Decidability of Circumscribed Description Logics Revisited

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Abstract. We revisit non-monotonic description logics based on circumscription (with preferences) and prove several decidability results for their satisfiability problem. In particular, we consider circumscribed description logics without the finite model property (DL-Lite$_{\mathcal{F}}$ and $\mathcal{ALCFI}$) and with fixed roles (DL-Lite$_{\mathcal{F}}$ and a fragment of DL-Lite$_{\mathcal{R}}$), improving upon previous decidability results that are limited to logics which have the finite model property and do not allow to fix roles during minimization.

1 Introduction

During the evolution from frame systems to description logics (DLs), nonmonotonic inferences and constructs (such as those supported by the LOOM system in the 1990s) have disappeared from the mainstream. However, a range of knowledge engineering requirements kept interest in nonmonotonic DLs alive, see e.g. [21,23,5] for more details. In fact, along the years all of the major nonmonotonic semantics have been adapted to DLs, including the integration of default rules and DLs [2,12,20,16,17], circumscription [7,22], and variations of autoepistemic logics, preferential semantics and rational closure [8,11,14,15,10]. In this paper, we focus on circumscription, which was first applied in the DL context by Gerd Brewka to whom this volume is dedicated [7]. The general idea of circumscription is to select a subclass of the classical models of the knowledge base by minimizing the extension of some selected predicates that represent abnormal situations. During minimization, the interpretation of the other predicates can be fixed or vary freely. To achieve a faithful modeling, in addition it is often necessary to allow a preference order on the minimized predicates, that is, if P_1 is preferred to P_2, then we allow the interpretation of P_2 to become larger (or change in an orthogonal way) if this allows the interpretation of P_1 to become smaller.

All these aspects of circumscription are incorporated in the *circumscription patterns* studied in [6,5], where a range of (un)decidability results for circumscribed DLs based on circumscription patterns has been obtained. The positive results are mostly obtained by using a filtration type of construction as known from modal logic [4], which is limited to logics that enjoy the finite model property. The negative results show that a main cause of undecidability is to allow

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role names (binary relations) to be minimized or fixed during minimization instead of minimizing/fixing only concept names (unary relations). However, many popular description logics such as those underlying the OWL ontology language recommended by the W3C do not enjoy the finite model property; moreover, minimizing/fixing roles would be useful as a modeling tool for applications.

In this paper, we contribute to a better understanding of the computational properties of circumscribed DLs without the finite model property and with fixed role names. Regarding the former, we deviate from the filtration approach and prove decidability by reduction to the (decidable) first-order theory of set systems with a binary predicate expressing that two sets have the same cardinality [13]. The reduction is inspired by reductions of inseparability problems for DL TBoxes to BAPA (Boolean Algebra with Presburger Arithmetic) from [19]. We note that the surprisingly close relationship between inseparability (and conservative extensions) of DL TBoxes and circumscribed DLs has been exploited to prove results in both areas before: complexity results for circumscribed DLs have been used to investigate the complexity of deciding inseparability and conservative extensions in [18]. Conversely, undecidability results for inseparability proved in [18] have been used in [5] to prove undecidability results for circumscribed $\mathcal{EL}$ TBoxes. Regarding fixed roles, we show that decidability results can be obtained for members of the DL-Lite family of inexpressive DLs. Considering two such members, we show that decidability results can be both obtained by reduction to the afore mentioned theory of set systems and by the original filtration-style method from [5].

In detail, our results are as follows (all referring to concept satisfiability relative to circumscribed knowledge bases as introduced in Section 2):

1. Circumscribed $\mathcal{ALCFI}$ without minimized roles and fixed roles is decidable where $\mathcal{ALCFI}$ is the basic DL $\mathcal{ALC}$ extended with functional and inverse roles. This extends the previous decidability results for DLs such as $\mathcal{ALCI}$ and $\mathcal{ALCQ}$ which enjoy the finite model property [6].
2. Circumscribed DL-Lite$^{\mathcal{F}}_{bool}$with fixed roles (but no minimized roles) is decidable where DL-Lite$^{\mathcal{F}}_{bool}$is DL-Lite with boolean concept connectives and functional roles. Note that, in addition, DL-Lite$^{\mathcal{F}}_{bool}$is another example of a decidable circumscribed DL without the finite model property.
3. Circumscribed DL-Lite$^{\mathcal{R}}_{bool}$ with fixed roles (but no minimized roles) is decidable if it is additionally assumed that no minimized or fixed role is subsumed by a varying role, where DL-Lite$^{\mathcal{R}}_{bool}$ is DL-Lite with boolean concept connectives and role inclusions.

2 Preliminaries

The alphabet of description logics (DLs) consists of three (pairwise disjoint) sets: a set $\mathsf{N_I}$ of *individual names*, denoted $a, b, \ldots$, a set $\mathsf{N_C}$ of *concept names*, denoted $A, B, \ldots$, and a set $\mathsf{N_R}$ of *role names*, denoted P. A *role*, denoted R, is either a role name or an *inverse role*, that is, an expression of the form P^-. As a convention, we set $R^- = P$ if $R = P^-$. We consider two members of the DL-Lite family of DLs [9,1]. The *concepts* C of DL-Lite$^{\mathcal{F}}_{bool}$ are defined inductively as follows:

$$B ::= \bot \quad | \quad \top \quad | \quad A_i \quad | \quad \exists R,$$
$$C ::= B \quad | \quad \neg C \quad | \quad C_1 \sqcap C_2.$$

The concepts of the form B are called *basic*. A *concept inclusion* in DL-Lite$^{\mathcal{F}}_{bool}$ is of the form $C_1 \sqsubseteq C_2$, where C_1 and C_2 are DL-Lite$^{\mathcal{F}}_{bool}$ concepts. A TBox $\mathcal{T}$ in DL-Lite$^{\mathcal{F}}_{bool}$ is a finite set of concept inclusions in DL-Lite$^{\mathcal{F}}_{bool}$ and *functionality assertions* $\mathsf{func}(R)$, where R is a role.

Concept inclusions in DL-Lite$^{\mathcal{R}}_{bool}$ are defined in the same way as concept inclusions in DL-Lite$^{\mathcal{F}}_{bool}$. A TBox $\mathcal{T}$ in DL-Lite$^{\mathcal{R}}_{bool}$ is a finite set of concept inclusions in DL-Lite$^{\mathcal{R}}_{bool}$ and *role inclusions* $R_1 \sqsubseteq R_2$, where R_1 and R_2 are roles.

