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From syllogism to common sense . . .

Exercise Sheet 7: Conditionals

To be discussed on 19 January 2012

Unfortunately, we made a mistake on Slides 27, 28 on conditionals, which we have corrected in the meantime. Due to this mistake, the original versions of Exercises 1, 2 are wrong. We have corrected them here, and added two exercises which show the combinations of "paradox avoided/not avoided" that were originally intended. For didactic reasons, you should proceed in the order 3a), 4b), 3b), 4a), 1, 2.

1. (Slide 28.) Show that the paradox of contraposition is *not* avoided

a) with strict implication and classical negation;

b) with intuitionistic implication and intuitionistic negation.

2. (Slide 28.) Show the same for *ex falso quodlibet*: $\models \Box\big((p \land \neg p) \to q\big)$ and (intuitionistically) $\models (p \land \neg p) \to q$.

3. (Slide 27.) Show that the paradox of an implication being true due to its antecedent being false

a) is avoided with strict impl. and classical negation: $\neg p \not\models \Box(p \to q)$;

b) is *not* avoided in intuitionistic logic, i.e., $\neg p \models p \to q$.

For a), it suffices to construct a Kripke frame (W, R), a valuation V and a world $w \in W$ such that $w \models \neg p$ but $w \not\models \Box(p \to q)$.

For b), show that, for any intutionistic Kripke frame M and any world x in M: whenever $M_x \models \neg p$, it holds that $M_x \models p \to q$.

4. (Slide 28.) Show the opposite for the the paradox "everything implies a true proposition":

a) $\models \Box\big(p \to (q \lor \neg q)\big)$ with strict impl. and classical negation

b) $\not\models p \to (q \lor \neg q)$ in intuitionistic logic

For a), show the following. For any Kripke frame (W, R), any valuation V that maps each of the propositional variables p, q to a set of worlds, and any world $w \in W$: $w \models \Box\big(p \to (q \lor \neg q)\big)$. Use the satisfaction condition that p ($\neg p$) is true in all worlds in $V(p)$ ($W \setminus V(p)$).

For b), construct an intuitionistic Kripke frame M and a world x in M with $M_x \not\models p \to (q \lor \neg q)$.