \text{Knee}
\end{align*}

$\subseteq$-Minimal module for $\Sigma$? \{(1), (2), (4), (5)\} and \{(1), (3), (5)\}

Note that a module for $\Sigma$ does not necessarily contain

- all axioms that use terms from $\Sigma$
- only axioms that only use terms from $\Sigma$
Bad news for expressive ontology languages?

Big, sad theorem [Ghilardi et al. 2006]

Let $\mathcal{O}_1, \mathcal{O}_2$ be ontologies in $\mathcal{L}$ and $\Sigma$ a signature.

Determining whether $\mathcal{O}_1 \equiv^\mathcal{L}_\Sigma \mathcal{O}_2$ is

- **$\text{ExpTime-complete}$** for $\mathcal{L} = \mathcal{EL}$
- **$2\text{ExpTime-complete}$** for $\mathcal{ALC} \leq \mathcal{L} \leq \mathcal{ALCQI}$, and
- **undecidable** for $\mathcal{L} \geq \mathcal{ALCQO}$, including OWL

(even if $\mathcal{O}_1, \mathcal{O}_2$ are in $\mathcal{ALC}$).
Consequences for modules of expressive DLs

Extracting modules is highly complex for expressive DLs.

What to do?

1. Give up? No: modules clearly too important
2. Reduce expressivity of logic? Yes! (Not covered here.)
3. Approximate for expressive logics? Yes – but from the right direction!

Next: 2 approximations, i.e., sufficient conditions for inseparability

1. based on semantic locality
2. based on syntactic locality

[Cuenca Grau et al. 2009]
Model-theoretic inseparability

Remember: $O_1 \equiv_{\Sigma} O_2$ if:
for all $\eta \in \mathcal{L}$ with $\text{sig}(\eta) \subseteq \Sigma$,

$$O_1 \models \eta \quad \text{iff} \quad O_2 \models \eta$$

Good news:

$$\uparrow$$

$$\{ \mathcal{I}|_{\Sigma} \mid \mathcal{I} \models O_1 \} = \{ \mathcal{I}|_{\Sigma} \mid \mathcal{I} \models O_2 \}$$

- i.e., $O_1$ and $O_2$ have the same models modulo $\Sigma$
  ($\mathcal{I}|_{\Sigma}$ is the restriction of $\mathcal{I}$ to $\Sigma$)
- shorthand: $O_1 \equiv_{\Sigma}^{\text{mod}} O_2$ (model-inseparable)
- this notion does not depend on $\mathcal{L}$

Bad news: $O_1 \equiv_{\Sigma}^{\text{mod}} O_2$ is undecidable already for $\mathcal{ALC}$!
Semantic locality

We can approximate model-inseparability, exploiting that \( \mathcal{M} \) is a subset of \( \mathcal{O} \)

\[
\mathcal{M} \equiv_{\Sigma} \mathcal{O}
\]

\[\updownarrow\]

every \( \mathcal{I} \models \mathcal{M} \) can be extended to \( \mathcal{J} \models \mathcal{O} \) with \( \mathcal{I}\mid_{\Sigma} = \mathcal{J}\mid_{\Sigma} \)

\[\uparrow\]

every \( \mathcal{I} \models \mathcal{M} \) can be extended to \( \mathcal{J} \models \mathcal{O} \) with \( \mathcal{I}\mid_{\Sigma} = \mathcal{J}\mid_{\Sigma} \)

and \( \forall X \notin \Sigma : X^J = \emptyset \)

\[\updownarrow\]

every \( \alpha \in \mathcal{O} \setminus \mathcal{M} \) is semantically local w.r.t. \( \Sigma \cup \text{sig}(\mathcal{M}) \): \( \alpha \), with all terms not in \( \Sigma \cup \text{sig}(\mathcal{M}) \) replaced by \( \bot \), is a tautology
From semantic to syntactic locality

- Semantic locality involves tautology check
  \( \sim \) can be tested using a reasoner
  \( \sim \) has the same complexity as standard reasoning

- A syntactic approximation that can be tested in poly-time:
  syntactic locality
  (describes “obviously” sem. local axioms via a grammar)

- Both notions lead to modules that are
  - \((\Sigma \cup \text{sig}(\mathcal{M}))\)-inseparable from \(\mathcal{O}\)
  - not necessarily minimal
Module extraction with locality

Module extraction algorithm:

- $\mathcal{M} \leftarrow \emptyset$
- While $\alpha$ not local w.r.t. $\Sigma \cup \text{sig}(\mathcal{M})$, for some $\alpha \in \mathcal{O} \setminus \mathcal{M}$, do $\mathcal{M} \leftarrow \mathcal{M} \cup \{\alpha\}$
- Output $\mathcal{M}$

Variations:

- this notion: (semantic/syntactic) $\perp$-module
- dual notion: (semantic/syntactic) $\top$-module
- smaller modules by nesting $\top$- and $\perp$-module extraction: $\top \perp^*$-modules
Summary locality-based modules

Locality-based modules . . .

• are “good approximations” of minimal modules because they guarantee $\mathcal{M} \equiv \sum \mathcal{O}$

• are not necessarily minimal (but in practice often small enough)

• can be extracted in polynomial time (syntactic locality)

• are even self-contained:

$$\mathcal{M} \equiv \sum \cup \text{sig}(\mathcal{M}) \mathcal{O}$$

and depleting:

$$\mathcal{O} \setminus \mathcal{M} \equiv \sum \cup \text{sig}(\mathcal{M}) \emptyset$$

and thus unique

• contain all justifications for any $\alpha$ with $\text{sig}(\alpha) \subseteq \Sigma$

$\sim \sim \text{Cheap is cheerful! :) }$
And now . . .

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Summary on modularity

- Inseparability/coverage is a guarantee relevant (not only) for reuse
- Approximation of minimal covering modules via locality
- Modules based on syntactic locality can be extracted efficiently in logics up to $\mathcal{SROIQ}$ (OWL 2)
- Tool support for extracting modules:
  - http://owl.cs.manchester.ac.uk/modularity
  - http://owlapi.sourceforge.net/
- This line of research is rather new for DLs and ontology languages, and many questions are (half)open.
See also . . .

. . . slides from ESSLLI 2013 course “Modularity in Ontologies”:
http://www.informatik.uni-bremen.de/~ts/teaching/2013_modularity/

. . . the references at the end of this presentation

We’re almost done! :)
Appendix and References
An import/reuse scenario

“Borrow” knowledge from external ontologies

- Provides access to well-established knowledge
- Doesn’t require expertise in external disciplines

This scenario is well-understood and implemented.
A collaboration scenario

Collaborative ontology development

- Developers work (edit, classify) locally
- Extra care at re-combination
- Prescriptive/analytic behaviour

This approach is mostly understood, but not implemented yet.
Understanding and/or structuring an ontology

Compute the modular structure of an ontology

1,000,000 axioms
Understanding and/or structuring an ontology

Compute the modular structure of an ontology

This is work in progress.
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