From syllogism to common sense: a tour through the logical landscape

Categorical syllogisms – Part 2

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Categorical propositions (CPs)

... state relations between classes:

<table>
<thead>
<tr>
<th>Example</th>
<th>General form</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>All politicians are liars</td>
<td>All $S$ is $P$</td>
<td>A Universal affirmation</td>
</tr>
<tr>
<td>No politicians are liars</td>
<td>No $S$ is $P$</td>
<td>E Universal negation</td>
</tr>
<tr>
<td>Some politicians are liars</td>
<td>Some $S$ is $P$</td>
<td>I Particular affirmation</td>
</tr>
<tr>
<td>Some politicians are not liars</td>
<td>Some $S$ is not $P$</td>
<td>O Particular negation</td>
</tr>
</tbody>
</table>

... can be interpreted via Venn diagrams:

All $S$ is $P$.
No $S$ is $P$.
Some $S$ is $P$.
Some $S$ is not $P$.

Proposition Quantity Quality Distributes
All $S$ is $P$. A Universal Affirmative $S$
No $S$ is $P$. E Universal Negative $S, P$
Some $S$ is $P$. I Particular Affirmative $S, P$
Some $S$ is not $P$. O Particular Negative $P$

Distribution: “Does the proposition make a statement about all members of $S$ or $P$?”
What happened so far?

Immediate inferences

... via square of opposition:

- Contraries
- Contradictories
- Subalternation
- Subcontraries

Further inferences:
- Conversion ($S \rightarrow P, P \rightarrow S$; not always successful)
- Obversion (Change quality, $P \rightarrow \neg P$)
- Contraposition ($S \rightarrow \neg P, P \rightarrow \neg S$; not always succ.)

Aristotelian versus Boolean interpretation

Aristotelian interpretation assumes existential import:

- $S$ is nonempty

Boolean interpretation rejects existential import:

- in $A$ and $E$, $S$ may be empty

In this part . . .

Part II

Categorical syllogisms (CSs)
Standard-form CSs

Venn-diagramme technique

Rules and fallacies

The valid CSs

Summary and outlook

And now . . .

Standard-form CSs

Venn-diagramme technique for testing CSs

Rules and fallacies

The valid CSs

Summary and outlook

Basic notions

Aim: more extended reasoning with CPs

Syllogism: deductive argument with 2 premises and 1 conclusion

Categorical syllogism:

- syllogism based on CPs
- deductive argument of 3 CPs
- all 3 CPs together contain 3 terms
- every term occurs in 2 propositions

Syllogisms are common, clear and easily testable. They are

one of the most beautiful and also one of the most important made by the human mind.

(GOTTFRID WILHELM LEIBNIZ, 1646–1716, German philosopher and mathematician, Hannover)

Examples

Major term, minor term, middle term

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.
Mood of a CS is the pattern of types of its three CPs, in the order major premise – minor premise – conclusions

A  All artists are egotists.
I  Some artists are paupers.  Mood AII
I  Therefore some paupers are egotists.

\[ \sim 4^3 = 64 \text{ moods} \]

Formal nature of the syllogistic argument

There are only \(4 \cdot 64 = 256\) possible forms of CSs.

Their validity can be exhaustively analysed and established.

Only a few will turn out to be valid.

Infinitely many (in-)valid syllogistic arguments can be obtained by replacing \(S, M, P\) in a(n in-)valid CS with “real-world” class descriptions.
Testing a form of CS for validity

... is very simple!

1. Draw three overlapping cycles for $S$, $P$, $M$:

![Diagram](image)

2. Mark the premises according to their types as earlier.

E.g.: **AAA-1**

- All $M$ is $P$.
- All $S$ is $M$.
- Therefore all $S$ is $P$.

3. Try to read off the conclusion without further marking.

Syllogism type is valid iff reading off was successful.

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Two cautions

1. Mark universal before particular premise.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.

2. If a particular premise speaks about two nonempty regions, put the $x$ on the boundary of these regions.

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

---

Example

What form does this syllogism have? Is it valid?

All dogs are mammals.
All cats are mammals.
Therefore all cats are dogs.

---

Examples

**AEE-1**

- All $M$ is $P$.
- No $S$ is $M$.
- Therefore no $S$ is $P$.

Invalid: diagramme does not exclude $S$ from $P$.

**EIO-4**

- Some $M$ is $S$.
- Some professional athletes are college graduates.
- Therefore some professional athletes are great scientists.

