Part I

What happened so far?
What happened so far?

## Categorical propositions (CPs)

... state relations between classes:

<table>
<thead>
<tr>
<th>Example</th>
<th>General form</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>All politicians are liars.</td>
<td>All $S$ is $P$.</td>
<td><strong>A</strong> Universal affirmation</td>
</tr>
<tr>
<td>No politicians are liars.</td>
<td>No $S$ is $P$.</td>
<td><strong>E</strong> Universal negation</td>
</tr>
<tr>
<td>Some politicians are liars.</td>
<td>Some $S$ is $P$.</td>
<td><strong>I</strong> Particular affirmation</td>
</tr>
<tr>
<td>Some politicians are not liars.</td>
<td>Some $S$ is not $P$.</td>
<td><strong>O</strong> Particular negation</td>
</tr>
</tbody>
</table>

... can be interpreted via Venn diagrams:

- All $S$ is $P$.
- No $S$ is $P$.
- Some $S$ is $P$.
- Some $S$ is not $P$. 

[Diagram images of Venn diagrams for each case]
Categorical propositions (CPs)

... have a quantity, quality and distribution:

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Quantity</th>
<th>Quality</th>
<th>Distributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>All $S$ is $P$.</td>
<td>A</td>
<td>Affirmative</td>
<td>$S$</td>
</tr>
<tr>
<td>No $S$ is $P$.</td>
<td>E</td>
<td>Negative</td>
<td>$S, P$</td>
</tr>
<tr>
<td>Some $S$ is $P$.</td>
<td>I</td>
<td>Affirmative</td>
<td>—</td>
</tr>
<tr>
<td>Some $S$ is not $P$.</td>
<td>O</td>
<td>Negative</td>
<td>$P$</td>
</tr>
</tbody>
</table>

Distribution: “Does the proposition make a statement about all members of $S$ or $P$?”
Immediate inferences

... via square of opposition:

![Square of Opposition Diagram]

Further inferences:
- Conversion ($S \leftrightarrow P, P \leftrightarrow S$; not always successful)
- Obversion (Change quality, $P \leftrightarrow non-P$)
- Contraposition ($S \leftrightarrow non-P, P \leftrightarrow non-S$; not always succ.)
Aristotelian versus Boolean interpretation

Aristotelian interpretation assumes existential import: 
$S$ is nonempty

Boolean interpretation rejects existential import: 
in $A$ and $E$, $S$ may be empty
Part II

Categorical syllogisms (CSs)
In this part . . .

2 Standard-form CSs

3 Venn-diagramme technique for testing CSs

4 Rules and fallacies

5 The valid CSs

6 Summary and outlook
And now . . .

2 Standard-form CSs

3 Venn-diagramme technique for testing CSs

4 Rules and fallacies

5 The valid CSs

6 Summary and outlook
### Basic notions

**Aim:** more extended reasoning with CPs

**Syllogism:** deductive argument with 2 premises and 1 conclusion

**Categorical syllogism:**
- syllogism based on CPs
- deductive argument of 3 CPs
- all 3 CPs together contain 3 terms
- every term occurs in 2 propositions

Syllogisms are common, clear and easily testable. They are *one of the most beautiful and also one of the most important made by the human mind.*

*(GOTTFRIED WILHELM LEIBNIZ, 1646–1716, German philosopher and mathematician, Hannover)*
Standard-form CSs

1. Premises and conclusion are standard CPs (A, E, I, O)
2. CPs are arranged in standard order:

\[
\begin{align*}
\cdots & S_1 \text{ is } \cdots P_1 \\
\cdots & S_2 \text{ is } \cdots P_2 \\
\therefore & \cdots S \text{ is } \cdots P
\end{align*}
\]

\(P\): major term, \(S\): minor term

Remember: 3 terms altogether, each in 2 propositions!
\(\sim S_1, S_2, P_1, P_2\) consist of \(P, S\) and a third term: the middle term

