

Two-Dimensional Description Logics of Context

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Abstract. We introduce an extension of Description Logics (DLs) for representing and reasoning about contextualized knowledge. Our formalism is inspired by McCarthy’s theory of formalizing contexts and based on two-dimensional semantics, with one dimension representing a usual object domain and the other a domain of contexts. Additionally, it is equipped with a second DL language for describing the context domain. As a result, we obtain a family of *two-sorted, two-dimensional* combinations of pairs of DLs.

1 Introduction

Description Logics (DLs) provide a clear and broadly accepted paradigm for reasoning about terminological knowledge. Under the standard Kripkean semantics, a DL ontology forces a unique, global view on the represented world, in which the ontology axioms are interpreted as universally true. This philosophy is well-suited as long as everyone can share the same conceptual perspective on the domain or there is no need for considering alternative viewpoints. Alas, this is hardly ever the case since a domain can be modeled differently depending on the intended use of an ontology. Consequently, effective representation and reasoning about knowledge pertaining to such multiple, heterogenous viewpoints becomes the primary objective for many practical applications [1,2].

The challenges above resemble clearly those problems that originally inspired J. McCarthy to introduce a theory of formalizing contexts in knowledge representation systems, as a way of granting them more generality [3,4]. The gist of his proposal is to replace logical formulas φ , as the basic knowledge carriers, with assertions $ist(c, \varphi)$ stating that φ is true in c , where c denotes an abstract first-order entity called a *context*, which on its own can be described in a first-order language. For instance:

$$ist(c, Heart(a)) \wedge \mathbf{HumanAnatomy}(c)$$

states that the object a is a heart in some context described as ***HumanAnatomy***. Based on this foundation, the theory advocates complex models of knowledge which are able to properly account for the local, context-specific scope of the represented knowledge, while at the same time provide an expressive apparatus

for modeling semantic interoperability of contexts, i.e. generic rules guiding the information flow between different contexts.

The importance of contextualized knowledge in DLs has been generally acknowledged, nevertheless the framework is still not supported with a dedicated theory of handling context-dependent information. In this direction, the most commonly considered perspectives are restricted to global integration of local ontologies [5,6] or modeling levels of abstraction as subsets of models of a DL ontology [7,8]. The purpose of this paper is to introduce a novel extension of DLs for reasoning with contextualized knowledge. Our proposal is systematically derived from two formal roots. On the one hand, by resorting to McCarthy’s theory we ground our approach in a longstanding tradition of formalizing contexts in AI. On the other, we build on top of two-dimensional DLs [9], which provide us with well-understood formal foundations. In particular, we extend the standard DL semantics with a second modal dimension, representing a possibly infinite domain of contexts. Additionally, our logics are equipped with a second DL language for describing the context domain. This way we obtain a family of *two-sorted*, *two-dimensional* combinations of pairs of DLs for reasoning about contextualized knowledge.

This paper is the workshop version of [10] and [11]. It extends the work presented there by discussing thoroughly the motivation underlying the formal design of the introduced DLs of contexts. We also review a number of expressive fragments of these logics and report the corresponding complexity results obtained and proven in the two papers.

2 Overview and formal motivation

Since its introduction, McCarthy’s theory of formalizing contexts has inspired a significant body of work in AI studying implementations of the approach in a variety of formalisms and applications [12,13,14,4,1,15]. The great appeal of this theory stems from the simplicity of the three major postulates it is based on:

1. Contexts are formal objects. More precisely, a *context* is anything that can be denoted by a first-order term and used meaningfully in a statement of the form $ist(c, \varphi)$, saying that formula φ is true (*ist*) in context c [3,4,12].

2. Contexts have properties and can be described. As first-order objects, contexts can be in a natural way described in a first-order language [14,4]. This allows for addressing them generically through quantified formulas such as $\forall x(C(x) \rightarrow ist(x, \varphi))$, expressing that φ is true in every context of type C .

3. Contexts are organized in relational structures. In the commonsense reasoning, contextual assumptions are dynamically and directionally altered [15,12], thus contexts are often accessed from other contexts. Formally, this can be captured by allowing nestings of the form $ist(c, ist(d, \varphi))$.

