From syllogism to common sense:
a tour through the logical landscape

Categorical syllogisms

Mehul Bhatt Oliver Kutz Thomas Schneider

10 November 2011

Towards a theory of deduction

- Deduction = drawing of inferences (logical conclusions) from a set of statements
- Focus here on deductive arguments (DAs) such as:
  “If A is true and B is true, then C is true”
  A, B: premises, C: conclusions
- Goal: analyse structure of DAs, determine when a DA is valid
- In general: DA is valid if “its premise(s) assure the truth of the conclusion with necessity”
  i.e., premise(s) can’t be true without the conclusion being true
- Theory of deduction explains relations between premises and conclusion in valid arguments

Why are we dealing with syllogism today?

- Syllogism goes back to ARISTOTLE:
  (ARISTOTLE: 384–322 BCE, Greek/Macedonian philosopher, “The philosopher”, founder of logic)
- Want to explore history and foundations of modern logic

Do ask whenever you have questions regarding content or language.

Part I

Categorical propositions (CPs)
In this part . . .

1. Classes and CPs
2. Kinds of CPs
3. Important characteristics of CPs
4. Relations between CPs, and immediate inferences
5. Traditional and modern interpretations of CPs
6. Summary

And now . . .

1. Classes and CPs
2. Kinds of CPs
3. Important characteristics of CPs
4. Relations between CPs, and immediate inferences
5. Traditional and modern interpretations of CPs
6. Summary

### Classes and relations

**Class** of objects:
Collection of objects with common characteristics – a set!

**Relations** between classes

- **C wholly included in D**  \( C \subseteq D \)
  - e.g.  \( C \): all dogs,  \( D \): all mammals

- **C partially included in D**  \( C \cap D \neq \emptyset \)
  - e.g.  \( C \): all athletes,  \( D \): all females

- **C, D have no members in common** (mutual exclusion, disjointness)
  - e.g.  \( C \): all triangles,  \( D \): all circles

### Categorical propositions and deductive arguments

**Categorical proposition (CP):** states relations between classes

**Deductive argument:** sequence of CPs

Example:

*No athletes are vegetarians.*

*All football players are athletes. Therefore no football players are vegetarians.*
### Classes and CPs

<table>
<thead>
<tr>
<th>Classes and CPs</th>
<th>Kinds</th>
<th>Characteristics</th>
<th>Relations</th>
<th>Interpretations</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>And now ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. **Classes and CPs**
2. **Kinds of CPs**
3. **Important characteristics of CPs**
4. **Relations between CPs, and immediate inferences**
5. **Traditional and modern interpretations of CPs**
6. **Summary**

---

### A Universal affirmation

**All S is P.**

I.e., the whole of $S$ is included in $P$.

**A-propositions assert** the relation of class inclusion *universally*.

Representation as Venn diagramme

(John Venn, 1834–1923, English mathematician/logician, Hull)

This traditional interpretation assumes that $S$ is not empty.

---

### E Universal negation

**No S is P.**

I.e., the whole of $S$ is excluded from $P$.

**E-propositions deny** the relation of class inclusion *universally*.

Representation as Venn diagramme:

Note: this is *not* symmetric in $S, P$ in the traditional interpretation.
I Particular affirmation

Some $S$ is $P$.

I.e., $S$ and $P$ have at least one member in common.

I-propositions affirm the relation of class intersection partially, for some particular member.

Representation as Venn diagramme:

![Venn diagram representing the relation between S and P in I Particular affirmation](image)

Note: this is symmetric in $S$, $P$!

O Particular negation

Some $S$ is not $P$.

I.e., At least one member of $S$ is excluded from $P$.

I-propositions deny the relation of class intersection partially, for some particular member.

Representation as Venn diagramme:

![Venn diagram representing the relation between S and P in O Particular negation](image)

Examples and exercises

Identify subject and predicate terms.

