The Modular Structure of an Ontology: an Empirical Study

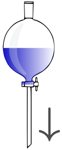
Chiara Del Vescovo Bijan Parsia Uli Sattler *Thomas Schneider*

School of Computer Science, University of Manchester, UK

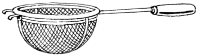
DL, 7 May 2010 WoMo, 11 May 2010

Three questions

What is my ontology about?



How many modules does my ontology have?



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How do we identify relevant modules?

And now . . .

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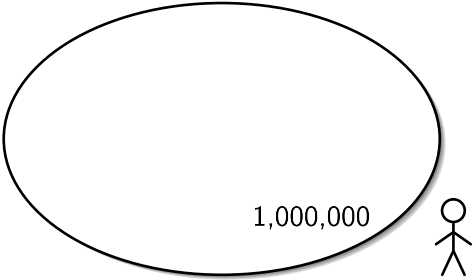
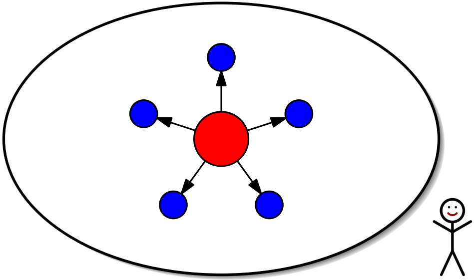
What is my ontology about?

We can't inspect all its axioms.



What is my ontology about?

We can inspect its modular structure, obtained a posteriori.



We bet Robert Stevens

- Ontology about periodic table of the chemical elements
- Logical structure $\approx$ intended modelling?
  - What is its modular structure?
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We bet Robert Stevens

- Ontology about periodic table of the chemical elements
- Logical structure ≈ intended modelling?
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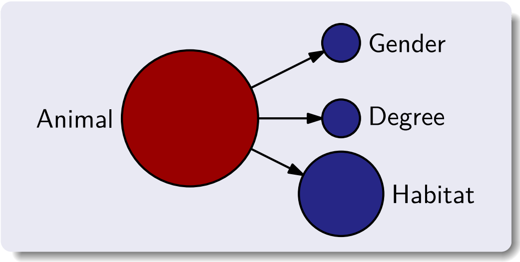
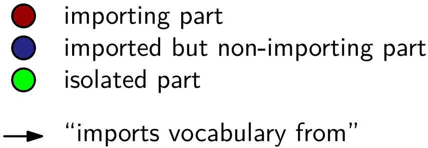
- Challenge: *automatic* partition into meaningful modules

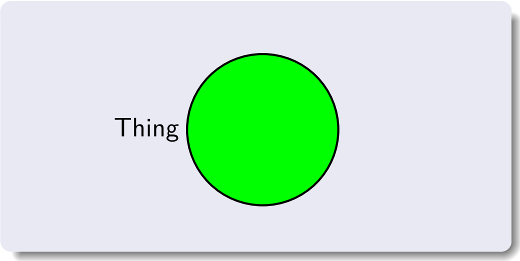
Modular structure with existing tools

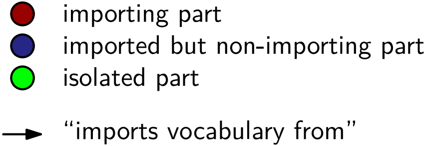
Partition of **Koala** via E-connections in Swoop





Partition for ontology **Periodic**





Locality-based modules (LBMs)

Module extraction service

Input: ontology $\mathcal{O}$; set Σ of terms from $\mathcal{O}$

Output: subset $\mathcal{M} = \text{mod}(\Sigma, \mathcal{O})$ of $\mathcal{O}$

Guarantee: for all axioms α with $\text{sig}(\alpha) \subseteq \Sigma$:

$$\mathcal{O} \models \alpha \quad \text{iff} \quad \mathcal{M} \models \alpha \qquad \textbf{(Coverage)}$$

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Modules providing coverage

- encapsulate knowledge about the *topic* Σ
- are important for modular import/reuse:
  "Give me all that $\mathcal{O}$ knows about the topic Σ"
- are hard to extract if minimality is required