The logics proposed in this paper originate as an attempt of adopting these principles in the framework of DLs. In the following paragraphs we discuss the

central design choices we made and the motivation behind them. We start from the basic semantic considerations on contexts and further trace their impact on the selection of specific logical languages.

The key to importing McCarthy’s theory into a knowledge representation framework is a faithful interpretation of his three postulates on the model-theoretic grounds of the framework. By doing so within the DL paradigm, we effectively commit ourselves to a specific sort of semantic structures that must be taken into account in order to express and interpret contextualized knowledge adequately. Figure 1 illustrates one such structure — a formal model of some application domain supporting multiple contexts of representation. As an intuitive example, consider here a formal description of a society of interconnected agents, each one sustaining his own viewpoint and focus on the represented world.

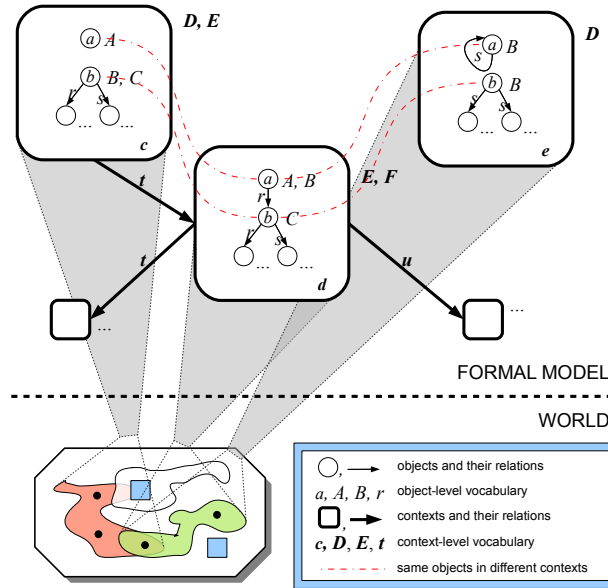


Fig. 1. A formal model of an application domain complying to McCarthy’s principles.

The model has two apparent levels. The *context-level* consists of context entities (postulate 1), which are possibly interlinked with accessibility relations (postulate 3) and described in a language containing individual names, concepts and relation names (postulate 2). For instance, context *c* is of type *D* and is related to *d* through the relation *t*. Intuitively, each context in the model can be seen as a box carrying a piece of the *object-level* representation. Clearly, instead of a unique global model of the object domain, we associate a single local model with every context. Naturally, these models might obviously differ from each other as each of them reflects a specific viewpoint on the object domain. Moreover, they might not necessarily cover the same fragment and aspects of

the application domain and not necessarily use the same fragment of the object language for describing it. For instance, objects a and b occur at the same time in contexts c, d, e , but in each of them they are described differently and remain in different relations to other objects.

The central insight emerging from this short analysis is that the semantic structures comprising the model theory of a reasonably expressive DL of context are inherently *two-dimensional*, with one dimension consisting of (domain) *objects* and the second — *contexts*.

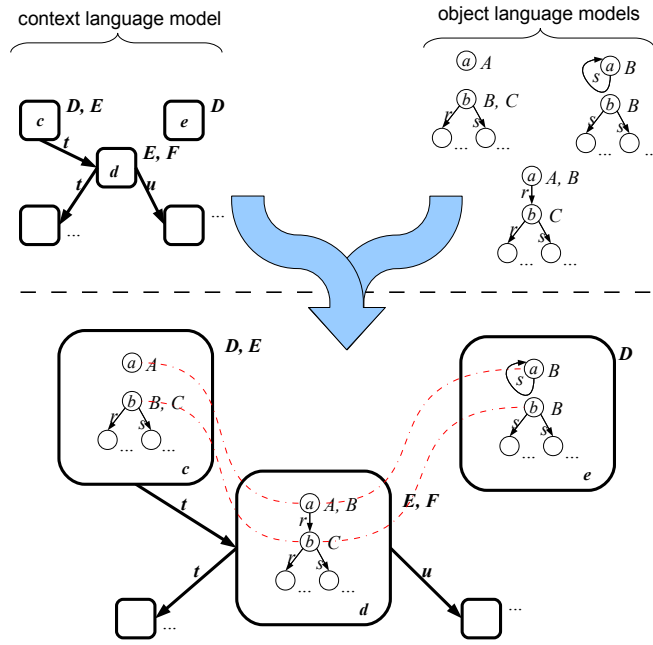


Fig. 2. Combining models of two DLs.