- No athletes who have ever accepted pay for participating in sports are amateurs.
- All satellites that are currently in orbit less than 1000 miles high are very delicate devices that cost many thousands of euros to manufacture.
- Some historians are extremely gifted writers whose works read like first-rate novels.
And now . . .

1. Classes and CPs
2. Kinds of CPs
3. Important characteristics of CPs
4. Relations between CPs, and immediate inferences
5. Traditional and modern interpretations of CPs
6. Summary

Distribution

Does a proposition make a statement about all members of S or P?

- **A** All S is P.
  - S is distributed: all members of S are included in P
  - P is not

- **E** No S is P.
  - S is distributed: all members of S are excluded from P
  - P is distributed: all members of P are excluded from S

- **I** Some S is P.
  - Neither S nor P are distributed.

- **O** Some S is not P.
  - P is distr.: all members of P excluded from a part of S
  - S is not

Quality and quantity

. . . lie in the names of the four types:

<table>
<thead>
<tr>
<th>Quality</th>
<th>Quantity</th>
<th>affirmative</th>
<th>negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>universal</td>
<td>All S is P.</td>
<td>Universal affirmation</td>
<td>No S is P.</td>
</tr>
<tr>
<td></td>
<td>Universal negation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>particular</td>
<td>Some S is P.</td>
<td>Particular affirmation</td>
<td>Some S is not P.</td>
</tr>
<tr>
<td></td>
<td>Particular negation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

~ General schema for propositions:

Quantifier (subject term) copula (predicate term)

All is (not)
No are (not)
Some were (not)
: will (not) be

Overview

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Quantity</th>
<th>Quality</th>
<th>Distributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>All S is P.</td>
<td>A</td>
<td>Universal</td>
<td>Affirmative</td>
</tr>
<tr>
<td>No S is P.</td>
<td>E</td>
<td>Universal</td>
<td>Negative</td>
</tr>
<tr>
<td>Some S is P.</td>
<td>I</td>
<td>Particular</td>
<td>Affirmative</td>
</tr>
<tr>
<td>Some S is not P.</td>
<td>O</td>
<td>Particular</td>
<td>Negative</td>
</tr>
</tbody>
</table>

- (Negative propositions distribute their predicate term; positive propositions don’t.)
- (Universal propositions distribute their subject term; particular propositions don’t.)
Identify quality, quantity, and contribution.

- Some presidential candidates will be sadly disappointed people.
- All those who died in Nazi concentration camps were victims of a cruel and irrational tyranny.
- Some recently identified unstable elements were not entirely accidental discoveries.

Contradictories and contraries

**Contradictories** are two props that are negations of each other, i.e., exactly one is true.

- A and O
- E and I

Contraries are 2 non-contradictory props that cannot both be true, i.e., if one is true, then the other is false.

("Werder will win the next game." ↔ "Werder will lose the next game.")

- A and E – only traditionally

Subcontraries and Subalternation

**Subcontraries**: 2 non-contradictory props that cannot both be false, i.e., if one is false, then the other is true.

("I’m at least as smart as you." ↔ “I’m at most as smart as you.”)

- I and O – only traditionally

A subaltern of a univ. CP is the partic. CP of the same quality, i.e., the univ. CP implies the partic. CP (only traditionally).

- I is the subalternate of A
- O is the subalternate of E
The square of opposition

All four relations in one diagram:

Can be used to draw

- immediate inferences
  - If A true, then E false, I true, O false.
  - If A false, then O true. (E, I undetermined)
  - If I true, then E false, (A, O undetermined)

- mediate inferences from a set of premises: syllogisms...

Further immediate inferences

... are drawn without using the square of opposition

Conversion exchanges subject and predicate term in a CP. Is only successful if the premise (traditionally) implies the conclusion.

<table>
<thead>
<tr>
<th>Premise</th>
<th>Successful?</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>A All S is P.</td>
<td>through limitation:</td>
<td>I Some S is P.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I Some S is P.</td>
<td>yes</td>
<td>E No P is S.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O Some S is not P.</td>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>

(Draw the Venn diagram to see why the direct conversion of A and O is not successful.)