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LBMs

- provide coverage and therefore encapsulation
- are not always minimal, but often of reasonable size
- can be efficiently computed
- have important robustness properties

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General remarks:

- Often $\text{sig}(\mathcal{M}) \neq \Sigma$
- Different seed signatures can lead to the same module

$$\begin{array}{ccc} & \text{sig}(\mathcal{M}) & \\ / & & \backslash \\ \Sigma_1 & \cdots & \Sigma_n \end{array}$$

Modular structure via LBMs

Want to extract *all (relevant)* LBMs in order to:

- obtain a finer-grained analysis
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Modular structure via LBMs

Want to extract *all (relevant)* LBMs in order to:

- obtain a finer-grained analysis
- guide users in choosing the right topic(s)
- draw conclusions on characteristics of an ontology:
  - To which extent does $\mathcal{O}$ cover its topics?
  - How strongly are certain terms connected in $\mathcal{O}$?
  - What is the axiomatic richness of $\mathcal{O}$?
  - Does $\mathcal{O}$ have superfluous parts?
  - Agreement between logical and intended intuitive modelling?

And now . . .

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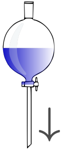

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Obvious lower and upper bounds

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- (We're working on tighter bounds.)
- Do real-life ontologies fall into the worst case?

An algorithm that extracts all modules

Results when applied to two small ontologies:

Ontology	#Ax	#Terms	**#mods**	Theor. Max.	Time
Koala	42	25	**3660**	33 554 432	9s
Mereology	44	25	**1952**	33 554 432	3min

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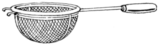
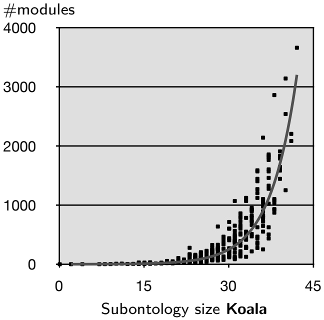
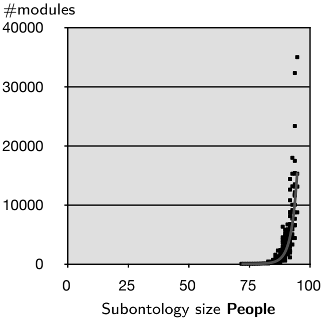
1. Are 3660 and 1952 “exponential” numbers? 
2. How to filter for interesting modules? 

Modularisation of subontologies

- Modularised randomly generated parts of 8 ontologies
- Example growth of module numbers:





Trendline equation: $y = O(1.2^x)$ $y = O(1.5^x)$

Confidence: 0.90 0.96

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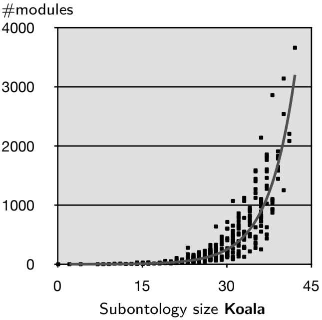

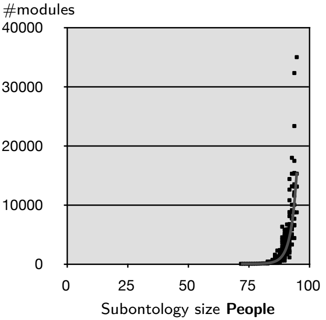

Trendline equation: $y = O(1.2^x)$ $y = O(1.5^x)$ **Exponential!**

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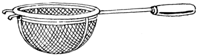

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How do we identify relevant modules?