Once we have identified the main characteristics of the intended semantic structures, the next step is to find convenient languages for speaking about them, and constraining their possible properties. By the assumption taken in this work, DLs are suitable formalisms for representing the object-level knowledge. The key challenge is then to extend them with additional syntactic means that would facilitate accommodating the context-level information. A first crucial observation in this direction is that contexts and their relations, as pictured above, correspond to Kripke frames, with possible worlds interpreted as context entities. It is commonly known that such frames can be combined in a product-like fashion with the standard DL interpretations, giving rise to two-dimensional semantics for DLs with additional modal operators [16]. These operators are

typically intended for modeling the evolution of the object knowledge across the states of the second dimension, for instance time points, as in temporal DLs [17]. Although this approach seems in general very encouraging, the caveat is that it does not offer a direct methodology for describing the elements of the second dimension. More precisely, we can easily augment a DL language with modal ‘*contextualization*’ operators \diamond, \square for traversing the context dimension of the models and quantifying over implicit context objects, but it is not clear how to explicitly assert properties of the accessed contexts — an essential point for obtaining a fine-grained contextualization machinery.

As a solution, we employ a second DL language for describing the context dimension. Thus, we obtain a *two-sorted* language with each sort interpreted over the respective dimension. The two languages are suitably integrated on the syntactic and semantic level, so that their models can be eventually combined as presented in Figure 2. The style of combination remains fully compatible with that underlying two-dimensional DLs described above. In fact, we are able to show that, depending on the choice of the integration mechanism, our logics are proper extensions of the well-known $(\mathbf{K}_n)_\mathcal{L}$ or $\mathbf{S5}_\mathcal{L}$ [16].

In the following sections, we first recap the basic DL nomenclature, next we formally define the syntax and semantics of the proposed DLs of context and give an overview of their expressiveness–complexity characterization. Finally, we consider intended application scenarios for the framework.

3 Description Logics of Context

A DL language \mathcal{L} is specified by a vocabulary $\Sigma = (N_C, N_R, N_I)$, where N_C is a set of *concept names*, N_R a set of *role names* and N_I a set of *individual names*, and a number of constructors for composing complex expressions. In this paper, we focus on the well-known DLs \mathcal{EL} , \mathcal{ALC} and \mathcal{ALCO} [18,19] and assume the reader is familiar with those formalism and the basic notions concerning DLs.

A *Description Logic of Context* $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ consists of a DL context language \mathcal{L}_C , supporting context descriptions, and an object language \mathcal{L}_O equipped with context operators for representing object knowledge relative to contexts. We introduce two families of such DLs, characterized by different types of context operators.

Definition 1 ($\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -context language). *The context language of $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ is a DL language \mathcal{L}_C over the vocabulary $\Gamma = (M_C, M_R, M_I)$, with a designated subset $M_I^* \subseteq M_I$.*

The set M_I^* contains context names. Following some common-sense intuitions, we consider contexts only as a subset of the domain of the context language. Indeed, certain elements of this domain might carry no object knowledge at all, and instead, serve only as individuals referred to in context descriptions (cf. Figure 1). This is often the case in applications concerned with provenance of knowledge [2]. For instance, a context c , associated with a single knowledge source, might be there described with an axiom *hasAuthor*(c , *henry*), where *henry* is an individual related to c , but obviously not a context *per se*.