The concepts C of the DL $\mathcal{ALCFI}$ are defined inductively as follows:

$$C \quad ::= \quad \bot \quad | \quad \top \quad | \quad A_i \quad | \neg C \quad | \quad C_1 \sqcap C_2 \quad | \quad \exists R.C.$$

Concept inclusions and TBoxes $\mathcal{T}$ in $\mathcal{ALCFI}$ are defined in the same way as TBoxes in DL-Lite$^{\mathcal{F}}_{bool}$, where concepts in DL-Lite$^{\mathcal{F}}_{bool}$ are replaced by concepts in $\mathcal{ALCFI}$.

An *ABox* $\mathcal{A}$ is a finite set of assertions of the form $A(a)$ and $P(a,b)$. We use $P^-(a,b)$ to denote the assertion $P(b,a)$. By $\mathsf{Ind}(\mathcal{A})$ we denote the set of individual names in $\mathcal{A}$. A *knowledge base* (KB, for short) is a pair $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ with a TBox $\mathcal{T}$ and an ABox $\mathcal{A}$.

The semantics of DL knowledge bases is defined as usual, see [3] for details. An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is given by its domain $\Delta^{\mathcal{I}}$ and an interpretation function that associates with every concept name A a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, with every role name P a relation $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and with every individual name a an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. We make the unique name assumption ($a^{\mathcal{I}} \neq b^{\mathcal{I}}$ if $a \neq b$). We denote by $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ the interpretation of a (complex) concept C in $\mathcal{I}$ and say that an interpretation $\mathcal{I}$ is *a model of a KB* $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ if

- $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$, for all $C \sqsubseteq D \in \mathcal{T}$;
- $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$, for all $R \sqsubseteq S \in \mathcal{T}$;
- $R^{\mathcal{I}}$ is a partial function, for all $\mathsf{func}(R) \in \mathcal{T}$;
- $a^{\mathcal{I}} \in A^{\mathcal{I}}$, for all $A(a) \in \mathcal{A}$;
- $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in P^{\mathcal{I}}$, for all $R(a,b) \in \mathcal{A}$.

Given a DL $\mathcal{L}$, *concept satisfiability relative to* $\mathcal{L}$ *KBs* is the following problem: given a concept C in $\mathcal{L}$ and a KB $\mathcal{K}$ in $\mathcal{L}$, decide whether there exists a model $\mathcal{I}$ of $\mathcal{K}$ such that $C^{\mathcal{I}} \neq \emptyset$. Concept satisfiability is NP-complete for DL-Lite$^{\mathcal{F}}_{bool}$ and DL-Lite$^{\mathcal{R}}_{bool}$, and ExpTime-complete for $\mathcal{ALCFI}$.

To define DLs with circumscription, we start by introducing circumscription patterns. Such a pattern describes how individual predicates are treated during minimization.

Definition 1 (Circumscription pattern, $<_{\mathsf{CP}}$). *A* circumscription pattern *is a tuple* CP *of the form* $(\prec, M, F, V)$, *where* $\prec$ *is a strict partial order over* M, *and* M*,* F*, and* V *are mutually disjoint and exhaustive subsets of* $\mathsf{N_C} \cup \mathsf{N_R}$*, the* minimized, fixed*, and* varying *predicates, respectively. By* $\preceq$*, we denote the reflexive closure of* $\prec$*. Define a preference relation* $<_{\mathsf{CP}}$ *on interpretations by setting* $\mathcal{I} <_{\mathsf{CP}} \mathcal{J}$ *iff the following conditions hold:*

1. $\Delta^{\mathcal{I}} = \Delta^{\mathcal{J}}$ *and, for all* $a \in \mathsf{N_I}$*,* $a^{\mathcal{I}} = a^{\mathcal{J}}$*,*
2. *for all* $p \in F$*,* $p^{\mathcal{I}} = p^{\mathcal{J}}$*,*
3. *for all* $p \in M$*, if* $p^{\mathcal{I}} \not\subseteq p^{\mathcal{J}}$ *then there exists* $q \in M$*,* $q \prec p$*, such that* $q^{\mathcal{I}} \subset q^{\mathcal{J}}$*,*
4. *there exists* $p \in M$ *such that* $p^{\mathcal{I}} \subset p^{\mathcal{J}}$ *and for all* $q \in M$ *such that* $q \prec p$*,* $q^{\mathcal{I}} = q^{\mathcal{J}}$*.*

A circumscribed knowledge base with circumscription pattern $\mathsf{CP} = (\prec, M, F, V)$ and KB $\mathcal{K}$ is denoted by $\mathsf{Circ_{CP}}(\mathcal{K})$. An interpretation $\mathcal{I}$ is a *model* of $\mathsf{Circ_{CP}}(\mathcal{K})$ if it is a model of $\mathcal{K}$ and no $\mathcal{J} <_{\mathsf{CP}} \mathcal{I}$ is a model of $\mathcal{K}$.

In this paper, we consider the decidability and complexity of *concept satisfiability relative to circumscibed KBs*: a concept C is *satisfiable* relative to a circumscribed KB $\mathsf{Circ_{CP}}(\mathcal{T}, \mathcal{A})$ if some model $\mathcal{I}$ of $\mathsf{Circ_{CP}}(\mathcal{T}, \mathcal{A})$ satisfies $C^{\mathcal{I}} \neq \emptyset$. By (concept) satisfiability problem relative circumscribed KBs we mean the problem to decide whether a given concept C is satisfiable relative to a given circumscribed KB. Other reasoning problems such as subsumption and instance checking relative to circumscribed KBs can be reduced to concept satisfiability relative to circumscribed KBs [6].

3 Decidability for DL-Lite$_{bool}^{\mathcal{F}}$

We show decidability of concept satisfiability relative to circumscribed DL-Lite$_{bool}^{\mathcal{F}}$ KBs with fixed roles and without minimized roles. Note that fixed roles easily lead to undecidability of concept satisfiability relative to circumscribed KBs, such as for the circumscribed version of the popular lightweight (and tractable) DL $\mathcal{EL}$ [5]. Also note that DL-Lite$_{bool}^{\mathcal{F}}$ does not have the finite model property. An example showing this is given by the KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, where

$$\mathcal{T} = \{A \sqsubseteq \exists P, \exists P^- \sqsubseteq \exists P, A \sqsubseteq \neg\exists P^-, \mathsf{func}(P^-)\}, \quad \mathcal{A} = \{A(a)\}.$$

It is easy to see that $\mathcal{K}$ is satisfiable but has no finite model. Thus, approaches to reasoning in circumscribed DLs that are based on filtration [6] cannot be employed in this case.

