From syllogism to common sense: a tour through the logical landscape

Categorical syllogisms – Part 2

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Part I

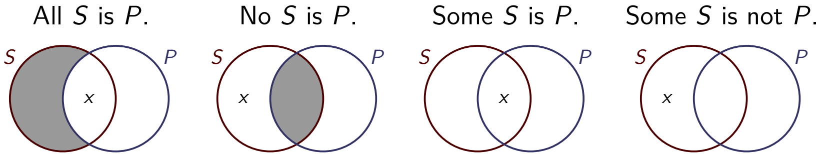
What happened so far?

Categorical propositions (CPs)

. . . state relations between classes:

Example	General form	Name
All policitians are liars.	All S is P.	**A** Universal affirmation
No policitians are liars.	No S is P.	**E** Universal negation
Some policitians are liars.	Some S is P.	**I** Particular affirmation
Some policitians are not liars.	Some S is not P.	**O** Particular negation

. . . can be interpreted via Venn diagrammes:



Categorical propositions (CPs)

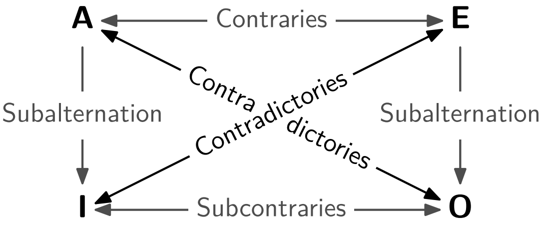
... have a quantity, quality and distribution:

Proposition		Quantity	Quality	Distributes
All S is P.	**A**	Universal	Affirmative	S
No S is P.	**E**	Universal	Negative	S, P
Some S is P.	**I**	Particular	Affirmative	—
Some S is not P.	**O**	Particular	Negative	P

Distribution: "Does the proposition make a statement about all members of S or P?"

Immediate inferences

. . . via square of opposition:



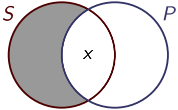
Further inferences:

- Conversion ($S \mapsto P$, $P \mapsto S$; not always successful)
- Obversion (Change quality, $P \mapsto$ non-P)
- Contraposition ($S \mapsto$ non-P, $P \mapsto$ non-S; not always succ.)

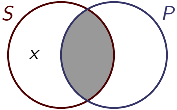
Aristotelian versus Boolean interpretation

Aristotelian interpretation assumes existential import:
S is nonempty

A All S is P.

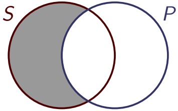


E No S is P.

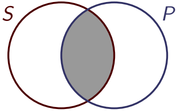


Boolean interpretation rejects existential import:
in **A** and **E**, S may be empty

A All S is P.



E No S is P.



Part II

Categorical syllogisms (CSs)

In this part . . .

And now . . .

Basic notions

Aim: more extended reasoning with CPs

Syllogism: deductive argument with 2 premises and 1 conclusion

Categorical syllogism:

- syllogism based on CPs
- deductive argument of 3 CPs
- all 3 CPs together contain 3 terms
- every term occurs in 2 propositions

Syllogisms are common, clear and easily testable. They are

> *one of the most beautiful and also one of the most important made by the human mind.*

(Gottfried Wilhelm Leibniz, 1646–1716, German philosopher and mathematician, Hannover)

Standard-form CSs

1. Premises and conclusion are standard CPs (**A**, **E**, **I**, **O**)
2. CPs are arranged in standard order:

$$\begin{array}{ll} & \ldots\ S_1 \text{ is } \ldots\ P_1 \\ & \ldots\ S_2 \text{ is } \ldots\ P_2 \\ \hline \therefore & \ldots\ S \text{ is } \ldots\ P \end{array}$$

P: major term, S: minor term

Remember: 3 terms altogether, each in 2 propositions!
$\rightsquigarrow$ S_1, S_2, P_1, P_2 consist of P, S and a third term: the middle term

Major premise contains P, M
Minor premise contains S, M

Examples

Major term, minor term, middle term

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.