Definition 2 ($\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -object language). Let \mathcal{L}_O be a DL language over the vocabulary $\Sigma = (N_C, N_R, N_I)$. The object language of $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ is the smallest language containing \mathcal{L}_O and closed under the constructors of \mathcal{L}_O and one of the two types — \mathfrak{F}_1 resp. \mathfrak{F}_2 — of concept-forming operators:

$$\langle \mathbf{r}.\mathbf{C} \rangle D \mid [\mathbf{r}.\mathbf{C}] D \quad (\mathfrak{F}_1)$$

$$\langle \mathbf{C} \rangle D \mid [\mathbf{C}] D \quad (\mathfrak{F}_2)$$

where \mathbf{C} and \mathbf{r} are a concept and a role of the context language and D is a concept of the object language.

Intuitively, the concept $\langle \mathbf{r}.\mathbf{C} \rangle D$ denotes all objects which are D in *some* context of type \mathbf{C} accessible from the current one through \mathbf{r} . Similarly, $[\mathbf{r}.\mathbf{C}] D$ denotes all objects which are D in *every* such context. In the case of the operators in \mathfrak{F}_2 , the concept $\langle \mathbf{C} \rangle D$ denotes all objects which are D in *some* context of type \mathbf{C} , whereas $[\mathbf{C}] D$ — all objects which are D in *every* such context. For example, $\langle \mathbf{neighbor}.\mathbf{Country} \rangle \mathbf{Citizen}$, refers to the concept $\mathbf{Citizen}$ in some context of type $\mathbf{Country}$ accessible through the $\mathbf{neighbor}$ relation from the current context. Analogically, $\langle \mathbf{HumanAnatomy} \rangle \mathbf{Heart}$ refers to the concept \mathbf{Heart} in some context of $\mathbf{HumanAnatomy}$.

Definition 3 ($\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -knowledge base). A $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -knowledge base (CKB) is a pair $\mathcal{K} = (\mathcal{C}, \mathcal{O})$, where \mathcal{C} is a set of axioms of the context language in any of the forms (\dagger) , and \mathcal{O} is a set of formulas of the form:

$$\mathbf{c} : \varphi \mid \mathbf{C} : \varphi$$

where φ is an axiom of the object language (a GCI or a concept/role assertion), $\mathbf{c} \in M_I^*$ and \mathbf{C} is a concept of the context language.

A formula $\mathbf{c} : \varphi$ states that the axiom φ holds in the context denoted by the name \mathbf{c} . Note that this corresponds directly to McCarthy's $ist(\mathbf{c}, \varphi)$. Axioms of the form $\mathbf{C} : \varphi$ assert the truth of φ in all contexts of type \mathbf{C} . For example, the formula $\mathbf{Country} : \langle \mathbf{neighbor}.\mathbf{Country} \rangle \mathbf{Citizen} \sqsubseteq \mathbf{NoVisaRequirement}$ states that in every country, the citizens of its neighbor countries do not require visas.

The semantics is given through $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -interpretations and $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -models, which combine the interpretations of \mathcal{L}_C with those of \mathcal{L}_O . We assume the semantics of \mathcal{EL} , \mathcal{ALC} and \mathcal{ALCO} to be defined in the standard way[18,19]. As explained before, the (possibly infinite) domain of contexts \mathfrak{C} is subsumed by the entire interpretation domain of the context language Θ . For technical reasons, we assume a constant object domain Δ for all contexts. This assumption, though often impractical, grants greater generality to the complexity results and can be easily relaxed to the varying domain case.

Definition 4 ($\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -interpretations). A $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -interpretation is a tuple $\mathfrak{M} = (\Theta, \mathfrak{C}, \cdot^{\mathcal{J}}, \Delta, \{\cdot^{\mathcal{I}(i)}\}_{i \in \mathfrak{C}})$, where:

1. $(\Theta, \cdot^{\mathcal{J}})$ is an interpretation of the context language, where Θ is a non-empty domain of individuals and $\cdot^{\mathcal{J}}$ an interpretation function, where:

- $\mathfrak{C} \subseteq \Theta$ is a non-empty domain of contexts,
 - $\mathfrak{c}^{\mathcal{J}} \in \mathfrak{C}$, for every $\mathfrak{c} \in M_I^*$,
2. $(\Delta, \cdot^{\mathcal{I}(i)})$, for every $i \in \mathfrak{C}$, is an interpretation of the object language, where Δ is a non-empty object domain and $\cdot^{\mathcal{I}(i)}$ an interpretation function of \mathcal{L}_O , such that:
- (\mathfrak{F}_1) for every $\langle \mathbf{r}, \mathbf{C} \rangle D$ and $[\mathbf{r}, \mathbf{C}] D$:
- $(\langle \mathbf{r}, \mathbf{C} \rangle D)^{\mathcal{I}(i)} = \{x \mid \exists j \in \mathfrak{C} : (i, j) \in \mathbf{r}^{\mathcal{J}} \wedge j \in \mathbf{C}^{\mathcal{J}} \wedge x \in D^{\mathcal{I}(j)}\}$,
 - $([\mathbf{r}, \mathbf{C}] D)^{\mathcal{I}(i)} = \{x \mid \forall j \in \mathfrak{C} : (i, j) \in \mathbf{r}^{\mathcal{J}} \wedge j \in \mathbf{C}^{\mathcal{J}} \rightarrow x \in D^{\mathcal{I}(j)}\}$.
- (\mathfrak{F}_2) for every $\langle \mathbf{C} \rangle D$ and $[\mathbf{C}] D$:
- $(\langle \mathbf{C} \rangle D)^{\mathcal{I}(i)} = \{x \mid \exists j \in \mathfrak{C} : j \in \mathbf{C}^{\mathcal{J}} \wedge x \in D^{\mathcal{I}(j)}\}$,
 - $([\mathbf{C}] D)^{\mathcal{I}(i)} = \{x \mid \forall j \in \mathfrak{C} : j \in \mathbf{C}^{\mathcal{J}} \rightarrow x \in D^{\mathcal{I}(j)}\}$.

Clearly, the difference between the context operators of type \mathfrak{F}_1 and \mathfrak{F}_2 lies in the choice of the relational structures involved in quantifying over the context domain. \mathfrak{F}_1 -operators bind contexts only along the roles of the context language (similarly to **K**-modalities), while \mathfrak{F}_2 -operators ignore these relationships and rest upon the universal relation over \mathfrak{C} (similarly to **S5**-modalities). This is reflected in the scope and the character of the distribution of the object knowledge over contexts in $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -models. For instance, in Figure 1, the concept $\langle \mathbf{t}, \mathbf{F} \rangle B$ is satisfied by object a only in context \mathfrak{c} , while $\langle \mathbf{F} \rangle B$ is satisfied by a in all contexts in the model. From the perspective of McCarthy’s theory, employing operators \mathfrak{F}_2 , rather than \mathfrak{F}_1 , is equivalent to scarfing postulate (3). This means that every two contexts in the model become in principle accessible to each other.

Definition 5 ($\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -models). A $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ -interpretation $\mathfrak{M} = (\Theta, \mathfrak{C}, \cdot^{\mathcal{J}}, \Delta, \{\cdot^{\mathcal{I}(i)}\}_{i \in \mathfrak{C}})$ is a model of a CKB $\mathcal{K} = (\mathcal{C}, \mathcal{O})$ iff:

- for every $\varphi \in \mathcal{C}$, $(\Theta, \cdot^{\mathcal{J}})$ satisfies φ ,
- for every $\mathfrak{c} : \varphi \in \mathcal{O}$, $(\Delta, \cdot^{\mathcal{I}(\mathfrak{c}^{\mathcal{J}})})$ satisfies φ ,
- for every $\mathbf{C} : \varphi \in \mathcal{O}$ and $i \in \mathfrak{C}$, if $i \in \mathbf{C}^{\mathcal{J}}$ then $(\Delta, \cdot^{\mathcal{I}(i)})$ satisfies φ .

As hinted before, there is a close connection between our DLs of context and the modal DLs $(\mathbf{K}_n)_{\mathcal{L}}$ and **S5** $_{\mathcal{L}}$. In particular, the former are proper extensions of $(\mathbf{K}_n)_{\mathcal{L}}$ resp. **S5** $_{\mathcal{L}}$. This relationship is formally established in Theorem 1.