We prove decidability by reduction to the first-order theory of set systems with a binary predicate expressing that two sets have the same cardinality, which is decidable [13]. Formally, the language SC of set systems with cardinality is defined as follows. Its *terms* are constructed from variables $X_1, X_2, \ldots$ (interpreted as sets) and constants $\mathbf{0}$ (the empty set) and $\mathbf{1}$ (the whole set) using the binary function symbols $\cap$ (intersection), $\cup$ (union), and the unary function symbol $\bar{\cdot}$ (complement). As usual, we prefer the infix notation for the binary function symbols and write, e.g., $X \cap Y$ instead of $\cap(X, Y)$. *Atomic SC formulas* are of the form

- $B_1 = B_2$ and $B_1 \subseteq B_2$, where B_1 and B_2 are terms;
- $|B_1| = |B_2|$ and $|B_1| \leq |B_2|$, where B_1 and B_2 are terms.

SC formulas are now constructed in the standard way using quantification, conjunction and negation. We are interested in the satisfiability of SC sentences in structures of the form $\mathfrak{A} = (2^{\Delta}, \cap, \cup, \bar{\cdot}, \emptyset, \Delta)$, where Δ is a non-empty set. We call such structures *SC structures*. An *SC model* $\mathfrak{M}$ consists of an SC structure $\mathfrak{A}$ and an *interpretation function* $X_i^{\mathfrak{M}} \subseteq \Delta$ of the variables X_i in $\mathfrak{A}$. The truth of SC sentences in an SC model is defined in the obvious way, for example,

- $\mathfrak{M} \models B_1 = B_2$ if $B_1^{\mathfrak{M}} = B_2^{\mathfrak{M}}$;
- $\mathfrak{M} \models |B_1| = |B_2|$ if $|B_1^{\mathfrak{M}}| = |B_2^{\mathfrak{M}}|$.

Decidability of satisfiability of SC sentences in SC models is proved in [13]:

Theorem 1. *Satisfiability of SC sentences is decidable.*

Suppose that a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, a circumscription pattern $\mathsf{CP} = (\prec, M, F, V)$, and a concept C_0 are given such that no role name is minimized in CP (that is, M contains no role names). We encode satisfiability of C_0 relative to $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{K})$ as a satisfiability problem for SC sentences.

Take for every concept name B in $\mathcal{K} \cup \{C_0\}$ and any B of the form $\exists P$ or $\exists P^-$ such that P occurs in $\mathcal{K} \cup \{C_0\}$, an SC variable X_B. Then define inductively for every subconcept C of $\mathcal{K} \cup \{C_0\}$ an SC term C^s:

$$B^s = X_B, \qquad \bot^s = \mathbf{0}, \qquad \top^s = \mathbf{1},$$
$$(\neg C)^s = \overline{C^s}, \qquad (C_1 \sqcap C_2)^s = C_1^s \cap C_2^s.$$

We also set

$$\mathcal{T}^s = \{C_1^s \subseteq C_2^s \mid C_1 \sqsubseteq C_2 \in \mathcal{T}\}.$$

If $\mathcal{T}$ and C_0 do not contain roles, then clearly C_0 is satisfiable relative to (uncircumscribed) $\mathcal{T}$ iff the SC sentence $\exists \boldsymbol{X} \left(\neg(C_0^s = \mathbf{0}) \wedge \bigwedge_{\alpha \in \mathcal{T}^s} \alpha\right)$ is satisfiable where $\boldsymbol{X}$ is the sequence of variables occurring in $\mathcal{T}^s$ or C_0^s. To extend this to an encoding of satisfiability of C_0 relative to (uncircumscribed) $\mathcal{T}$ with roles, it is sufficient to state that $X_{\exists P}$ is empty iff $X_{\exists P^-}$ is empty for every role name P and to state for functional roles R that the cardinality of $X_{\exists R}$ is not smaller that the cardinality of $X_{\exists R^-}$. Thus, we extend $\mathcal{T}^s$ to $\mathcal{T}^{s,e}$ by adding the following SC formulas to $\mathcal{T}^s$:

$$(\neg(X_{\exists P} = \mathbf{0}) \leftrightarrow \neg(X_{\exists P^-} = \mathbf{0})),$$

for every role name P in $\mathcal{K} \cup \{C_0\}$, and

$$|X_{\exists R}| \geq |X_{\exists R^-}|$$

for every role R with $\mathsf{func}(R) \in \mathcal{T}$. We prove that C_0 is satisfiable relative to $\mathcal{T}$ iff the SC formula $\varphi = \exists \boldsymbol{X} \left(\neg(C_0^s = \mathbf{0}) \wedge \bigwedge_{\alpha \in \mathcal{T}^{s,e}} \alpha\right)$ is satisfiable. First let $\mathcal{I}$ be a model of $\mathcal{T}$ such that $C_0^{\mathcal{I}} \neq \emptyset$. Define an SC structure $\mathfrak{M}$ based on $\mathfrak{A} = (2^{\Delta}, \cap, \cup, \bar{\cdot}, \emptyset, \Delta)$ by setting $\Delta = \Delta^{\mathcal{I}}$, $X_A^{\mathfrak{M}} = A^{\mathcal{I}}$ for all concept names A, and $X_{\exists R}^{\mathfrak{M}} = \{d \in \Delta \mid \exists d' \ (d, d') \in R^{\mathcal{I}}\}$ for all roles R. It is readily checked that $\mathfrak{M}$ satisfies φ. Conversely, assume that a model $\mathfrak{M}$ based on $\mathfrak{A} = (2^{\Delta}, \cap, \cup, \bar{\cdot}, \emptyset, \Delta)$ satisfies φ. Define $\mathcal{I}$ by setting $\Delta^{\mathcal{I}} = \Delta$,

- $A^{\mathcal{I}} = X_A^{\mathfrak{M}}$ for all concept names A;
- $P^{\mathcal{I}} = X_{\exists P}^{\mathfrak{M}} \times X_{\exists P^-}^{\mathfrak{M}}$ for all roles P with neither $\mathsf{func}(P)$ nor $\mathsf{func}(P^-)$ in $\mathcal{T}$;
- and defining $R^{\mathcal{I}}$ as a surjective function with domain $X_{\exists R}^{\mathfrak{M}}$ and range $X_{\exists R^-}^{\mathfrak{M}}$ if $\mathsf{func}(R) \in \mathcal{T}$ (such a function exists since $|X_{\exists R}^{\mathfrak{M}}| \geq |X_{\exists R^-}^{\mathfrak{M}}|$ for every role R with $\mathsf{func}(R) \in \mathcal{T}$).

