Exercise 1 (15 points). Let $\Sigma = \{a, b, c\}$. Define Büchi tree automata which recognize the following tree languages:

1. the set of all trees $t$ such that: for all positions $p$, if $t(p) = a$ then $t(p') = a$ for all $p \sqsubseteq p'$

2. the set of all trees in which every occurrence of $a$ on a path is immediately followed by $bb$

3. the set of all trees in which every path contains at least one $c$

Exercise 2 (15 points). Let $\Sigma = \{a, b\}$. Define Müller tree automata which recognize the following tree languages:

1. the set of all trees containing exactly two occurrences of $b$

2. the set of all trees all of whose paths have either finitely many $a$’s or contain only $a$’s

3. the set of all trees where for every position $p$, either $p1$ or $p2$ is labelled by $a$

Exercise 3 (10 points). Let $\Sigma = \{a, b\}$. Define parity tree automata which recognize the following tree languages:

1. the set of all trees which have a path $b^\omega$

2. the set of all trees whose paths all have finitely many occurrences of $bb$

Exercise 4 (15 points). Prove that every Müller-recognizable tree language is recognized by a Müller automaton which is complete and contains a single initial state.

Exercise 5 (35 points). The projection of a $\Sigma_1 \times \Sigma_2$-tree $\tau$ over the alphabet $\Sigma_1$ is the tree $pr_1(\tau)$ defined such that $pr_1(\tau)(v) = q_1$ when $\tau(v) = (q_1, q_2)$. The projection of a language $L$ of $\Sigma_1 \times \Sigma_2$-trees onto $\Sigma_1$ is the set $\{pr_1(\tau) \mid \tau \in L\}$ of $\Sigma_1$-trees.
Prove that the set of Müller-recognizable tree languages is closed under union, intersection, and projection.

Is the set of Büchi-recognizable tree languages also closed under these operations?

**Exercise 6** (10 points). Explain briefly why the transformation from Müller word automata into equivalent Büchi word automata from Chapter 2 cannot be used to transform Müller tree automata into equivalent Büchi tree automata.