Theorem 1. If $M_I = M_C = \emptyset$ then $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ with context operators (only) of type \mathfrak{F}_1 resp. \mathfrak{F}_2 is a notational variant of $(\mathbf{K}_n)_{\mathcal{L}_O}$ resp. **S5** $_{\mathcal{L}_O}$ with global axioms.

Proof sketch. Observe that all formulas are of the form $\top : \varphi$. First, replace every $\langle \mathbf{r}, \top \rangle$ with \diamond_r and $[\mathbf{r}, \top]$ with \square_r , resp. every $\langle \top \rangle$ with \diamond and $[\top]$ with \square . Next, replace every $\top : \varphi \in \mathcal{O}$ with φ . It is easy to see that the semantics of $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ coincides with that of $(\mathbf{K}_n)_{\mathcal{L}_O}$ resp. **S5** $_{\mathcal{L}_O}$. Note that an axiom is global iff it is satisfied in all possible \mathbf{K}_n -worlds resp. **S5**-worlds. \square

As our main technical contributions in [10] and [11], we obtained a wide panorama of complexity results for reasoning in DLs of context using particular combinations of DLs for \mathcal{L}_C and \mathcal{L}_O , and different types of context operators. We summarize these results in Table 1, and shortly elaborate on them below.

context operators	$\mathcal{L}_O \backslash \mathcal{L}_C$	$\mathcal{E}\mathcal{L}$	$\mathcal{A}\mathcal{L}\mathcal{C}, \mathcal{A}\mathcal{L}\mathcal{C}\mathcal{O}$
	type \mathfrak{F}_1	$\mathcal{A}\mathcal{L}\mathcal{C}, \mathcal{A}\mathcal{L}\mathcal{C}\mathcal{O}$	2EXPTIME-complete
type \mathfrak{F}_2	$\mathcal{E}\mathcal{L}$	PTime	EXPTIME-hard
	$\mathcal{A}\mathcal{L}\mathcal{C}$	EXPTIME-complete	NEXPTIME-complete
	$\mathcal{A}\mathcal{L}\mathcal{C}\mathcal{O}$	NEXPTIME-complete	NEXPTIME-complete

Table 1. Complexity of reasoning in $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$.

The results reveal that the computational properties of the proposed logics are predominantly affected by the choice of the context operators. More precisely, reasoning in $\mathfrak{C}_{\mathcal{L}_O}^{\mathcal{L}_C}$ with \mathfrak{F}_1 -operators is harder than with \mathfrak{F}_2 -operators. This behavior can be explained by the fact that such difference in the complexity is essentially present already between the underlying logics $(\mathbf{K}_n)_{\mathcal{L}}$ and $\mathbf{S5}_{\mathcal{L}}$ [10,20].

In the case of DLs of context with \mathfrak{F}_1 -operators, we first established the 2EXPTIME lower bound for the satisfiability problem for $(\mathbf{K}_n)_{\mathcal{A}\mathcal{L}\mathcal{C}}$ w.r.t. to global TBoxes and only local roles. The proof is a reduction of the *word problem* for an exponentially bounded, alternating Turing machine. This result turned out to be quite surprising since it could be expected that without rigid roles the satisfiability problem can be straightforwardly reduced to satisfiability in fusion models. This in turn would have to yield an EXPTIME upper bound by means of the standard techniques. However, as the following example for $(\mathbf{K}_n)_{\mathcal{A}\mathcal{L}\mathcal{C}}$ demonstrates, this strategy fails.

$$(\dagger) \diamond_i C \sqcap \exists r. \Box_i \perp \quad (\ddagger) \exists succ_i. C \sqcap \exists r. \forall succ_i. \perp$$

Although (\dagger) clearly does not have a model, its reduction (\ddagger) to a fusion language, where modal operators are translated to restrictions on fresh $\mathcal{A}\mathcal{L}\mathcal{C}$ roles, is satisfiable. The reason is that while in the former case the information about the structure of the \mathbf{K} -frame is global for all individuals, in the latter it becomes local. The r -successor in (\ddagger) is simply not ‘aware’ that it should actually have a $succ_i$ -successor. The matching 2EXPTIME upper bound is proven by using the *quasistate elimination* technique, similar to the proofs for certain products of modal logics [9].