One can check that $\mathcal{I}$ satisfies $\mathcal{T}$ and that $C_0^{\mathcal{I}} \neq \emptyset$.

To encode circumscription, we define a second translation C^m of every subconcept C in $\mathcal{K} \cup \{C_0\}$. C^m is defined in exactly the same way as C^s except that we use fresh SC variables Y_B instead of the SC variables X_B used in the translation C^s. We define $\mathcal{T}^m$ and $\mathcal{T}^{m,e}$ in exactly the same way as $\mathcal{T}^s$ and $\mathcal{T}^{s,e}$ with X_B replaced by Y_B.

Assume now that the ABox $\mathcal{A}$ is empty. Then we can encode satisfiability of C_0 relative to $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{K})$ in a straightforward way by considering satisfiability of the SC sentence

$$\exists \boldsymbol{X} \Big(\neg(C_0^s = \mathbf{0}) \wedge \bigwedge_{\alpha \in \mathcal{T}^{s,e}} \alpha \ \wedge\ \forall \boldsymbol{Y} (\boldsymbol{Y} <_{\mathsf{CP}} \boldsymbol{X} \rightarrow \neg \bigwedge_{\alpha \in \mathcal{T}^{m,e}} \alpha)\Big) \qquad (1)$$

where $\boldsymbol{X}$ is as above, $\boldsymbol{Y}$ is the sequence of variables occuring in $\mathcal{T}^m$ and $\boldsymbol{Y} <_{\mathsf{CP}} \boldsymbol{X}$ stands for the conjunction of

$$X_B = Y_B,$$

for each concept name B in F and B of the form $\exists P$ or $\exists P^-$ with $P \in F$,

$$\bigwedge_{A \in M} ((Y_A \not\subseteq X_A) \rightarrow \bigvee_{B \in M, B \prec A} (Y_B \subset X_A)),$$

and

$$\bigvee_{A \in M} ((Y_A \subset X_A) \wedge \bigwedge_{B \in M, B \prec A} (Y_B = X_B)).$$

We now extend the encoding above to KBs with non-empty ABox $\mathcal{A}$. To encode the ABox, take for every individual name $a \in \mathsf{Ind}(\mathcal{A})$ an SC variable X_a and define the set of SC formulas $\mathcal{A}^s$ as follows:

(A1) $|X_a| = 1$ for all $a \in \mathsf{Ind}(\mathcal{A})$, where $|X_a| = 1$ abbreviates the conjunction of $|X_a| > |\mathbf{0}|$ and $\forall X((X \subset X_a) \rightarrow (X = \mathbf{0}))$.
(A2) $X_a \cap X_b = \mathbf{0}$ for $a \neq b$ and $a, b \in \mathsf{Ind}(\mathcal{A})$. These formulas encode the unique name assumption.
(A3) $X_a \subseteq X_A$ if $A(a) \in \mathcal{A}$ for $a \in \mathsf{Ind}(\mathcal{A})$.
(A4) $X_a \subseteq X_{\exists P}$ if $P(a, b) \in \mathcal{A}$ for some b.
(A5) $X_a \subseteq X_{\exists P^-}$ if $P(b, a) \in \mathcal{A}$ for some b.
(A6) $\mathsf{0} = \mathbf{1}$ if there exists a role R with $\mathsf{func}(R) \in \mathcal{T}$ and a, b, b' with $b \neq b'$ such that $R(a, b), R(a, b') \in \mathcal{A}$.
(A7) If $\mathsf{func}(R) \in \mathcal{T}$ and $\mathsf{func}(R^-) \notin \mathcal{T}$, then let $\mathcal{X}_R$ be the set of $a \in \mathsf{Ind}(\mathcal{A})$ such that there exists b with $R(a, b) \in \mathcal{A}$ and let $\mathcal{Y}_R$ be the set of $b \in \mathsf{Ind}(\mathcal{A})$ such that there exists a with $R(a, b) \in \mathcal{A}$. Include

$$|X_{\exists R} \setminus (\bigcup_{a \in \mathcal{X}_R} X_a)| \geq |X_{\exists R^-} \setminus (\bigcup_{a \in \mathcal{Y}_R} X_a)|$$

in $\mathcal{A}^s$. (Note that for such R we can remove from $\mathcal{T}^s$ the formulas $|X_{\exists R}| \geq |X_{\exists R^-}|$ since they are implied.)

Define $\mathcal{A}^m$ analogously to $\mathcal{A}^s$ with X_B replaced by Y_B (note that we do *not* introduce fresh variables Y_a since the interpretation of individual names is fixed). Set $\mathcal{K}^s = (\mathcal{T}^{s,e}, \mathcal{A}^s)$ and $\mathcal{K}^m = (\mathcal{T}^{m,e}, \mathcal{A}^m)$. Now, it is readily checked that C_0 is satisfiable relative to $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{K})$ if the following SC sentence is satisfiable:

$$\exists \boldsymbol{X} \left(\neg(C_0^s = \mathbf{0}) \wedge \bigwedge_{\alpha \in \mathcal{K}^s} \alpha \quad \wedge \quad \forall \boldsymbol{Y} (\boldsymbol{Y} <_{\mathsf{CP}} \boldsymbol{X} \rightarrow \neg \bigwedge_{\alpha \in \mathcal{K}^m} \alpha) \right) \qquad (2)$$

We have proved the following result:

Theorem 2. *Satisfiability of concepts relative to circumscribed DL-Lite$_{bool}^{\mathcal{F}}$ KBs without minimized roles is decidable.*

4 Decidability for $\mathcal{ALCFI}$

We show decidability of concept satisfiability for circumscribed $\mathcal{ALCFI}$ KBs without minimized and fixed roles. The proof is again by reduction to the theory of set systems with a binary predicate expressing that two sets have the same cardinality. Note that decidability of concept satisfiability for circumscribed KBs without minimized and fixed roles has been proved using filtration in [6] for DLs with the finite model property such as $\mathcal{ALCI}$ and $\mathcal{ALCF}$. As an extension of DL-Lite$_{bool}^{\mathcal{F}}$, $\mathcal{ALCFI}$ does not have the finite model property.