Examples and exercises

Of the following sets of 4 propositions,

- assume that the first is true.
- assume that the first is false.

What can we conclude about the truth or falsehood of the others?

All successful executives are intelligent people.
No successful executives are intelligent people.
Some successful executives are intelligent people.
Some successful executives are not intelligent people.

Some college professors are not entertaining lecturers.
All college professors are entertaining lecturers.
No college professors are entertaining lecturers.
Some college professors are entertaining lecturers.

Further immediate inferences

Complements of S and P are "non-P" and "non-S"  
Think of set complements. Clearly, non-non-X = X.

Do not mistake complements with complementary terms:

- non-hero ≠ coward
- non-elephant ≠ mouse
- non-lecturer ≠ student

Obversion leads to a logically equivalent CP  
by changing the quality and replacing P with non-P.

<table>
<thead>
<tr>
<th>Premise</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>A All S is P.</td>
<td>E No S is non-P.</td>
</tr>
<tr>
<td>E No S is P.</td>
<td>A All S is non-P.</td>
</tr>
<tr>
<td>I Some S is P.</td>
<td>O Some S is non-non-P.</td>
</tr>
<tr>
<td>O Some S is not P.</td>
<td>I Some S is non-P.</td>
</tr>
</tbody>
</table>

(Draw the Venn diagram to see why the direct conversion of A and O is not successful.)
### Further immediate inferences

**Contraposition** replaces $S$ with non-$P$, and $P$ with non-$S$.

<table>
<thead>
<tr>
<th>Premise</th>
<th>Successful?</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong> All $S$ is $P$.</td>
<td>yes</td>
<td><strong>A</strong> All non-$P$ is non-$S$.</td>
</tr>
<tr>
<td><strong>E</strong> No $S$ is $P$.</td>
<td>through limitation:</td>
<td><strong>O</strong> Some $S$ is not $P$.</td>
</tr>
<tr>
<td></td>
<td>no</td>
<td><strong>O</strong> Some non-$P$ is not non-$S$.</td>
</tr>
<tr>
<td><strong>I</strong> Some $S$ is $P$.</td>
<td>yes</td>
<td><strong>O</strong> Some non-$P$ is not non-$S$.</td>
</tr>
<tr>
<td><strong>O</strong> Some $S$ is not $P$.</td>
<td>yes</td>
<td><strong>O</strong> Some non-$P$ is not non-$S$.</td>
</tr>
</tbody>
</table>

(Draw the Venn diagram to see why the direct contraposition of **E** and **I** is not successful.)

### Aristotelian interpretation and existential import

**Aristotelian interpretation** (until now called “traditional”) assumes: whenever we speak of the subject class $S$, it cannot be empty.

- $\sim$ **A** All $S$ is $P$. implies **I** Some $S$ is $P$.  
- **E** No $S$ is $P$. implies **O** Some $S$ is not $P$.

Propositions made under this assumption have existential import.

**Aristotelian interpretation** in Venn diagrams:

![Aristotelian interpretation](image)

### Boolean interpretation

(ARTHUR C. BOOLE, 1815–64, English mathematician, Cork/Ireland)

**Boolean interpretation** rejects existential import.

- **A** All $S$ is $P$. means: If there is such a thing as an $S$, it is always in $P$.  
- **E** No $S$ is $P$. means: If there is such a thing as an $S$, it is not in $P$.

**Boolean interpretation** in Venn diagrams:

![Boolean interpretation](image)

Modern logic adopts **Boolean interpretation**. Arguments relying on exist. import commit the **existential fallacy**.
The "modern" square of opposition

... admits less inferences:

\[ \begin{array}{c}
A \\
\downarrow \\
\text{Contraries} \\
O \\
\uparrow \\
E \\
\downarrow \\
\text{Contraries} \\
I \\
\end{array} \]

And now...