Mood of a CS

Mood of a CS is the pattern of types of its three CPs,
in the order major premise – minor premise – conclusions

A	All artists are egotists.	
I	Some artists are paupers.	Mood **AII**
I	Therefore some paupers are egotists.	

$\leadsto 4^3 = 64$ moods

Figure of a CS

Figure of a CS: combination of order of S, M, P in the premises:

$$\begin{array}{ll} & \text{No } P \text{ is } M \\ & \text{Some } S \text{ is not } M \\ \hline \therefore & \text{All } S \text{ is } P \end{array} \quad \text{has figure} \quad \begin{array}{r} P\text{–}M \\ S\text{–}M \\ \hline \therefore S\text{–}P \end{array}$$

$\rightsquigarrow$ 4 figures:

$$(1)\ \begin{array}{r} M\text{–}P \\ S\text{–}M \\ \hline \therefore S\text{–}P \end{array} \qquad (2)\ \begin{array}{r} P\text{–}M \\ S\text{–}M \\ \hline \therefore S\text{–}P \end{array} \qquad (3)\ \begin{array}{r} M\text{–}P \\ M\text{–}S \\ \hline \therefore S\text{–}P \end{array} \qquad (4)\ \begin{array}{r} P\text{–}M \\ M\text{–}S \\ \hline \therefore S\text{–}P \end{array}$$

Formal nature of the syllogistic argument

There are only $4 \cdot 64 = 256$ possible *forms* of CSs.

Their validity can be exhaustively analysed and established.

Only a few will turn out to be valid.

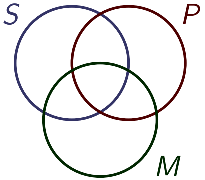
Infinitely many (in-)valid syllogistic arguments can be obtained by replacing S, M, P in a(n in-)valid CS with "real-world" class descriptions.

And now . . .

Testing a form of CS for validity

... is very simple!

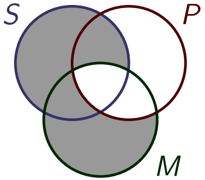
1. Draw three overlapping cycles for S, P, M:



2. Mark the premises according to their types as earlier.

E.g.: **AAA**-1

$$\begin{array}{l} \text{All } M \text{ is } P. \\ \text{All } S \text{ is } M. \\ \hline \therefore \text{All } S \text{ is } P. \end{array}$$



3. Try to read off the conclusion without further marking.
Syllogism type is valid iff reading off was successful.

Example

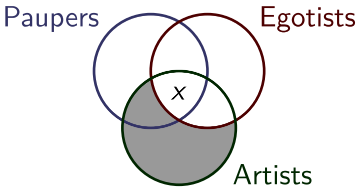
What form does this syllogism have? Is it valid?

All dogs are mammals.
All cats are mammals.
Therefore all cats are dogs.

Two cautions

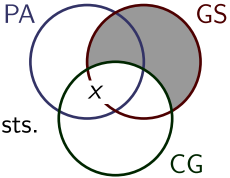
(1) Mark universal before particular premise.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.



(2) If a particular premise speaks about two nonempty regions, put the x on the boundary of these regions.

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

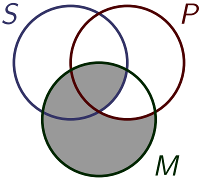


Examples

AEE-1

All M is P.
No S is M.
$\therefore$ No S is P.

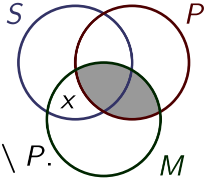
Invalid: diagramme does not exclude S from P.



EIO-4

No P is M.
Some M is S.
$\therefore$ Some S is not P.

Valid: diagramme gives a particular instance of $S \setminus P$.



And now . . .

An alternative characterisation of validity of CSs

... via rules that focus on the *form* of the syllogism

Rule 1: Avoid four terms.