Consider a circumscribe $\mathcal{ALCFI}$ KB $\mathcal{K} = \mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$ where the pattern $\mathsf{CP} = (\prec, M, F, V)$ has no minimized or fixed role names, and a $\mathcal{ALCFI}$-concept C_0. We encode satisfiability of C_0 relative to $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{K})$ as a satisfiability problem for an SC sentence.

Take for every concept name B in $\mathcal{K} \cup \{C_0\}$ and any concept B of the form $\exists P.C$ or $\exists P^-.C$ which occurs in $\mathcal{K} \cup \{C_0\}$, an SC variable X_B. Then define inductively for every subconcept C of $\mathcal{K} \cup \{C_0\}$ an SC term C^s as before:

$$B^s = X_B, \qquad \bot^s = \mathbf{0}, \qquad \top^s = \mathbf{1},$$
$$(\neg C)^s = \overline{C^s}, \qquad (C_1 \sqcap C_2)^s = C_1^s \cap C_2^s.$$

By $\mathsf{sub}(\mathcal{K} \cup \{C_0\})$ we denote the closure under single negation of the subconcepts that occur in $\mathcal{K} \cup \{C_0\}$. A *type* $\mathbf{t}$ is a subset of $\mathsf{sub}(\mathcal{K} \cup \{C_0\})$ such that

- $\bot \notin \mathbf{t}$ and $\top \in \mathbf{t}$;
- $\neg C \in \mathbf{t}$ iff $C \notin \mathbf{t}$, for all $\neg C \in \mathsf{sub}(\mathcal{K} \cup \{C_0\})$;
- $C_1 \sqcap C_2 \in \mathbf{t}$ iff $C_1, C_2 \in \mathbf{t}$, for all $C_1 \sqcap C_2 \in \mathsf{sub}(\mathcal{K} \cup \{C_0\})$.

We use $\mathbf{t}^s$ as an abbreviation for the SC term $\bigcap_{C \in \mathbf{t}} C^s$. To encode the behavior of roles we, intuitively, decompose roles R into roles $R_{\mathbf{t},\mathbf{t}'}$ such that two individuals d, d' are in relation $R_{\mathbf{t},\mathbf{t}'}$ iff they are in relation R and d is in $\mathbf{t}$ and d' is in $\mathbf{t}'$.

We cannot directly talk about $R_{\mathbf{t},\mathbf{t}'}$ in SC and so we introduce variables denoting the domain and range of $R_{\mathbf{t},\mathbf{t}'}$, respectively: for any pair $\mathbf{t},\mathbf{t}'$ of types and any role R introduce an SC variable $X_{R,\mathbf{t},\mathbf{t}'}$. Intuitively $X_{R,\mathbf{t},\mathbf{t}'}$ stands for all individuals which are in $\mathbf{t}$ and which are in the relation R to an individual in $\mathbf{t}'$. Define $\mathcal{T}^r$ as the union of $\{C_1^s \subseteq C_2^s \mid C_1 \sqsubseteq C_2 \in \mathcal{T}\}$ and the following SC formulas:

(a) $\mathbf{t}^s \cap X_{R,\mathbf{t}',\mathbf{t}''} = \mathbf{0}$ if $\mathbf{t} \neq \mathbf{t}'$, for all types $\mathbf{t},\mathbf{t}'$. These formulas state that an individual in $\mathbf{t}$ cannot be in the domain of $R_{\mathbf{t}',\mathbf{t}''}$ for $\mathbf{t} \neq \mathbf{t}'$.
(b) $\mathbf{t}^s \subseteq \bigcup_{C\in\mathbf{t}'} X_{R,\mathbf{t},\mathbf{t}'}$ if $\exists R.C \in \mathbf{t}$. These formulas state that if d is in $\mathbf{t}$ and $\mathbf{t}$ contains some $\exists R.C$, then d must be in relation R to some d' in $\mathbf{t}'$ with $C \in \mathbf{t}'$.
(c) $\mathbf{t}^s \cap X_{R,\mathbf{t},\mathbf{t}'} = \mathbf{0}$ if $\neg\exists R.C \in \mathbf{t}$ and $C \in \mathbf{t}'$.
(d) $X_{R,\mathbf{t},\mathbf{t}'} \cap X_{R,\mathbf{t},\mathbf{t}''} = \mathbf{0}$ if R is functional and $\mathbf{t}' \neq \mathbf{t}''$.

Now we extend $\mathcal{T}^r$ to $\mathcal{T}^{r,e}$ by adding the following SC formulas to $\mathcal{T}^r$:

$$(\neg(X_{P,\mathbf{t},\mathbf{t}'} = \mathbf{0}) \leftrightarrow \neg(X_{P^-,\mathbf{t}',\mathbf{t}} = \mathbf{0})),$$

for every role name P in $\mathcal{K} \cup \{C_0\}$, and

$$|X_{R,\mathbf{t},\mathbf{t}'}| \geq |X_{R^-,\mathbf{t}',\mathbf{t}}|$$

for every role R with $\mathsf{func}(R) \in \mathcal{T}$. We show that C_0 is satisfiable relative to $\mathcal{T}$ iff the SC sentence $\exists \boldsymbol{X} \left(\neg(C_0^s = \mathbf{0}) \wedge \bigwedge_{\alpha\in\mathcal{T}^{r,e}} \alpha\right)$ is satisfiable where $\boldsymbol{X}$ is the sequence of variables occurring in $\mathcal{T}^{r,e}$ or C_0^s.