Unification of similar modules

- Identify sets $\mathfrak{M} = \{\mathcal{M}_1, \ldots, \mathcal{M}_k\}$ of modules that differ in only few axioms
- Replace $\mathfrak{M}$ with $\bigcup \mathfrak{M}$ and $\bigcap \mathfrak{M}$

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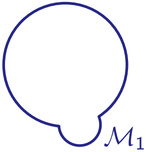

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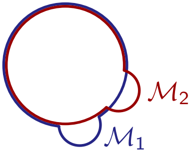

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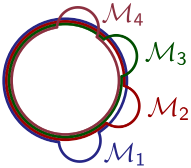

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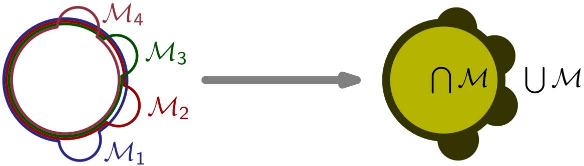

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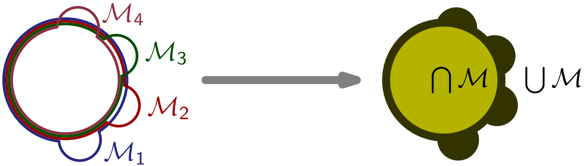
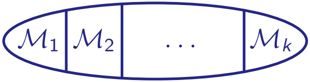


- Outcome for **Koala**: no significant reduction in module numbers

Genuine and fake modules

- Identify modules that are an agglomeration of other modules and contain no new information
- $\mathcal{M}$ is *fake* if there is partition $\mathcal{M} = \mathcal{M}_1 \uplus \cdots \uplus \mathcal{M}_k$ with pairwise disjoint $\mathrm{sig}(\mathcal{M}_i)$.



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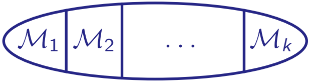
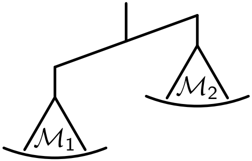


- All other modules are *genuine*.
- Outcome for **Koala**:
  66% of the 3660 modules are genuine.

Weight analysis

. . . by scalesperson Chiara ;-)



Weight analysis

- Number of terms in the module m

$\mathrm{sig}(\mathcal{M})$

Weight analysis

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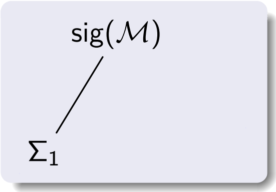

Weight analysis

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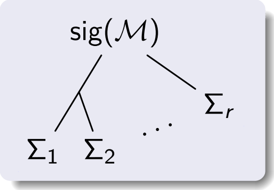

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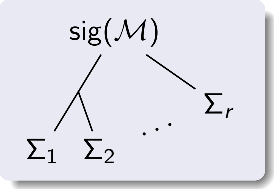
$\rightsquigarrow$ PullingPower($\mathcal{M}$)



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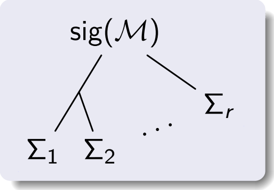
$\rightsquigarrow$ PullingPower($\mathcal{M}$) $\frac{m}{s}$



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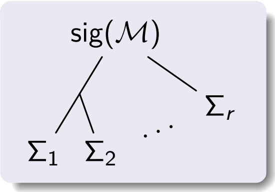
$\leadsto$ PullingPower($\mathcal{M}$) $\frac{m}{s}$

$\leadsto$ Cohesion($\mathcal{M}$)



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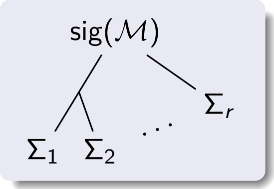

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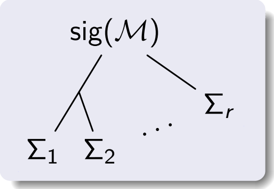
Weight($\mathcal{M}$) $w = \frac{r \cdot m}{s^2}$