Major premise contains \(P, M\)
Minor premise contains \(S, M\)
Examples

Major term, minor term, middle term

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.
Mood of a CS is the pattern of types of its three CPs, in the order major premise – minor premise – conclusions

A  All artists are egotists.
I  Some artists are paupers.               Mood All
I  Therefore some paupers are egotists.

\[ 4^3 = 64 \text{ moods} \]
Figure of a CS: combination of order of $S$, $M$, $P$ in the premises:

\[
\begin{align*}
\text{No } P & \text{ is } M \\
\text{Some } S & \text{ is not } M \quad \text{has figure} \quad P-M \\
\therefore \quad \text{All } S & \text{ is } P \quad S-M \\
\therefore \quad S & \text{ is } P \\
\end{align*}
\]

$\sim \sim$ 4 figures:

(1) $M-P$ 
\[
\begin{align*}
M-P \\
S-M \\
\therefore \quad S & \text{ is } P \\
\end{align*}
\]

(2) $P-M$ 
\[
\begin{align*}
P-M \\
S-M \\
\therefore \quad S & \text{ is } P \\
\end{align*}
\]

(3) $M-P$ 
\[
\begin{align*}
M-P \\
M-S \\
\therefore \quad S & \text{ is } P \\
\end{align*}
\]

(4) $P-M$ 
\[
\begin{align*}
P-M \\
M-S \\
\therefore \quad S & \text{ is } P \\
\end{align*}
\]
There are only $4 \cdot 64 = 256$ possible \textit{forms} of CSs.

Their validity can be exhaustively analysed and established.

Only a few will turn out to be valid.

Infinitely many (in-)valid syllogistic arguments can be obtained by replacing $S, M, P$ in a(n in-)valid CS with “real-world” class descriptions.
And now . . .

2 Standard-form CSs

3 Venn-diagram technique for testing CSs

4 Rules and fallacies

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6 Summary and outlook
Testing a form of CS for validity

... is very simple!

1. Draw three overlapping cycles for $S$, $P$, $M$:

2. Mark the premises according to their types as earlier.
   E.g.: **AAA-1**
   
   All $M$ is $P$.
   All $S$ is $M$.
   $\therefore$ All $S$ is $P$.

3. Try to read off the conclusion without further marking.
   Syllogism type is valid iff reading off was successful.
Example

What form does this syllogism have? Is it valid?

All dogs are mammals.
All cats are mammals.
Therefore all cats are dogs.
Two cautions

(1) Mark universal before particular premise.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.

(2) If a particular premise speaks about two nonempty regions, put the x on the boundary of these regions.

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.
Examples

**AEE-1**

- All *M* is *P*.
- No *S* is *M*.

\[ \therefore \text{No } S \text{ is } P. \]

Invalid: diagramme does not exclude *S* from *P*.

**EIO-4**

- No *P* is *M*.
- Some *M* is *S*.

\[ \therefore \text{Some } S \text{ is not } P. \]

Valid: diagramme gives a particular instance of *S \ M*.
<table>
<thead>
<tr>
<th></th>
<th>Standard-form CSs</th>
<th>Venn-diagramme technique</th>
<th>Rules and fallacies</th>
<th>The valid CSs</th>
<th>Summary and outlook</th>
</tr>
</thead>
</table>

And now ...
An alternative characterisation of validity of CSs

... via rules that focus on the form of the syllogism

**Rule 1: Avoid four terms.**
- With $\geq 4$ terms, it's no syllogism at all
- Beware of equivocations!
  (two occurrences of the same word with different meanings)

*And the Lord spake, saying, “First shalt thou take out the Holy Pin. Then, shalt thou count to three. No more. No less. Three shalt be the number thou shalt count, and the number of the counting shall be three. Four shalt thou not count, neither count thou two, excepting that thou then proceed to three. Five is right out. Once at the number three, being the third number to be reached, then, lobbest thou thy Holy Hand Grenade of Antioch towards thy foe, who, being naughty in My sight, shall snuff it.”*

(from “Monty Python and the Holy Grail”, 1975)
Rule 2: Distribute the middle term in at least one premise.