First let $\mathcal{I}$ be a model of $\mathcal{T}$ such that $C_0^{\mathcal{I}} \neq \emptyset$. Define an SC model $\mathfrak{M}$ based on $\mathfrak{A} = (2^\Delta, \cap, \cup, \bar{\cdot}, \emptyset, \Delta)$ by setting $\Delta = \Delta^{\mathcal{I}}$, $X_A^{\mathfrak{M}} = A^{\mathcal{I}}$ for all concept names A, $X_{\exists R.C}^{\mathfrak{M}} = \{d \in \Delta \mid \exists d' \in C^{\mathcal{I}} \text{ and } (d,d') \in R^{\mathcal{I}}\}$ for all $\exists R.C \in \mathsf{sub}(\mathcal{K}, \cup\{C_0\})$, and

$$X_{R,\mathbf{t},\mathbf{t}'}^{\mathfrak{M}} = \{d \in (\mathbf{t}^s)^{\mathfrak{M}} \mid \exists d' \in (\mathbf{t}'^s)^{\mathfrak{M}} \text{ and } (d,d') \in R^{\mathcal{I}}\},$$

for all roles R and types $\mathbf{t},\mathbf{t}'$. It is readily checked that $\mathfrak{M}$ satisfies φ. Conversely, assume that a model $\mathfrak{M}$ based on $\mathfrak{A} = (2^\Delta, \cap, \cup, \bar{\cdot}, \emptyset, \Delta)$ satisfies φ. Define $\mathcal{I}$ by setting $\Delta^{\mathcal{I}} = \Delta$,

- $A^{\mathcal{I}} = X_A^{\mathfrak{M}}$ for all concept names A;
- $P^{\mathcal{I}} = \bigcup_{\mathbf{t},\mathbf{t}'} X_{P,\mathbf{t}',\mathbf{t}}^{\mathfrak{M}} \times X_{P^-,\mathbf{t},\mathbf{t}'}^{\mathfrak{M}}$ for all roles P with $\mathsf{func}(P), \mathsf{func}(P^-) \notin \mathcal{T}$;
- $R^{\mathcal{I}}$ is the union of surjective functions $f_{\mathbf{t},\mathbf{t}'}$ with domain $X_{R,\mathbf{t},\mathbf{t}'}^{\mathfrak{M}}$ and range $X_{R^-,\mathbf{t}',\mathbf{t}}^{\mathfrak{M}}$ if $\mathsf{func}(R) \in \mathcal{T}$ (where $\mathbf{t},\mathbf{t}'$ range over all types).

One can check that $\mathcal{I}$ satisfies $\mathcal{T}$ and that $C_0^{\mathcal{I}} \neq \emptyset$.

To encode circumscription, we again define a second translation C^n of every subconcept C in $\mathcal{K} \cup \{C_0\}$. C^n is defined in exactly the same way as C^s except that we use fresh SC variables Y_B instead of the SC variables X_B used in the translation C^s. We also introduce fresh SC variables $Y_{R,\mathbf{t},\mathbf{t}'}$ for every role R and types $\mathbf{t}$, $\mathbf{t}'$. Now define $\mathcal{T}^n$ and $\mathcal{T}^{n,e}$ in exactly the same way as $\mathcal{T}^r$ and $\mathcal{T}^{r,e}$, where the variables X are replaced by the corresponding variables Y.

Assume again that the ABox $\mathcal{A}$ is empty. Then we can encode satisfiability of C_0 relative to $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{K})$ in a straightforward way by considering satisfiability of the SC sentence

$$\exists \boldsymbol{X} \left(\neg(C_0^s = \mathbf{0}) \wedge \bigwedge_{\alpha \in \mathcal{T}^{r,e}} \alpha \quad\wedge\quad \forall \boldsymbol{Y}(\boldsymbol{Y} <^a_{\mathsf{CP}} \boldsymbol{X} \rightarrow \neg \bigwedge_{\alpha \in \mathcal{T}^{n,e}} \alpha\right)) \tag{3}$$

where $\boldsymbol{X}$ is as above, $\boldsymbol{Y}$ is the sequence of variables occuring in $\mathcal{T}^m$ and now $\boldsymbol{Y} <^a_{\mathsf{CP}} \boldsymbol{X}$ is obtained from $\boldsymbol{Y} <_{\mathsf{CP}} \boldsymbol{X}$ by taking the equations $X_A = Y_A$ for concept names $A \in F$ only. (The remaining equations involving $X_{\exists R}$ do not make sense here.)

We extend the encoding above to KBs with non-empty ABox $\mathcal{A}$. Take again for every individual name $a \in \mathsf{Ind}(\mathcal{A})$ an SC variable X_a and define a set $\mathcal{A}^r$ of SC formulas by taking the formulas in (A1), (A2), (A3), and (A6) from above as well as the following:

– for all $R(a,b) \in \mathcal{A}$ and all types $\mathbf{t}_1, \mathbf{t}_2$ include

$$(X_a \subseteq \mathbf{t}_1^s) \wedge (X_b \subseteq \mathbf{t}_2^s) \rightarrow (X_a \subseteq X_{R,\mathbf{t}_1,\mathbf{t}_2}),$$

into $\mathcal{A}^r$.

– Assume, as in (A7), that $\mathsf{func}(R) \in \mathcal{T}$ and $\mathsf{func}(R^-) \notin \mathcal{T}$. Let $\mathcal{X}_R$ be the set of $a \in \mathsf{Ind}(\mathcal{A})$ such that there exists b with $R(a,b) \in \mathcal{A}$ and let $\mathcal{Y}_R$ be the set of $b \in \mathsf{Ind}(\mathcal{A})$ such that there exists a with $R(a,b) \in \mathcal{A}$. Include for all types $\mathbf{t}, \mathbf{t}'$ the formula

$$|X_{R,\mathbf{t},\mathbf{t}'} \setminus (\bigcup_{a \in \mathcal{X}_R} X_a)| \geq |X_{R^-,\mathbf{t}',\mathbf{t}} \setminus (\bigcup_{a \in \mathcal{Y}_R} X_a)|$$

into $\mathcal{A}^r$.