1. Classes and CPs
2. Kinds of CPs
3. Important characteristics of CPs
4. Relations between CPs, and immediate inferences
5. Traditional and modern interpretations of CPs
6. Summary

Summary

Categorical propositions
- relate classes of objects
- come in four types A, E, I, O
- have a quality, quantity, and distribution
- can be related in the square of opposition
- can be used to draw simple inferences
- can be interpreted with existential import (Aristotelian) or without (Boolean)

From now on, we'll interpret CPs in the Boolean way.

Part II

Categorical syllogisms (CSs)
Basic notions

Aim: more extended reasoning with CPs

**Syllogism:** deductive argument with 2 premises and 1 conclusion

**Categorical syllogism:**
- syllogism based on CPs
- deductive argument of 3 CPs
- all 3 CPs together contain 3 terms
- every term occurs in 2 propositions

Syllogisms are common, clear and easily testable. They are
one of the most beautiful and also one of the most
important made by the human mind.

(GOTTFRIED WILHELM LEIBNIZ, 1646–1716, German philosopher
and mathematician, Hannover)
Examples

Major term, minor term, middle term

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.

Mood of a CS

Mood of a CS is the pattern of types of its three CPs, in the order major premise – minor premise – conclusions

A  All artists are egotists.
I  Some artists are paupers.                      Mood All
I  Therefore some paupers are egotists.          \[ \rightarrow 4^3 = 64 \text{ moods} \]

Figure of a CS

Figure of a CS: combination of order of $S, M, P$ in the premises:

$$\begin{array}{c}
\text{No } P \text{ is } M \\
\text{Some } S \text{ is not } M \\
\therefore \text{All } S \text{ is } P
\end{array} \quad \text{has figure} \quad
\begin{array}{c}
P \rightarrow M \\
S \rightarrow M \\
\therefore S \rightarrow P
\end{array}$$

\[ \sim \rightarrow 4 \text{ figures:} \]

(1) \[ \frac{M}{S} \rightarrow P \]  (2) \[ \frac{P}{S} \rightarrow M \]  (3) \[ \frac{M}{S} \rightarrow M \]  (4) \[ \frac{P}{M} \rightarrow S \]

\[ \therefore S \rightarrow P \]  \[ \therefore S \rightarrow P \]  \[ \therefore S \rightarrow P \]  \[ \therefore S \rightarrow P \]

Formal nature of the syllogistic argument

There are only $4 \cdot 64 = 256$ possible forms of CSs.

Their validity can be exhaustively analysed and established.

Only a few will turn out to be valid.

Infinitely many (in-)valid syllogistic arguments can be obtained by replacing $S, M, P$ in a(n in-)valid CS with "real-world" class descriptions.
And now . . .

- Standard-form CSs
- Venn-diagramme technique for testing CSs
- Rules and fallacies
- The valid CSs
- Summary and outlook

Testing a form of CS for validity

... is very simple!

1. Draw three overlapping cycles for $S$, $P$, $M$:

2. Mark the premises according to their types as earlier.

   E.g.: AAA-1
   
   All $M$ is $P$.
   All $S$ is $M$.
   ∴ All $S$ is $P$.

3. Try to read off the conclusion without further marking.
   Syllagism type is valid if reading off was successful.

Example

What form does this syllogism have? Is it valid?

All dogs are mammals.
All cats are mammals.
Therefore all cats are dogs.

Two cautions

(1) Mark universal before particular premise.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.

(2) If a particular premise speaks about two nonempty regions, put the $x$ on the boundary of these regions.

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.
### Examples

**AEE-1**

<table>
<thead>
<tr>
<th>S</th>
<th>P</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>All M is P.</td>
<td>No S is M.</td>
<td>∴ No S is P.</td>
</tr>
</tbody>
</table>

Invalid: diagramme does not exclude S from P.

**EIO-4**

<table>
<thead>
<tr>
<th>S</th>
<th>P</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>No P is M.</td>
<td>Some M is S.</td>
<td>∴ Some S is not P.</td>
</tr>
</tbody>
</table>

Valid: diagramme gives a particular instance of S \ P.

