Exercise 1 (20 points). Construct deterministic finite automata over the alphabet \( \Sigma = \{a, b\} \) which recognize the following languages:

1. The set of words with an even number of \( a \)'s.
2. The set of words which contain no substring \( aaa \).
3. The set of words in which every \( a \) is immediately followed by a \( b \).
4. The set of words which terminate in \( bb \).
5. The set of words whose third-to-last symbol is \( a \).

For the last language, also give a simpler NFA.

Exercise 2 (20 points). Prove or disprove: if the language \( L \) is regular, then so is the language \( \text{Reverse}(L) = \{w^R \mid w \in L\} \) (where \( w^R \) is the word \( w \) in reverse).

Exercise 3 (20 points). Let \( L \) be a language over \( \Sigma \). Then we define \( \text{DeleteOne}(L) \) to be the language \( \{xz \mid xyz \in L, y \in \Sigma\} \), i.e. the set of words from \( L \) with one symbol deleted. Prove that if \( L \) is regular, then so is \( \text{DeleteOne}(L) \).

Exercise 4 (20 points). Let \( L \) be a language. Then we define \( \text{Half}(L) \) as the set of strings \( x \) such that there is a string \( y \) with \( |x| = |y| \) and \( xy \in L \). Prove that if \( L \) is regular, then so is \( \text{Half}(L) \).

Exercise 5 (20 points). Show the following two languages are not regular. For the first, use the pumping lemma, and for the second, use Myhill-Nerode.

- \( \{www \mid w \in \{a, b\}^*\} \)
- \( \{a^ib^ja^k \mid k > i + j\} \)