Regarding DLs of context with \mathfrak{F}_2 -operators, for $\mathcal{L}_O \in \{\mathcal{A}\mathcal{L}\mathcal{C}, \mathcal{A}\mathcal{L}\mathcal{C}\mathcal{O}\}$ and $\mathcal{L}_C \in \{\mathcal{A}\mathcal{L}\mathcal{C}, \mathcal{A}\mathcal{L}\mathcal{C}\mathcal{O}\}$, we encounter a jump from EXPTIME to NEXPTIME-completeness. The non-determinism involved can be interpreted by the need of guessing the interpretation of the context language first, before finding the model of the object component of the combination. In particular, the lower bound is obtained by an encoding of the $2^n \times 2^n$ tiling problem, known to be NEXPTIME-complete [9]. In the case of $\mathcal{L}_O = \mathcal{A}\mathcal{L}\mathcal{C}\mathcal{O}$ and $\mathcal{L}_C = \mathcal{E}\mathcal{L}$ this jump can be explained by the interaction of nominals and the context operators, in fact this enables to encode the $2^n \times 2^n$ tiling problem, as in the previous cases. For the upper bounds for $\mathcal{L}_O \in \{\mathcal{A}\mathcal{L}\mathcal{C}, \mathcal{A}\mathcal{L}\mathcal{C}\mathcal{O}\}$ we devise a variant of a *type elimination algorithm*, whereas for $\mathcal{L}_O = \mathcal{E}\mathcal{L}$ a *completion algorithm* in the style of [21].

4 Application scenarios

There are two natural application scenarios for the DLs of context. First, they can be used as native representation languages dedicated to modeling and reasoning about knowledge of inherently contextualized nature. Alternatively, the framework can be used to support an external ‘integration’ layer over standard DL ontologies. Observe, that a collection of DL ontologies $\mathcal{O}_1, \dots, \mathcal{O}_n$ in some language $\mathcal{L}_{\mathcal{O}}$ can be seen as a set of formulas $\mathcal{O} = \{c_i : \varphi \mid \varphi \in \mathcal{O}_i, i \in (1, n)\}$ in $\mathfrak{C}_{\mathcal{L}_{\mathcal{O}}}^{\mathcal{L}_{\mathcal{O}}}$, where every ontology corresponds to a unique context name. Consequently, the extra expressive power of $\mathfrak{C}_{\mathcal{L}_{\mathcal{O}}}^{\mathcal{L}_{\mathcal{O}}}$ can be utilized for imposing interoperability constraints over those ontologies. Arguably, the first type of use might be of interest for knowledge-intensive/expert applications, while the second one seems appealing from the perspective of integrating information on the Semantic Web. We support the two cases with small examples, based on different types of context operators, and explain some possible inferences.

Contextualized knowledge base. Consider a simple representation of knowledge about the legal status of people, contextualized with respect to geographic locations. We define a CKB $\mathcal{K} = (\mathcal{C}, \mathcal{O})$, consisting of the context (geographic) ontology \mathcal{C} and the object (people) ontology \mathcal{O} , as follows:

$\mathcal{C} :$	<i>Country</i> (<i>germany</i>)	(1)
	<i>neighbor</i> (<i>france</i> , <i>germany</i>)	(2)
$\mathcal{O} :$	<i>germany</i> : $\exists \text{hasParent.Citizen}(\text{john})$	(3)
	<i>Country</i> : $\exists \text{hasParent.Citizen} \sqsubseteq \text{Citizen}$	(4)
	<i>france</i> : $\langle \text{neighbor.Citizens} \rangle \text{Citizen} \sqsubseteq \text{NoVisaRequirement}$	(5)

Visibly, ***france*** and ***germany*** play here the role of contexts, described in the context language by axioms (1) and (2). In the context of ***germany***, it is known that *john* has a parent who is a citizen (3). Since in every ***Country*** context — thus including ***germany*** — the concept $\exists \text{hasParent.Citizen}$ is subsumed by *Citizen* (4), therefore it must be true that *john* is an instance of *Citizen* in ***germany***. Finally, since ***germany*** is related to ***france*** via the role ***neighbor***, it follows that *john* (assuming rigid interpretation of this name across contexts) has to be an instance of *NoVisaRequirement* in the context of ***france*** (5). A sample $\mathfrak{C}_{\mathcal{L}_{\mathcal{O}}}^{\mathcal{L}_{\mathcal{O}}}$ -model of \mathcal{K} is depicted in Figure 3.



