Homework 1

Finite Automata on Infinite Words and Trees   Winter semester, 2009-2010

Note: Graphical representations of automata are accepted.

Exercise 1 (20 points)**.** Construct deterministic finite automata over the alphabet $\Sigma = \{a, b\}$ which recognize the following languages:

1. The set of words with an even number of a's.
2. The set of words which contain no substring aaa.
3. The set of words in which every a is immediately followed by a b.
4. The set of words which terminate in bb.
5. The set of words whose third-to-last symbol is a.

For the last language, also give a simpler NFA.

Exercise 2 (20 points)**.** Prove or disprove: if the language L is regular, then so is the language $\textsc{Reverse}(L) = \{w^R \mid w \in L\}$ (where w^R is the word w in reverse).

Exercise 3 (20 points)**.** Let L be a language over Σ. Then we define $\textsc{DeleteOne}(L)$ to be the language $\{xz \mid xyz \in L, y \in \Sigma\}$, i.e. the set of words from L with one symbol deleted. Prove that if L is regular, then so is $\textsc{DeleteOne}(L)$.

Exercise 4 (20 points)**.** Let L be a language. Then we define $\textsc{Half}(L)$ as the set of strings x such that there is a string y with $|x| = |y|$ and $xy \in L$. Prove that if L is regular, then so is $\textsc{Half}(L)$.

Exercise 5 (20 points)**.** Show the following two languages are not regular. For the first, use the pumping lemma, and for the second, use Myhill-Nerode.

- $\{www \mid w \in \{a, b\}^*\}$
- $\{a^i b^j a^k \mid k > i + j\}$