Homework 2
Finite Automata on Infinite Words and Trees  Winter semester, 2009-2010

Note: Graphical representations of automata are accepted.

Exercise 1 (25 points). Consider the alphabet $\Sigma = \{0, 1\}$. Construct Büchi automata which accept each of the following languages:

1. $\{w \mid$ the symbol 0 appears in $w$ exactly twice$\}$
2. $\{w \mid$ every 0 which appears in $w$ is followed immediately by 11$\}$
3. $\{w \mid w$ does not contain the substring 000 $\}$
4. $\{w \mid w$ contains finitely many substrings 11 $\}$
5. $\{w \mid w$ contains finitely many substrings 11 but infinitely many 1’s $\}$

Exercise 2 (25 points). Prove or disprove the following two statements:

1. The set of languages recognized by deterministic Büchi automata is closed under union.
2. The set of languages recognized by deterministic Büchi automata is closed under intersection.

Exercise 3 (25 points). Suppose that $L$ is a Büchi recognizable language over the alphabet $\Sigma = \Sigma_1 \times \Sigma_2 = \{(a, b) \mid a \in \Sigma_1, b \in \Sigma_2\}$

Prove that the languages $\text{PR}_1(L)$ and $\text{PR}_2(L)$ defined by

$\text{PR}_1 = \{u \in \Sigma_1^\omega \mid \exists v \in \Sigma_2^\omega \text{ such that } (u[0], v[0])(u[1], v[1])(u[2], v[2]) \ldots \in L\}$

$\text{PR}_2 = \{v \in \Sigma_2^\omega \mid \exists u \in \Sigma_1^\omega \text{ such that } (u[0], v[0])(u[1], v[1])(u[2], v[2]) \ldots \in L\}$

are both Büchi recognizable.

Exercise 4 (25 points). Given a Büchi recognizable language SELECT over alphabet $\{1, 2\}$ and two Büchi recognizable languages $L_1, L_2$ over alphabet $\{a, b\}$, prove that the following language FUSION is also Büchi recognizable:

$\text{FUSION} = \{\sigma \mid$ there exists $\alpha \in L_1$ and $\beta \in L_2$ and $\gamma \in \text{SELECT}$ such that $\sigma[i] = \alpha[i]$ if $\gamma[i] = 1$, and $\sigma[i] = \beta[i]$ if $\gamma[i] = 2\}$