On Finite and Unrestricted Query Entailment beyond $\mathcal{SQ}$ with Number Restrictions on Transitive Roles*

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We study query entailment in extensions of the description logic (DL) $\mathcal{SQ}$ allowing number restrictions ($\mathcal{Q}$) to be applied to transitive roles ($\mathcal{S}$). Most previous work on query entailment in expressive DLs, such as $\mathcal{SHIQ}$ or $\mathcal{SHOQ}$, forbid the interaction of number restrictions and transitive roles [2–4], but it is required in areas like biomedicine, e.g., to restrict the number of certain parts an organ has. For instance, one can express that the human heart has exactly one mitral valve, which has to be shared by its left and right atrium [5]. Allowing for the interaction of $\mathcal{S}$ and $\mathcal{Q}$ is dangerous in the sense that even modest extensions of $\mathcal{SQ}$, such as with role inclusions or inverse roles, lead to an undecidable satisfiability problem [6]. Decidability of satisfiability in $\mathcal{SQ}$ and in its extension with nominals was shown several years ago [6, 7], but only recently tight computational complexity bounds were established [8]. Even more recently, decidability for entailment of regular path queries over $\mathcal{SQ}$ knowledge bases was established. More precisely, based on a novel *tree-like model property* of $\mathcal{SQ}$ it was possible to devise an automata-based decision procedure yielding a tight 2ExpTime upper bound [5].

The objective of our investigation is to paint a more complete picture of query entailment in DLs with number restrictions on transitive roles. We pursue the following two specific goals.

1. We aim at understanding the limits of decidability of query entailment for such DLs. To this end, we investigate the extensions of $\mathcal{SQ}$ by *nominals* ($\mathcal{SOQ}$) and *controlled inverse roles* ($\mathcal{SIQ}^-$), where we allow number restrictions on inverse non-transitive roles and only existential restrictions on inverse transitive roles. As query language, we consider *positive existential regular path queries*, thus capturing the common languages of conjunctive queries and regular path queries.
2. We initiate the study of *finite* query entailment for $\mathcal{SIQ}^-$ and $\mathcal{SOQ}$, where one is interested in reasoning only over finite models. This distinction is crucial because in database applications, both database instances and the models they represent are commonly assumed to be finite. The study of

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finite query entailment in $\mathcal{SQ}$ is interesting since, due to the presence of transitivity, $\mathcal{SQ}$ lacks *finite controllability*, and therefore unrestricted and finite entailment do not coincide. Interestingly, most previous works on finite query entailment consider logics lacking finite controllability because of number restrictions and inverse roles [9–12]. The study of finite query entailment in logics with transitivity (without number restrictions on transitive roles) started only recently [13–15]. Here, we focus on finite entailment of positive existential queries in $\mathcal{SOQ}$ and of instance queries in $\mathcal{SIQ}^{-}$.

Contributions. We start by showing a *tree-like model property* for both $\mathcal{SOQ}$ and $\mathcal{SIQ}^{-}$. More specifically, we carefully extend and adapt the *canonical* tree decompositions that were introduced for $\mathcal{SQ}$ in previous work [5] to also incorporate the presence of controlled inverses and nominals. Next, we prove that if a query is not entailed by a knowledge base (KB), then there is a counter-model with a canonical tree decomposition of small width. This tree-like model property is the basis for automata-based approaches to unrestricted and finite query entailment in the remainder of the paper. First, we construct tree automata to optimally decide entailment of regular path queries over $\mathcal{SOQ}$ and $\mathcal{SIQ}^{-}$ KBs in 2ExpTime. We move then to finite entailment of positive existential queries over $\mathcal{SOQ}$ KBs, showing again an optimal 2ExpTime upper bound. To this end, we look at more refined canonical tree decompositions, which ensure the existence of a finite counter model. In other words, we reduce finite query entailment to entailment over models with this special canonical tree decomposition. Finally, we investigate the complexity for unrestricted and finite instance query (IQ) entailment in $\mathcal{SIQ}^{-}$. In particular, we show that IQ entailment is 2ExpTime-hard both in the finite and in the unrestricted case. We found this surprising since it is rarely the case that IQ entailment becomes more difficult when inverses are added to the logic. Moreover, the result provides an orthogonal reason for 2ExpTime-hardness for conjunctive query entailment in $\mathcal{SIQ}^{-}$ [16]. We complement this lower bound with a matching upper bound in the unrestricted case, thus confirming the conjecture that satisfiability in $\mathcal{SIQ}^{-}$ is decidable [6]. In the finite case, we show a 2ExpTime-upper bound for KBs using a single transitive role. Note that $\mathcal{SIQ}^{-}$ with a single transitive role is a notational variant of the *graded modal logic with converse* $\mathbf{K4}(\Diamond_{\geq}, \Diamond^{-})$. Thus, our result entails 2ExpTime-completeness for global consequence in $\mathbf{K4}(\Diamond_{\geq}, \Diamond^{-})$, which was only known to be decidable [17].

Outlook. This study provides a step towards a complete picture of query entailment in DLs with number restrictions on transitive roles. There are several natural next steps involving finite entailment. For $\mathcal{SIQ}^{-}$ the first thing to do is to cover the full logic; the second thing is to go beyond instance queries. In general, it would be interesting to study data complexity and consider transitive closure instead of transitivity.

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