Fig. 3. A $\mathfrak{C}_{\mathcal{L}_{\mathcal{O}}}^{\mathcal{L}_{\mathcal{O}}}$ -model of the CKB \mathcal{K} .

Interoperability constraints over DL ontologies. Consider an architecture such as the NCBO BioPortal project³, which gathers numerous published bio-health ontologies, and categorizes them via thematic tags, e.g.: *Cell*, *Health*, *Anatomy*, etc., organized in a meta-ontology. The intention of the project is to facilitate the reuse of the collected resources in new applications. Note, that the division between the context and the object language is already present in the architecture of the BioPortal, which can be immediately utilized to state, e.g.:

$$\begin{array}{l}
 \mathcal{C} : \mathbf{HumanAnatomy} \sqsubseteq \mathbf{Anatomy} \quad (1) \\
 \mathcal{O} : \top : \langle \mathbf{HumanAnatomy} \rangle \mathit{Heart} \sqsubseteq [\mathbf{Anatomy}] \mathit{HumanHeart} \quad (2) \\
 \mathbf{Anatomy} : \mathit{Heart} \sqsubseteq \mathit{Organ} \quad (3)
 \end{array}$$

where (2) maps the concept *Heart* from any *HumanAnatomy* ontology to the concept *HumanHeart* in every *Anatomy* ontology; (3) imposes the axiom $\mathit{Heart} \sqsubseteq \mathit{Organ}$ of an upper anatomy ontology over all *Anatomy* ontologies, which due to axiom (1) carries over to all *HumanAnatomy* ontologies.

In general, $\mathfrak{C}_{\mathcal{L}^c}^{\mathcal{L}^c}$ provides logic-based explications of some interesting notions, relevant to the problem of semantic interoperability of ontologies, such as:

concept alignment: $\top : \langle \mathbf{A} \rangle C \sqsubseteq [\mathbf{B}] D$

every instance of *C* in any ontology of type *A* is *D* in every ontology of type *B*

semantic importing: $c : \langle \mathbf{A} \rangle C \sqsubseteq D$

every instance of *C* in any ontology of type *A* is *D* in ontology *c*

upper ontology axiom: $\mathbf{A} : C \sqsubseteq D$

axiom $C \sqsubseteq D$ holds in every ontology of type *A*

5 Conclusions

The problems of 1) representing inherently contextualized knowledge within the paradigm of DLs and 2) reasoning with multiple heterogenous, but semantically interoperating DL ontologies, are both interesting and important issues, motivated by numerous practical application scenarios. It is our belief that these two challenges are in fact two sides of the same coin and, consequently, they should be approached within the same, unifying formal framework. In this paper, we have proposed two novel families of two-dimensional DLs of context. Arguably, these logics achieve the objective declared above to a great extent, by providing sufficient syntactic and semantic means to support both functionalities, seamlessly integrated within one formalism.

As our results show, such two-dimensional extension of the DL framework does not necessarily entail an increase in the computational complexity of reasoning, as for e.g. $\mathfrak{C}_{\mathcal{L}^c}^{\mathcal{L}^c}$ and $\mathfrak{C}_{\mathcal{A}^c}^{\mathcal{L}^c}$ with \mathfrak{F}_2 -operators, nor does it affect the generally adopted knowledge representation methodology of DLs. We therefore consider the approach a worthwhile subject to further research. In particular, it is essential to investigate how certain notions and problems central to the practical use and maintenance of multi-context knowledge systems (e.g. handling local inconsistencies) can be meaningfully restated within the presented framework.

³ See <http://bioportal.bioontology.org/>.

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