---

**An alternative characterisation of validity of CSs**

... via rules that focus on the form of the syllogism

**Rule 1:** Avoid four terms.
- With ≥ 4 terms, it’s no syllogism at all
- Beware of equivocations!
  (two occurrences of the same word with different meanings)

And the Lord spake, saying, “First shalt thou take out the Holy Pin. Then, shalt thou count to three. No more. No less. Three shalt be the number thou shalt count, and the number of the counting shall be three. Four shalt thou not count, neither count thou two, excepting that thou then proceed to three. Five is right out. Once at the number three, being the third number to be reached, then, lobbeth thou thy Holy Hand Grenade of Antioch towards thy foe, who, being naughty in My sight, shall snuff it.”

(from “Monty Python and the Holy Grail”, 1975)

**Rule 2:** Distribute the middle term in at least one premise.
(One proposition must refer to all members of M.)

Example: All Russians were revolutionists.
All anarchists were revolutionists.
Therefore all anarchists were Russians.

Fallacy: middle term “revolutionists” doesn’t link S, P
- Russians are included in a part of revolutionists
- Anarchists are included in a part of revolutionists, possibly a different part!

Fallacy of the undistributed middle
Watch your distribution

Rule 3: Any term distributed in the conclusion must be distributed in the premises.
Intuition: if premises speak about some members of a class, we cannot conclude anything about all members of that class.
Example: All dogs are mammals.
No cats are dogs.
Therefore no cats are mammals.

Fallacy: "mammals" is distributed in the conclusion, but not in the major premise.

Fallacy of illicit process (here: illicit process of the major term)

Don’t turn neg into pos; don’t be so Aristotelian

Rule 5: If ≥ 1 premise is negative, the conclusion must be neg.
Example: No poets are accountants.
Some artists are poets.
Therefore some artists are accountants.

Fallacy of drawing an affirmative conclusion from a neg. premise

Rule 6: From two universal premises, no particular conclusion may be drawn.
Example: All household pets are domestic animals.
No unicorns are domestic animals.
Therefore some unicorns are not household pets.

Existential fallacy (which is not a fallacy in the Aristotelian interpretation)

Two negative premises are bad

Rule 4: Avoid two negative premises.
ô 2 negative premises
¬ → 2× class exclusion between S, M and between P, M
ô No power to enforce any relation between S, P
ô Try all nine possibilities in a Venn diagramme!

Fallacy of exclusive premises

And now . . .
The 15 valid forms of syllogisms

AAAA-1 Barbara Datisi
EAE-1 Celarent EAE-4 Camenes
AII-1 Darri EIO-4 Fresison
EIO-1 Ferio OAO-4 Bokardo
AEE-2 Camestres AEE-4 Camenes
EAE-2 Cesare IAI-4 Dimaris
AEO-2 Baroko EIO-4 Fresison
EIO-2 Festino

Summary

Categorical syllogisms are deductive arguments consisting of 3 CPs
require a certain amount of interaction between the terms in their CPs
come in 4 figures and 64 moods
can be tested for validity using Venn diagrams or rules/fallacies

There are 15 valid forms of syllogisms in Boolean interpretation,
24 in Aristotelian interpretation

And now...

Standard-form CSs

Venn-diagramme technique for testing CSs

Rules and fallacies

The valid CSs

Summary and outlook

Contents is taken from Chapters 5, 6 of
I. Copi, C. Cohen, K. McMahon:

SUUB Magazin 02 E 2115
Link to available copies

Chapter 7:
transform arguments of everyday speech into syllogistic form,
possible difficulties

http://www.theotherscience.com/syllogism-machine
Try with examples from Pages 41, 47, 53, 55
Contents is taken from Chapters 5, 6 of
I. Copi, C. Cohen, K. McMahon:
SUUB Magazin 02 E 2115
Link to available copies

Chapter 7:
transform arguments of everyday speech into syllogistic form,
possible difficulties

Thank you.