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From syllogism to common sense . . .

Exercise Sheet 4: Propositional Logic

To be discussed on 8 December 2011

1. Prove that the following formulas are tautologies using the deduction theorem.

a) $(p \to q \to r) \to (p \to q) \to (p \to r)$

b) $(p \to q) \to (q \to r) \to (p \to r)$

2. Show the following derivabilities using natural deduction. Consider $\alpha \vee \beta$ as an abbreviation for $\neg(\neg\alpha \wedge \neg\beta)$ and $\alpha \to \beta$ as an abbreviation for $\neg(\alpha \wedge \neg\beta)$.

a) $\neg\neg p \ \vdash \ p$

b) $\alpha \vee \top \ \vdash \ \top$

c) $\alpha \to \beta \to \gamma \ \vdash \ \beta \to \alpha \to \gamma$

3. Prove the correctness of the following non-basic rule of natural deduction:

$$\frac{X \vdash \alpha}{X \vdash \alpha \vee \beta, \ \beta \vee \alpha}$$

That is, find a sequence $S_0, \ldots, S_n$ with $S_n = X \vdash \alpha \vee \beta$ where, in addition to the basic rules, each S_i may be $X \vdash \alpha$. As above, consider $\alpha \vee \beta$ as an abbreviation for $\neg(\neg\alpha \wedge \neg\beta)$.

4. Using the rule from the previous exercise, show via natural deduction that $\vdash \alpha \vee \neg\alpha$.