Homework 3

Finite Automata on Infinite Words and Trees   Winter semester, 2009-2010

Note: Graphical representations of automata are accepted.

Exercise 1 (30 points)**.** Consider the alphabet $\Sigma = \{0, 1\}$. Construct Müller, Rabin, and Streett automata which accept each of the following languages:

1. $\{w \mid$ the symbol 0 appears in w exactly twice$\}$
2. $\{w \mid$ every 0 which appears in w is followed immediately by 11$\}$
3. $\{w \mid$ w contains finitely many 1's $\}$
4. $\{w \mid w$ contains finitely many substrings 11 $\}$
5. $\{w \mid$ either w contains only 1's, or w contains infinitely many 0's and infinitely many 1's$\}$

Exercise 2 (20 points)**.** Let $\Sigma = \{a, b, c\}$. Let $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$ be the Müller automata whose states and transitions are as pictured in Figure 1, and whose acceptance conditions are defined as follows: $\mathcal{F}_1 = \{\{2, 3\}\}$,

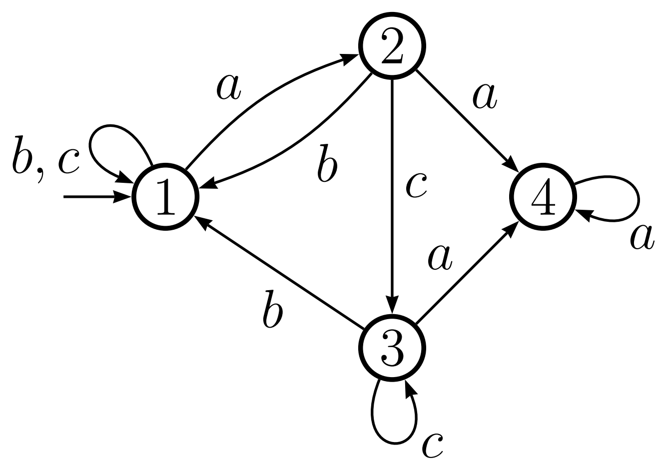

Figure 1: Graph representation of the automata in Exercise 2.

$\mathcal{F}_2 = \{\{1\}, \{1, 4\}\}$, $\mathcal{F}_3 = \{\{3\}, \{1, 2\}\}$, and $\mathcal{F}_4 = \{\{1, 2, 3\}\}$. Determine the languages $L(\mathcal{A}_1)$, $L(\mathcal{A}_2)$, $L(\mathcal{A}_3)$, and $L(\mathcal{A}_4)$. You may represent the languages by ω-regular expressions (e.g. $(aa+bb)^*(ab)^\omega$), but try to keep the expressions as simple as possible, and explain your reasoning.

Exercise 3 (15 points)**.** Prove that every Müller-recognizable language is a Boolean combination (union, intersection, complementation) of languages $\overrightarrow{W}_i$, where each W_i is a FA-recognizable language (of finite words).

Exercise 4 (25 points)**.** A parity automaton is a tuple $(Q, \Sigma, T, \{q_I\}, c : Q \rightarrow \mathbb{N})$, where T is a transition function. A run r of a parity automaton is successful if and only if $\max\{c(q) | q \in \mathsf{Inf}(r)\}$ is even.

1. Show how to construct Müller, Rabin, and Streett automata which recognize the same language as a given parity automaton. Explain briefly how your constructions work.
2. Prove that parity automata are closed under negation.

Exercise 5 (10 points)**.** *Review of complementation of finite word automata.* Use the power set construction to convert the following NFA into an equivalent DFA.