- With $\geqslant 4$ terms, it's no syllogism at all
- Beware of equivocations!
  (two occurrences of the same word with different meanings)

  And the Lord spake, saying, "First shalt thou take out the Holy Pin. Then, shalt thou count to three. No more. No less. Three shalt be the number thou shalt count, and the number of the counting shall be three. Four shalt thou not count, neither count thou two, excepting that thou then proceed to three. Five is right out. Once at the number three, being the third number to be reached, then, lobbest thou thy Holy Hand Grenade of Antioch towards thy foe, who, being naughty in My sight, shall snuff it."

  (from "Monty Python and the Holy Grail", 1975)

Distribute your middle term

Rule 2: Distribute the middle term in at least one premise.

(One proposition must refer to *all* members of M.)

Example: All Russians were revolutionists.
All anarchists were revolutionists.
Therefore all anarchists were Russians.

Fallacy: middle term "revolutionists" doesn't link S, P

- Russians are included in a part of revolutionists
- Anarchists are included in a part of revolutionists, possibly a *different part!*

Fallacy of the undistributed middle

Watch your distribution

Rule 3: Any term distributed in the conclusion must be distributed in the premises.

Intuition: if premises speak about *some* members of a class,
we cannot conclude anything about *all* members of that class.

Example: All dogs are mammals.
No cats are dogs.
Therefore no cats are mammals.

Fallacy: "mammals" is distributed in the conclusion,
but not in the major premise.

Fallacy of illicit process (here: illict process of the major term)

Two negative premises are bad

Rule 4: Avoid two negative premises.

- 2 negative premises
  $\rightsquigarrow$ $2\times$ class exclusion between S, M and between P, M
- No power to enforce any relation between S, P
- Try all nine possibilities in a Venn diagramme!

Example: No artists are accountants.
Some poets are not accountants.
Therefore some poets are not artists.

Fallacy of exclusive premises

Don't turn neg into pos

Rule 5: If $\geqslant 1$ premise is negative, the conclusion must be neg.

- Affirmative conclusion $\hat{=}$ one of S, P is (wholly or partly) contained in the other.
- Can only be inferred if premises assert existence of M which contains one of S, P and is contained in the other
- Class inclusion only via affirmative propositions

Example: No poets are accountants.
Some artists are poets.
Therefore some artists are accountants.

Fallacy of drawing an affirmative conclusion from a neg. premise

Don't be so Aristotelian

Rule 6: From two universal premises, no particular conclusion may be drawn.

Example: All household pets are domestic animals.
No unicorns are domestic animals.
Therefore some unicorns are not household pets.

Existential fallacy (*not* a fallacy in the Aristotelian interpretation)

And now . . .

The 15 valid forms of syllogisms

AAA-1	B**a**rb**a**r**a**	**AII**-3	Datisi
EAE-1	C**e**l**a**r**e**nt	**IAI**-3	Disamis
AII-1	Darii	**EIO**-3	Ferison
EIO-1	Ferio	**OAO**-3	Bokardo
AEE-2	Camestres	**AEE**-4	Camenes
EAE-2	Cesare	**IAI**-4	Dimaris
AOO-2	Baroko	**EIO**-4	Fresison
EIO-2	Festino		

And now . . .

Summary

Categorical syllogisms . . .

- are deductive arguments consisting of 3 CPs
- require a certain amount of interaction between the terms in their CPs
- come in 4 figures and 64 moods
- can be tested for validity using Venn diagrammes or rules/fallacies

There are 15 valid forms of syllogisms in Boolean interpretation, 24 in Aristotelian interpretation

It's almost play time:
`http://www.theotherscience.com/syllogism-machine`

Try with examples from Pages ▸ 41 ▸ 47 ▸ 53 ▸ 55 ▸ 56

Literature and outlook

Contents is taken from Chapters 5, 6 of

I. Copi, C. Cohen, K. McMahon:
Introduction to Logic, 14th ed., Prentice Hall, 2011.

SUUB Magazin 02 E 2115
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Thank you.