Define $\mathcal{A}^n$ analogously to $\mathcal{A}^r$ with variables X replaced by the corresponding variables Y. Set $\mathcal{K}^r = (\mathcal{T}^{r,e}, \mathcal{A}^r)$ and $\mathcal{K}^n = (\mathcal{T}^{n,e}, \mathcal{A}^n)$. Now, it is readily checked that C_0 is satisfiable relative to $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{K})$ if the following SC sentence is satisfiable:

$$\exists \boldsymbol{X} \left(\neg(C_0^s = \mathbf{0}) \wedge \bigwedge_{\alpha \in \mathcal{K}^r} \alpha \quad\wedge\quad \forall \boldsymbol{Y}(\boldsymbol{Y} <^a_{\mathsf{CP}} \boldsymbol{X} \rightarrow \neg \bigwedge_{\alpha \in \mathcal{K}^n} \alpha\right)) \tag{4}$$

We have proved the following result:

Theorem 3. *Satisfiability of concepts relative to circumscribed $\mathcal{ALCFI}$ KBs without minimized and fixed roles is decidable.*

5 Decidability for DL-Lite$_{bool}^{\mathcal{R}}$

We prove decidability of concept satisfiability relative to circumscribed DL-Lite$_{bool}^{\mathcal{R}}$ knowledge bases with fixed roles and without minimized roles under the additional assumption that no varying role is subsumed by a fixed role. In contrast to the previous two sections, our approach is to use a filtration-style technique to establish a finite (in fact, single exponential) model property. To capture the mentioned syntactic restriction, we call a circumscribed KB $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$ in DL-Lite$^{\mathcal{R}}_{bool}$ *role-layered* if for each role inclusion $R \sqsubseteq S \in \mathcal{T}$ either $R \in F$ or $S \in V$.

For a concept C_0, ABox $\mathcal{A}$, and TBox $\mathcal{T}$, we denote by $\mathsf{cl}(C_0, \mathcal{T}, \mathcal{A})$ the set of subconcepts of concepts in C_0, $\mathcal{A}$, and $\mathcal{T}$. The *concept-size* of C_0 and a KB $(\mathcal{T}, \mathcal{A})$ is the cardinality of $\mathsf{cl}(C_0, \mathcal{T}, \mathcal{A})$.

Lemma 1. *Let C_0 be a concept in DL-Lite$^{\mathcal{R}}_{bool}$ and $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$ a KB in DL-Lite$^{\mathcal{R}}_{bool}$ that is role-layered and does not contain minimized roles. If C_0 is satisfiable relative to $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$, then it is satisfied in a model $\mathcal{J}$ of $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$ with $|\Delta^{\mathcal{J}}| \leq 2^n + |\mathsf{Ind}(\mathcal{A})|$, where n is the concept size of C_0 and $(\mathcal{T}, \mathcal{A})$.*

Proof. Let $\mathcal{I}$ be a model of $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$ satisfying C_0. Set $\mathsf{Ind}^{\mathcal{I}}(\mathcal{A}) = \{a^{\mathcal{I}} \mid a \in \mathsf{Ind}(\mathcal{A})\}$. Define on $\Delta^{\mathcal{I}}$ the equivalence relation $\sim$ by setting $d \sim d'$ iff

$$\{C \in \mathsf{cl}(C_0, \mathcal{T}, \mathcal{A}) \mid d \in C^{\mathcal{I}}\} = \{C \in \mathsf{cl}(C_0, \mathcal{T}, \mathcal{A}) \mid d' \in C^{\mathcal{I}}\}$$

and $d, d' \notin \mathsf{Ind}^{\mathcal{I}}(\mathcal{A})$ or $d = d'$ (this is needed to respect the unique name assumption). We use $[d]$ to denote the equivalence class of d w.r.t. $\sim$. Let $\mathcal{J}$ be the following interpretation:

$$\begin{aligned} \Delta^{\mathcal{J}} &= \{[d] \mid d \in \Delta^{\mathcal{I}}\} \\ A^{\mathcal{J}} &= \{[d] \mid d \in A^{\mathcal{I}}\} \\ P^{\mathcal{J}} &= \{([d_1], [d_2]) \mid \exists d \in [d_1], d' \in [d_2] \text{ s.t. } (d, d') \in P^{\mathcal{I}}\} \\ a^{\mathcal{J}} &= [a^{\mathcal{I}}]. \end{aligned}$$

We show that $\mathcal{J}$ is a model of $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$ that satisfies C_0. It is standard to show the following by induction on C:

Claim 1. For all $d \in \Delta^{\mathcal{I}}$ and $C \in \mathsf{cl}(C_0, \mathcal{T}, \mathcal{A})$: $d \in C^{\mathcal{I}}$ iff $[d] \in C^{\mathcal{J}}$.

Claim 1 implies that $\mathcal{J}$ satisfies C_0 and is a model of the KB $(\mathcal{T}, \mathcal{A})$. To prove that $\mathcal{J}$ is a model of $\mathsf{Circ}_{\mathsf{CP}}(\mathcal{T}, \mathcal{A})$, it thus remains to show that $\mathcal{J}$ is minimal w.r.t. $<_{\mathsf{CP}}$. Assume for a proof by contradiction that there exists a model $\mathcal{J}'$ of $\mathcal{T}$ and $\mathcal{A}$ such that $\mathcal{J}' <_{\mathsf{CP}} \mathcal{J}$. Define $\mathcal{I}'$ as follows:

$$\begin{aligned} \Delta^{\mathcal{I}'} &= \Delta^{\mathcal{I}} \\ A^{\mathcal{I}'} &= \bigcup_{[d] \in A^{\mathcal{J}'}} [d] \\ P^{\mathcal{I}'} &= \bigcup_{([d_1],[d_2]) \in P^{\mathcal{J}'}} [d_1] \times [d_2] \text{ if } P \in V \\ P^{\mathcal{I}'} &= P^{\mathcal{I}} \text{ if } P \in F \\ a^{\mathcal{I}'} &= a^{\mathcal{I}}. \end{aligned}$$

Observe that, by construction, each fixed concept name A has the same interpretation in $\mathcal{I}$ and $\mathcal{I}'$.

Claim 2. Let $d, d' \in \Delta^{\mathcal{I}'}$ and let R be a role occurring in $\mathcal{T}$. Then

1. if $R \in V$, then $(d, d') \in R^{\mathcal{I}'}$ iff $([d], [d']) \in R^{\mathcal{J}'}$;
2. if $R \in F$, then $(d, d') \in R^{\mathcal{I}'}$ implies $([d], [d']) \in R^{\mathcal{J}'}$.

