Algebraic Properties of Qualitative Spatio-Temporal Calculi[1]

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[1]Thanks for discussions at the "Spatial Reasoning Teatime".

And now . . .

Qualitative spatio-temporal representation and reasoning

= Symbolic way to

- represent spatio-temporal knowledge
- and draw inferences from it

- Common approach:
  define set $\mathcal{R}$ of relations to describe spatial relationships
  - use $\mathcal{R}$ as primitives for representation
  - employ techniques from constraint and qualitative reasoning to reason about the primitives
  - various domains, typically infinite

Applications by domain

Time interval relations

- Medical diagnostics *Simplified Allen*
- Law texts *Allen-13*
- Business and manufacturing: diagnostics *Allen-13*

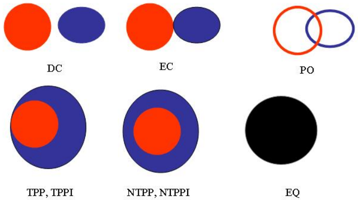
Relation	Symbol	Inverse	Meaning
x before y	b	bi	
x meets y	m	mi	
x overlaps y	o	oi	
x during y	d	di	
x starts y	s	si	
x finishes y	f	fi	
x equal y	eq	eq	

Applications by domain

Positions, regions (topology)

- Planning, robotics, navigation — *RCC, Block Algebra, ROC*
- Natural language processing — *Rectangle Alg., ...*
- Image understanding — *9-int, CarDir, RCC, Allen*
- GIS, spatial query answering — *9-int, CarDir, RCC*
- Traffic tracking — *9-intersection*
- CAD and manufacturing — *LR, RCC*

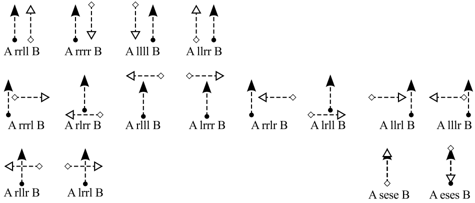
RCC-8

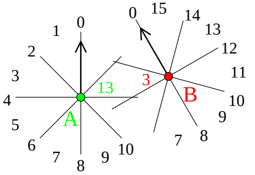


Applications by domain

Moving point objects, directional information

- Robotics, navigation, motion planning — *OPRA, Dipole Flipflop, StarVars*
- GIS, spatial query answering — *QTC*
- Traffic tracking — *Dipole*
- Ambient intelligence, smart environments (scene analysis, task modelling) — *OPRA, RCC*





. . .

Dipole DRA_{fp}

OPRA_4

Representation and reasoning tasks required

- Knowledge representation
- Data interpretation
- Inference
  - Constraint-based reasoning
    (CSP-SAT, -ENT, -MOD, -MIN)
  - Neighbourhood-based reasoning
    (Relaxing constraints, continuity constraints, dominance space)
  - Logical reasoning
    (Deduction, abduction)
  - Learning
    (Inductive logic programming)

What is a qualitative calculus?

Some answers from the literature

- A weak representation of a non-associative relation algebra
  [Egenhofer & Rodríguez 1999; Ligozat et al. 2003; Ligozat & Renz 2004]
- A system of relations forming a constraint algebra
  [Nebel & Scivos 2002]

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Commonly agreed ingredients

- Set $\mathcal{R}$ of relations
- Operations $\cup, \cap, \bar{}, \circ, \breve{}$ with certain properties
  (closure, algebraic properties)
- Mapping to a domain with certain properties
  (e.g., JEPD)

And in reality?

Zoo of qualitative calculi

- 36 implemented in SparQ; many more in the literature
- "classical" calculi, usually with strong algebraic properties (e.g., Allen-13, RCC-8)
- more recent calculi, often with weaker algebraic properties (e.g., Cardinal Direction Relations, Rectangular Cardinal Relations)

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Research question

To what extent do existing calculi meet the imposed requirements?

$\rightsquigarrow$ If not, what are minimal requirements?

$\rightsquigarrow$ How can we classify calculi according to their properties?

On the agenda today

- Revisit and generalise the definition of a qualitative calculus
- Identify notions of algebras that cover existing calculi
- Discuss relevance of algebraic properties for spatial reasoning
- Evaluate the algebraic properties of existing calculi
  $\rightsquigarrow$ derive improved generic reasoning procedure
- Examine information-preservation properties of calculi during reasoning

And now . . .