Example: **Koala**

$\text{Animal} \sqsubseteq\ \geqslant 1\ \text{hasHabitat}$
$\text{Animal} \sqsubseteq\ =1\ \text{hasGender}$
$\text{DryEucalyptForest} \sqsubseteq \text{Forest}$
$\text{Female} \equiv \exists\ \text{hasGender}.\{\text{female}\}$
$\text{Forest} \sqsubseteq \text{Habitat}$
$\text{GraduateStudent} \sqsubseteq \text{Student}$
$\text{GraduateStudent} \sqsubseteq \exists\ \text{hasDegree}.(\{\text{BA}\} \sqcup \{\text{BS}\})$
$\text{Koala} \sqsubseteq \exists\ \text{hasHabitat}.\text{DryEucalyptForest}$
$\text{Koala} \sqsubseteq \text{Marsupials}$
$\text{Koala} \sqsubseteq \exists\ \text{isHardworking}.\{\text{false}\}$
$\text{KoalaWithPhD} \equiv \text{Koala} \sqcap \exists\ \text{hasDegree}.\{\text{PhD}\}$
$\text{Male} \equiv \exists\ \text{hasGender}.\{\text{male}\}$
$\text{Marsupials} \sqsubseteq \text{Animal}$
$\text{Marsupials} \sqsubseteq \neg\ \text{Person}$
$\text{Parent} \equiv \text{Animal} \sqcap \geqslant 1\ \text{hasChildren}$
$\text{Parent} \sqsubseteq \text{Animal}$
$\text{Person} \sqsubseteq \text{Animal}$
$\text{Person} \sqsubseteq \neg\ \text{Marsupials}$
$\text{Quokka} \sqsubseteq \exists\ \text{isHardworking}.\{\text{true}\}$
$\text{Quokka} \sqsubseteq \text{Marsupials}$
$\text{Rainforest} \sqsubseteq \text{Forest}$
$\text{TasmanianDevil} \sqsubseteq \text{Marsupials}$
$\text{University} \sqsubseteq \text{Habitat}$
$\exists\ \text{hasChildren} \sqsubseteq \text{Animal}$
$\top \sqsubseteq \forall\ \text{hasChildren}.\text{Animal}$
$\exists\ \text{hasDegree} \sqsubseteq \text{Person}$

$\top \sqsubseteq \forall\ \text{hasDegree}.\text{Degree}$
$\top \sqsubseteq\ \leqslant 1\ \text{hasGender}$
$\exists\ \text{hasGender} \sqsubseteq \text{Animal}$
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$\exists\ \text{hasHabitat} \sqsubseteq \text{Animal}$
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$\top \sqsubseteq\ \leqslant 1.\text{isHardWorking}$
$\text{BA} : \text{Degree}$
$\text{BS} : \text{Degree}$
$\text{MA} : \text{Degree}$
$\text{PhD} : \text{Degree}$
$\text{female} : \text{Gender}$
$\text{male} : \text{Gender}$

$\text{MaleStudentWith3Daughters} \equiv \text{Student} \sqcap \forall\ \text{hasChildren}.\text{Female} \sqcap \exists\ \text{hasGender}.\{\text{male}\} \sqcap =1\ \text{hasChildren}$
$\text{Student} \equiv \text{Person} \sqcap \exists\ \text{hasHabitat}.\text{University} \sqcap \exists\ \text{isHardworking}.\{\text{true}\}$

Example: **Koala**

MaleStudentWith3Daughters,
isHardWorking, University,
Student, Parent, hasChildren

17 axioms

{Student, Parent}
{Student, hasChildren}
{MaleStudentWith3Daughters}
{hasChildren, University, isHardWorking}
{Parent, University, isHardWorking}

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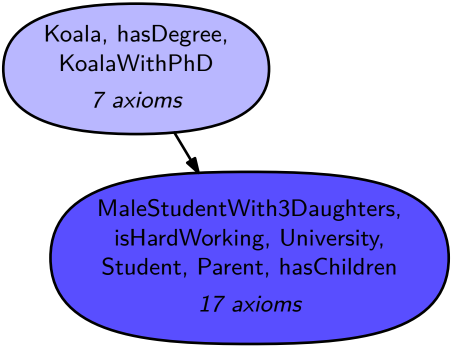