Valid: diagramme gives a particular instance of $S \setminus P$. 
An alternative characterisation of validity of CSs

... via rules that focus on the form of the syllogism

Rule 1: Avoid four terms.
- With \( \geq 4 \) terms, it’s no syllogism at all
- Beware of equivocations!
  (two occurrences of the same word with different meanings)

> And the Lord spake, saying, “First shalt thou take out the Holy Pin. Then, shalt thou count to three. No more. No less. Three shalt be the number thou shalt count, and the number of the counting shall be three. Four shalt thou not count, neither count thou two, excepting that thou then proceed to three. Five is right out. Once at the number three, being the third number to be reached, then, lobbeth thou thy Holy Hand Grenade of Antioch towards thy foe, who, being naughty in My sight, shall sniff it.”

(from “Monty Python and the Holy Grail”, 1975)

Rule 2: Distribute the middle term in at least one premise.
(One proposition must refer to all members of \( M \).)

Example: All Russians were revolutionists.
All anarchists were revolutionists.
Therefore all anarchists were Russians.

Fallacy: middle term “revolutionists” doesn’t link \( S, P \)
- Russians are included in a part of revolutionists
- Anarchists are included in a part of revolutionists, possibly a different part!

Fallacy of the undistributed middle

Rule 3: Any term distributed in the conclusion must be distributed in the premises.

Intuition: if premises speak about some members of a class, we cannot conclude anything about all members of that class.

Example: All dogs are mammals.
No cats are dogs.
Therefore no cats are mammals.

Fallacy: “mammals” is distributed in the conclusion, but not in the major premise.

Fallacy of illicit process (here: illicit process of the major term)
Two negative premises are bad

Rule 4: Avoid two negative premises.

- 2 negative premises
  $\sim 2 \times$ class exclusion between $S, M$ and between $P, M$
- No power to enforce any relation between $S, P$
- Try all nine possibilities in a Venn diagramme!

Example: No artists are accountants.
Some poets are not accountants.
Therefore some poets are not artists.

Fallacy of exclusive premises

Rule 5: If $\geq 1$ premise is negative, the conclusion must be neg.

- Affirmative conclusion $\equiv$ one of $S, P$ is (wholly or partly) contained in the other.
- Can only be inferred if premises assert existence of $M$ which contains one of $S, P$ and is contained in the other
- Class inclusion only via affirmative propositions

Example: No poets are accountants.
Some artists are poets.
Therefore some artists are accountants.

Fallacy of drawing an affirmative conclusion from a neg. premise

Don’t be so Aristotelian

Rule 6: From two universal premises, no particular conclusion may be drawn.

Example: All household pets are domestic animals.
No unicorns are domestic animals.
Therefore some unicorns are not household pets.

Existential fallacy (not a fallacy in the Aristotelian interpretation)
The 15 valid forms of syllogisms

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<tr>
<td>AAA-1</td>
<td>AII-3</td>
<td>Datisi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EAE-1</td>
<td>IAII-3</td>
<td>Disamis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AII-1</td>
<td>EIO-3</td>
<td>Ferson</td>
<td></td>
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</tr>
<tr>
<td>EIO-1</td>
<td>OAO-3</td>
<td>Bokardo</td>
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<tr>
<td>AEE-2</td>
<td>Camstres</td>
<td>Camenes</td>
<td></td>
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</tr>
<tr>
<td>EAE-2</td>
<td>Cesare</td>
<td>Dimaris</td>
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<tr>
<td>AOO-2</td>
<td>Baroko</td>
<td>Fresison</td>
<td></td>
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</tr>
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<td>EIO-2</td>
<td>Festino</td>
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Summary

Categorical syllogisms . . .
- are deductive arguments consisting of 3 CPs
- require a certain amount of interaction between the terms in their CPs
- come in 4 figures and 64 moods
- can be tested for validity using Venn diagrams or rules/fallacies

There are 15 valid forms of syllogisms in Boolean interpretation, 24 in Aristotelian interpretation

It’s almost play time:
http://www.theotherscience.com/syllogism-machine
Try with examples from Pages 41, 47, 53, 55, 56

Contents is taken from Chapters 5, 6 of
I. Copi, C. Cohen, K. McMahon:
SUUB Magazin 02 E 2115
Link to available copies
Chapter 7:
transform arguments of everyday speech into syllogistic form, possible difficulties
Literature and outlook

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Thank you.