Calculi à la Ligozat & Renz (1)

Basic notions

- **Universe** (domain) $\mathcal{U}$: spatio-temporal entities
- Set $\mathcal{R}$ of **base relations** over $\mathcal{U}$
  - Uncertain information $\rightsquigarrow$ union of base relations
  - Restriction to binary relations in this work
- $\mathcal{R}$ is **JEPD**: jointly exhaustive and pairwise disjoint
  - Jointly exhaustive: $\mathcal{R}$ covers $\mathcal{U} \times \mathcal{U}$

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Partition scheme

- Pair $(\mathcal{U}, \mathcal{R})$ with $\mathcal{R}$ being JEPD
- $\mathcal{R}$ contains the identity relation id
  and is closed under converse $\breve{}$

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- Set of symbolic relations
- Plus interpretation $\varphi =$ mapping to a partition scheme
- Plus symbolic operations $\breve{}, \diamond$ (converse, *weak* composition)

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  - $\breve{}, \diamond$ usually given by tables

These requirements are strong

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$\leadsto$ **Weaken requirements to partition schemes and calculi!**

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(Analogous definition for other reasoning problems)

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Common techniques for solving QCSPs

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Common techniques for solving QCSPs

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- Algebraic closure (a-closure)
  - sufficient condition for consistency guaranteed by "$\supseteq$" of abstract composition
  - For some calculi, a-closure known to be necessary too
- Composition of *arbitrary* relations R, S is uniquely determined by the composition results of the base relations in R, S (composition table)

Existing qualitative spatio-temporal calculi

	Name	Domain	#BR
	9-Intersection	simple 2D regions	8
●	Allen's interval relations	intervals (order)	13
●	Block Algebra	n-dimensional blocks	13^n
●	Cardinal Dir. Calculus CDC	directions (point abstr.)	9
	Cardinal Dir. Relations CDR	regions	218
	CycOrd, binary $\mathrm{CYC_b}$	oriented lines	4
●	Dependency Calculus	points (partial order)	5
	Dipole Calculus[a] $\mathrm{DRA_f}$	directions from line segm.	72
	$\mathrm{DRA_{fp}}$	directions from line segm.	80
	DRA-connectivity	connectivity of line segm.	7
	Geometric Orientation	relative orientation	4
	INDU	intervals (order, rel. dur.n)	25
	OPRA_m, $m = 1, \ldots, 8$ (Oriented Point Rel. Algebra)	oriented points	$4m \cdot (4m + 1)$
●	Point Calculus	points (total order)	3
	Qualitat. Traject. Calc. $\mathrm{QTC_{B11}}$	moving point obj.s in 1D	9
	$\mathrm{QTC_{B12}}$	"	17
	$\mathrm{QTC_{B21}}$	moving point obj.s in 2D	9
	$\mathrm{QTC_{B22}}$	"	27
	$\mathrm{QTC_{C12}}$	"	81
	$\mathrm{QTC_{C22}}$	"	305
●	Region Connection Calc. RCC-5	regions	5
●	RCC-8	regions	8
●	Rectangular Cardinal Rel.s RDR	regions	36
	Star Algebra $\mathrm{STAR_4}$	directions from a point	9

a-closure decides consistency

And now . . .

Why relation algebras?

If a calculus is a relation algebra (RA), then . . .

- certain optimisations in reasoners are permitted
- e.g., associativity of $\diamond$ ensures fast processing of paths:
  - if a QCSP contains xRy, ySz, zTu,
    then compute relation between x, u "from left to right"
  - without associativity, compute $(R \diamond S) \diamond T$ **and** $R \diamond (S \diamond T)$

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RAs have been considered for spatio-temporal calculi before.

[Ligozat & Renz 2004, Düntsch 2005, F. Mossakowski 2007]

What is a relation algebra?

R_1	$r \cup s = s \cup r$	$\cup$-commutativity
R_2	$r \cup (s \cup t) = (r \cup s) \cup t$	$\cup$-associativity
R_3	$\overline{\bar{r} \cup \bar{s}} \cup \overline{\bar{r} \cup s} = r$	Huntington's axiom
R_4	$r \diamond (s \diamond t) = (r \diamond s) \diamond t$	$\diamond$-associativity
R_5	$(r \cup s) \diamond t = (r \diamond t) \cup (s \diamond t)$	$\diamond$-distributivity
R_6	$r \diamond \mathsf{id} = r$	identity law
R_7	$(r^{\smile})^{\smile} = r$	$^{\smile}$-involution
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- **NA** non-associative RA $\mathsf{R}_1, \ldots, \mathsf{R}_{10}$ *minus* R_4
- **WA, SA** weakly/semi-associative RA weakenings of R_4