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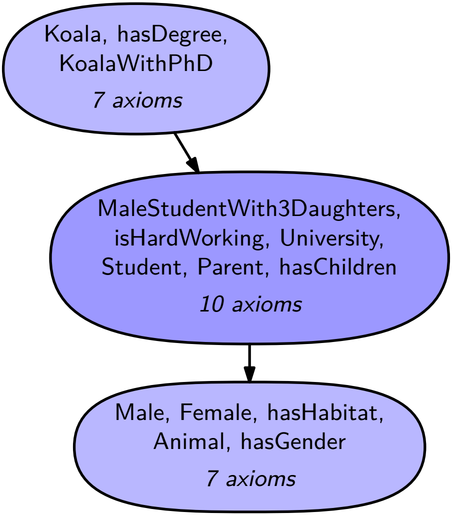
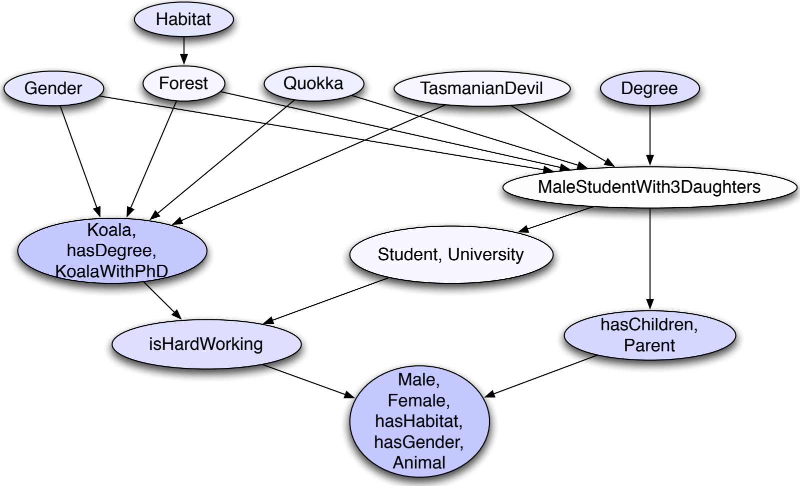


Example: **Koala**

After the first 12 heaviest modules . . .

Example: **Koala**



Work in progress

Is it necessary for the weight analysis to compute all modules?

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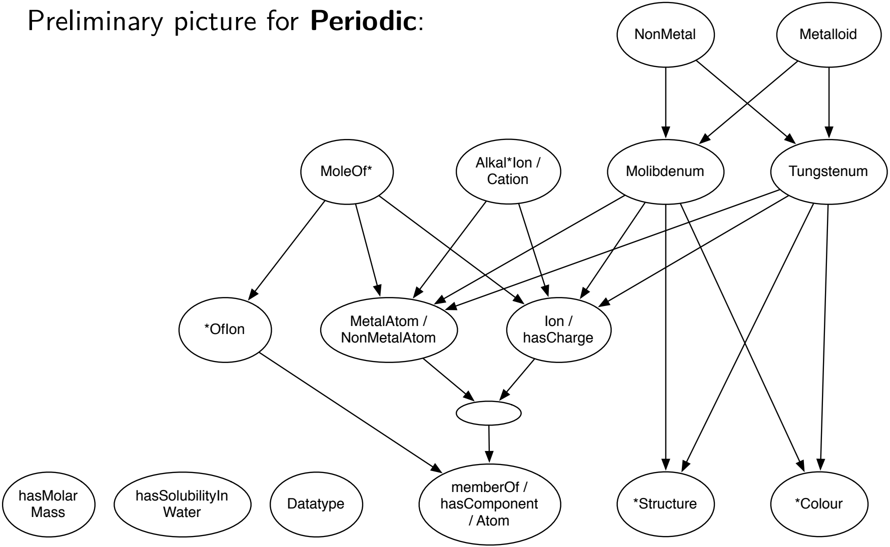
Remember: Weight($\mathcal{M}$) $w = \frac{r \cdot m}{s^2}$

$\rightsquigarrow$ search within modules with very small seed signatures

Work in progress

Preliminary picture for **Periodic**:



Outlook

- Find heaviest modules without computing all modules
- How many modules can ontologies have?
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Thank you.