(One proposition must refer to **all** members of \( M \).)

Example: All Russians were revolutionists.
All anarchists were revolutionists.
Therefore all anarchists were Russians.

**Fallacy:** middle term “revolutionists” doesn’t link \( S, P \)
- Russians are included in a part of revolutionists
- Anarchists are included in a part of revolutionists, possibly a different part!

**Fallacy of the undistributed middle**
Rule 3: Any term distributed in the conclusion must be distributed in the premises.

Intuition: if premises speak about some members of a class, we cannot conclude anything about all members of that class.

Example: All dogs are mammals.
No cats are dogs.
Therefore no cats are mammals.

Fallacy: “mammals” is distributed in the conclusion, but not in the major premise.

Fallacy of illicit process (here: illicit process of the major term)
Rule 4: Avoid two negative premises.

- 2 negative premises
  \[ \Rightarrow 2 \times \text{class exclusion between } S, M \text{ and between } P, M \]
- No power to enforce any relation between \( S, P \)
- Try all nine possibilities in a Venn diagramme!

Example: No artists are accountants.
Some poets are not accountants.
Therefore some poets are not artists.

Fallacy of exclusive premises
Rule 5: If \( \geq 1 \) premise is negative, the conclusion must be neg.

- Affirmative conclusion \( \hat{=} \) one of \( S, P \) is (wholly or partly) contained in the other.
- Can only be inferred if premises assert existence of \( M \) which contains one of \( S, P \) and is contained in the other
- Class inclusion only via affirmative propositions

Example: No poets are accountants.
Some artists are poets.
Therefore some artists are accountants.

Fallacy of drawing an affirmative conclusion from a neg. premise
Don’t be so Aristotelian

Rule 6: From two universal premises, no particular conclusion may be drawn.

Example: All household pets are domestic animals.
No unicorns are domestic animals.
Therefore some unicorns are not household pets.

Existential fallacy (not a fallacy in the Aristotelian interpretation)
And now . . .

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The 15 valid forms of syllogisms

<table>
<thead>
<tr>
<th>AAA-1</th>
<th>EAE-1</th>
<th>AII-1</th>
<th>EIO-1</th>
<th>AEE-2</th>
<th>EAE-2</th>
<th>AOO-2</th>
<th>EIO-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barbara</td>
<td>Celarent</td>
<td>Darii</td>
<td>Ferio</td>
<td>Camestres</td>
<td>Cesare</td>
<td>Baroko</td>
<td>Festino</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>AII-3</th>
<th>IAI-3</th>
<th>EIO-3</th>
<th>OAO-3</th>
<th>AEE-4</th>
<th>IAI-4</th>
<th>EIO-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Datisi</td>
<td>Disamis</td>
<td>Ferison</td>
<td>Bokardo</td>
<td>Camenes</td>
<td>Dimaris</td>
<td>Fresison</td>
</tr>
</tbody>
</table>
And now . . .

2 Standard-form CSs

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Summary

Categorical syllogisms ...

- are deductive arguments consisting of 3 CPs
- require a certain amount of interaction between the terms in their CPs
- come in 4 figures and 64 moods
- can be tested for validity using Venn diagrammes or rules/fallacies

There are 15 valid forms of syllogisms in Boolean interpretation, 24 in Aristotelian interpretation

It’s almost play time:
http://www.theotherscience.com/syllogism-machine

Try with examples from Pages

Thomas Schneider
Literature and outlook

Contents is taken from Chapters 5, 6 of

I. Copi, C. Cohen, K. McMahon:  

SUUB Magazin 02 E 2115
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Chapter 7:  
transform arguments of everyday speech into syllogistic form,  
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Thank you.