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Calculi à la Ligozat & Renz: based on NA's (by definition)

Testing algebraic properties of calculi

Research questions

1. Which calculi correspond to RAs (NAs, WAs, SAs)?
2. Which weaker algebra notions correspond to other calculi?

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Experimental setup

- Corpus: 31 calculi listed before
- Used HETS (Heterogeneous Tool Set) to test
  - the SparQ implementation of each calculus
  - against CASL specifications of the RA axioms (+ weakenings)
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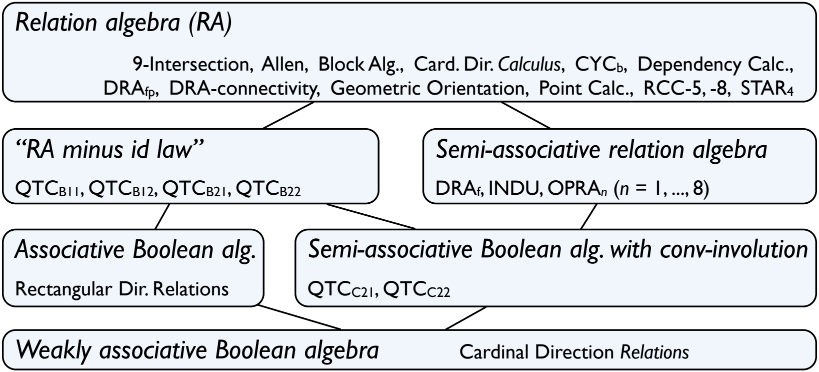
- Parallel tests via SparQ's built-in function `analyze-calculus`

Test results per calculus

Calculus	R_4	SA	WA	R_6	R_{6l}	R_7	R_9	PL	R_{10}
Allen	✓	✓	✓	✓	✓	✓	✓	✓	✓
Block Algebra	✓	✓	✓	✓	✓	✓	✓	✓	✓
Cardinal Direction *Calculus*	✓	✓	✓	✓	✓	✓	✓	✓	✓
CYC_b, Geometric Orientation	✓	✓	✓	✓	✓	✓	✓	✓	✓
DRA_fp, DRA-conn.	✓	✓	✓	✓	✓	✓	✓	✓	✓
Point Calculus	✓	✓	✓	✓	✓	✓	✓	✓	✓
RCC-5, Dependency Calc.	✓	✓	✓	✓	✓	✓	✓	✓	✓
RCC-8, 9-Intersection	✓	✓	✓	✓	✓	✓	✓	✓	✓
STAR_4	✓	✓	✓	✓	✓	✓	✓	✓	✓
DRA_f	19	✓	✓	✓	✓	✓	✓	✓	✓
INDU	12	✓	✓	✓	✓	✓	✓	✓	✓
OPRA_n, $n \leqslant 8$	21–91[b]	✓	✓	✓	✓	✓	✓	✓	✓
$\mathrm{QTC}_\mathrm{Bxx}$	✓	✓	✓	89–100		✓	✓	✓	✓
$\mathrm{QTC}_\mathrm{C21}$	55	✓	✓	99	99	✓	2	<1	1
$\mathrm{QTC}_\mathrm{C22}$	79	✓	✓	99	99	✓	3	<1	1
Rectang. Direction Relations	✓	✓	✓	97	92	89	66	7	52
Cardinal Direction *Relations*	28	17	✓	99	99	98	12	<1	88

Test results per algebra notion



(Abstract partition scheme yields Boolean algebra with distributivity)

What do we gain from these results?

In the paper: discussion on relevance of axioms to reasoning

$\leadsto$ Theoretical underpinning of optimisations implemented

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  - $\rightsquigarrow$ reduce memory consumption by 50%

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Many base relations

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Research questions:

- How well do calculi with many relations make use of the potentially higher information content?
- Does the information content differ between the 6 groups of calculi established in the algebraic study?

Measuring information content

To be measured:

- How much additional information is obtained by applying $\diamond$?

  Observe xRy, ySz

  Compute $R \diamond S$ and conclude $x(R \diamond S)z$

  $\rightsquigarrow$ Is it worthwhile to observe xTz too?