For Point 1, assume first that $R \in V$. Let $(d, d') \in R^{\mathcal{I}'}$. By construction $(d, d') \in [d_1] \times [d_2]$, for some $([d_1], [d_2]) \in R^{\mathcal{J}'}$. Clearly, $[d_1] = [d]$ and $[d_2] = [d']$. The converse direction is by construction. For Point 2, assume $R \in F$ and let $(d, d') \in R^{\mathcal{I}'}$. Then $(d, d') \in R^{\mathcal{I}}$. By construction $([d], [d']) \in R^{\mathcal{J}}$. Then, using $R \in F$ and the semantics it follows that $([d], [d']) \in R^{\mathcal{J}'}$.

Claim 3: For all $d \in \Delta^{\mathcal{I}'}$ and $C \in \mathsf{cl}(C_0, \mathcal{T}, \mathcal{A})$: $d \in C^{\mathcal{I}'}$ iff $[d] \in C^{\mathcal{J}'}$.

The proof is by induction on the structure of C, where the interesting case is $C = \exists R$. If $R \in V$, Claim 3 follows directly from Point 1 of Claim 2. Assume that $R \in F$. By Point 2 of Claim 2, $d \in (\exists R)^{\mathcal{I}'}$ implies $[d] \in (\exists R)^{\mathcal{J}'}$. Conversely, assume that $[d] \in (\exists R)^{\mathcal{J}'}$. Clearly, we have that $([d], [d']) \in R^{\mathcal{J}'}$, for some $[d'] \in \Delta^{\mathcal{J}'}$. Since $R \in F$, $([d], [d']) \in R^{\mathcal{J}}$, i.e. $[d] \in (\exists R)^{\mathcal{J}}$. By Claim 1, $d \in (\exists R)^{\mathcal{I}}$ and using that $R \in F$ we obtain that $d \in (\exists R)^{\mathcal{I}'}$.

We now prove that $\mathcal{I}'$ is a model of $\mathcal{T}$ and $\mathcal{A}$. Indeed, if $d \in C_1^{\mathcal{I}'} \setminus C_2^{\mathcal{I}'}$ for some $C_1 \sqsubseteq C_2 \in \mathcal{T}$, then, by Claim 3, $[d] \in C_1^{\mathcal{J}'} \setminus C_2^{\mathcal{J}'}$ which contradicts the assumption that $\mathcal{J}'$ is a model of $\mathcal{T}$. Let $R \sqsubseteq S \in \mathcal{T}$ and assume that $(d, d') \in R^{\mathcal{I}'} \setminus S^{\mathcal{I}'}$. If R and S are varying, by Point 2 of Claim 2 we obtain that $([d], [d']) \in R^{\mathcal{J}'} \setminus S^{\mathcal{J}'}$ in contradiction to $\mathcal{J}'$ being a model of $\mathcal{T}$. If R and S are fixed, then $(d, d') \in R^{\mathcal{I}} \setminus S^{\mathcal{I}}$ in contradiction to $\mathcal{I}$ being a model of $\mathcal{T}$. Finally, if R is fixed and S varying, by Point 2 of Claim 2, $([d], [d']) \in R^{\mathcal{J}'}$ and Point 1 implies that $([d], [d']) \notin S^{\mathcal{J}'}$, again a contradiction. These three cases are exhaustive since our circumscribed knowledge base is role-layered. Therefore, $\mathcal{I}'$ is a model of $\mathcal{T}$. That $\mathcal{I}'$ is a model of $\mathcal{A}$ follows directly from the construction of $\mathcal{I}'$.

Finally, notice that for each $A \in \mathsf{N_C}$, $A^{\mathcal{I}} \odot A^{\mathcal{I}'}$ iff $A^{\mathcal{J}} \odot A^{\mathcal{J}'}$, where $\odot = \subseteq, \supseteq$. Consequently, since $M \subseteq \mathsf{N_C}$, $\mathcal{J}' <_{\mathsf{CP}} \mathcal{J}$ implies $\mathcal{I}' <_{\mathsf{CP}} \mathcal{I}$. Therefore, $\mathcal{I}$ is not a model of $\mathsf{Circ_{CP}}(\mathcal{T}, \mathcal{A})$ and we have derived a contradiction. □

The single exponential model property just proved implies the following decidability result.

Theorem 4. *Satisfiability of concepts relative to circumscribed role-layered DL-Lite$^{\mathcal{R}}_{bool}$ KBs without minimized roles is decidable.*

Note that we also obtain a NExp$^{\text{NP}}$-upper bound for checking concept satisfiability: given C_0 and $\mathsf{Circ_{CP}}(\mathcal{T}, \mathcal{A})$ guess a model $\mathcal{I}$ with $|\Delta^{\mathcal{I}}| \leq 2^n + |\mathsf{Ind}(\mathcal{A})|$, where n is the concept size of C_0 and $(\mathcal{T}, \mathcal{A})$ and then check using an NP-oracle whether $\mathcal{I}$ is a model of C_0 and $\mathsf{Circ_{CP}}(\mathcal{T}, \mathcal{A})$.

6 Open Problems

We briefly discuss some computational problems regarding DLs with circumscription that remain open.

- First note that we have not proved any new results for circumscription patterns with minimized roles. In particular, the decidability and complexity of circumscribed reasoning in DL-Lite$^{\mathcal{F}}_{bool}$ and DL-Lite$^{\mathcal{R}}_{bool}$ with minimized roles remains open.
- Our concern in this was paper was decidability of reasoning in circumscribed DLs without the finite model property and/or fixed roles instead of a detailed complexity analysis. Thus, the complexity of reasoning in circumscribed DL-Lite$^{\mathcal{F}}_{bool}$ KBs with fixed roles (and without minimized roles), the complexity of reasoning in circumscribed $\mathcal{ALCFI}$ KBs without fixed and minimized roles, and the complexity of reasoning in role-layered circumscribed DL-Lite$^{\mathcal{R}}_{bool}$ KBs without minimized roles remains open. For $\mathcal{ALCFI}$, we conjecture concept satisfiability to be $\mathsf{NExp}^{\mathsf{NP}}$-complete. Note that, in this case, hardness follows from the $\mathsf{NExp}^{\mathsf{NP}}$-lower bound for $\mathcal{ALC}$ established in [6].
- It remains open whether the condition of being role-layered is necessary for obtaining the finite model property/decidability result for DL-Lite$^{\mathcal{R}}_{bool}$.
- Finally, it would be of great interest to extend our results to more expressive ontology and query languages and, for example, to consider the decidability and complexity of conjunctive query answering relative to circumscribed KBs.

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