- Generalise this to chains $R_1 \diamond \cdots \diamond R_k$

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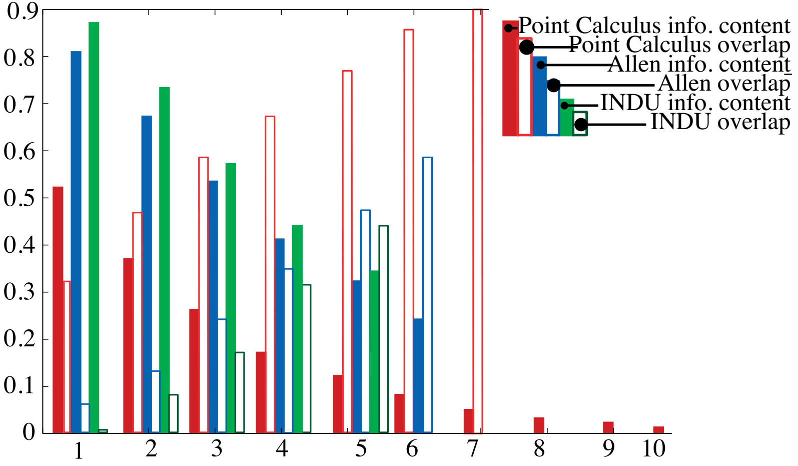
Test results

- Tested 24 calculi, for *k* up to 14
- Graphical view for Point Calculus, Allen-13 and INDU:



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- No observable relation between IC/IO and algebraic properties

And now ...

Conclusion

We have . . .

- weakened requirements to spatio-temporal calculi to accommodate existing calculi
- discussed algebraic properties of binary calculi
- classified existing calculi according to their algebraic properties
- measured the information preservation by existing calculi

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Thank you.

Quantitative account: IC for all calculi

Calculus	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Allen	92.3	81.4	66.8	52.8	41.1	31.8	24.5	18.9	14.5	11.2	8.6	6.6	5.1	3.9	3.0
Block Algebra	99.4	96.5	89.0	77.7	65.3	53.4	43.0	34.1	27.0	21.1	16.4	12.8	9.9	7.7	5.9
CDC	88.9	76.8	60.4	44.5	31.6	21.9	14.9	10.1	6.8	4.6	3.1	2.0	1.4	0.9	0.6
CYC_b	75.0	62.5	46.9	32.8	21.9	14.1	8.8	5.4	3.2	1.9	1.1	0.6	0.4		
DRA_{fp}	98.8	89.9	69.0	45.0	25.8	13.4	6.5	3.0	1.3	0.6	0.2				
DRA-con	85.7	74.6	59.0	43.4	30.4	20.5	13.5	8.7	5.6	3.5	2.2	1.3	0.8	0.5	0.3
Point Calculus	66.7	51.9	37.0	25.5	17.3	11.6	7.8	5.2	3.5	2.3	1.5	1.0	0.7	0.5	
RCC-5	80.0	56.8	34.9	19.7	10.6	5.5	2.7	1.3	0.6	0.3					
RCC-8	87.5	62.3	38.0	21.1	11.0	5.5	2.6	1.2	0.6	0.3					
STAR_4	88.9	66.9	45.0	28.5	17.4	10.3	6.0	3.5	2.0	1.1	0.6	0.4			
DRA_f	98.6	90.6	70.4	46.3	26.7	13.9	6.7	3.0	1.3	0.6	0.2				
INDU	96.0	86.9	72.5	57.5	44.1	33.2	24.7	18.2	13.4	9.9	7.2	5.3	4.0	2.9	2.1
OPRA_1	95.0	82.0	55.8	30.8	14.5	6.2	2.4	0.9	0.3						
OPRA_2	98.6	90.3	64.1	32.9	13.0	4.3	1.3	0.3							
OPRA_3	99.4	93.1	71.4	40.2	16.7	5.6									
OPRA_4	99.6	94.6	76.7	48.0											
QTC_{B11}	88.9	90.0	93.2	95.8	97.5	98.6	99.1	99.5	99.7						
QTC_{B12}	94.1	91.2	90.5	91.3	92.8	94.2	95.6								
QTC_{B21}	88.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
QTC_{B22}	96.3	51.9	37.0	25.5	17.3	11.6	7.8	5.2	3.5	2.3	1.5	1.0	0.7	0.5	0.3
QTC_{C21}	98.8	92.5	76.6	68.6	69.5	73.0	76.5	79.4	81.8	83.7	85.2	86.4	87.4		
QTC_{C22}	99.5	95.1	78.0	69.3	51.2										
RDR	97.2	82.6	63.2	45.7	32.0	22.0	15.0	10.1	6.8	4.6	3.1	2.0	1.4	0.9	0.6
CDR	99.5	78.8	60.9	48.9	39.6	32.1	26.1	21.2	17.2	14.0	11.4	9.3	7.6	6.2	5.1