Querying the Unary Negation Fragment with Regular Path Expressions

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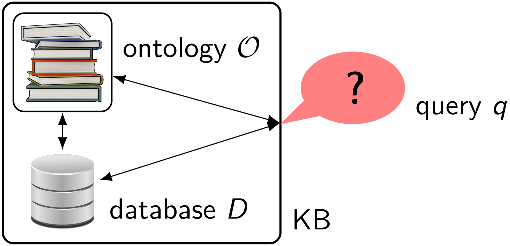
Ontology-mediated query answering



Ontology-mediated query (OMQ):

triple $Q = (\mathcal{O}, \Sigma, q)$ with Σ the data signature

OMQ evaluation:

Given Q and D over Σ, does $D \cup \mathcal{O} \models q$ hold?

OMQ containment $Q_1 \subseteq Q_2$:

Given $Q_1 = (\mathcal{O}_1, \Sigma, q_1)$ and $Q_2 = (\mathcal{O}_2, \Sigma, q_2)$,

does $D \cup \mathcal{O}_1 \models q_1$ imply $D \cup \mathcal{O}_2 \models q_2$ for all D over Σ?

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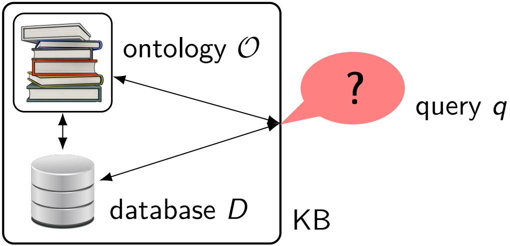

Typical ontology languages:

- Description logics (DLs)
- Languages based on existential rules (aka Datalog+/–)

Typical query languages: CQs, UCQs

OMQ evaluation in 2ExpTime (data complexity: coNP)

Unary Negation and regular expressions

We study: an expressive language that subsumes

- popular ontology languages
  (including transitive roles and/or regular expressions on roles)
- popular query languages
  (U)CQs, (U)C2RPQs

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UNFO$^{\text{reg}}$ = UNFO + regular path expressions on binary relations

$$\varphi \; ::= \; P(\bar{x}) \mid \qquad x = y \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \exists x \varphi \mid \neg\varphi(x)$$

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$$E \ ::= \ R \mid R^{-} \mid E \cup E \mid E \cdot E \mid E^{*} \mid \varphi(x)?$$

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UC2RPQ = UCQ with atoms $E(x, y)$ as above (UC2RPQs as tests)
"union of conjunctive 2-way regular path queries"

UNFO$^{\text{reg}}$: a powerful language

UNFO$^{\text{reg}}$ can express ...

- **many popular MLs and DLs**
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- **C2RPQs $\rightsquigarrow$ queries are for free**
  OMQ evaluation and containment reduce to (un)satisfiability:

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  - $D \cup \mathcal{O} \models q$ iff $\mathcal{O} \wedge D \wedge \neg q$ is unsatisfiable
  - $(\mathcal{O}, \Sigma_{\text{full}}, q_1) \subseteq (\mathcal{O}, \Sigma_{\text{full}}, q_2)$ iff $\mathcal{O} \wedge q_1 \wedge \neg q_2$ is unsatisfiable

Our results

Theorem 1

(1) Satisfiability in $\mathrm{UNFO}^{\mathrm{reg}}$ is 2ExpTime-complete.

(2) Model checking is $\mathrm{P}^{\mathrm{NP}[O(\log^2 n)]}$-complete.

Same complexity as for UNFO [ten Cate & Segoufin, LMCS'13].

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Theorem 2

For OMQs with with $\mathcal{O}$ in UNFO^{reg} and q UC2RPQ,

(3) evaluation is 2ExpTime-compl. (combined), coNP-compl. (data);

(4) containment w.r.t. Σ_{full} is 2ExpTime-complete.

2ExpTime upper bounds follow from Theorem 1 (1).

All lower bounds inherited from $(\mathcal{ALCI}, \text{CQ})$.

[Lutz IJCAR'08; Calvanese et al. KR'06]

Overview

1. Results for Satisfiability
2. Results for OMQ Evaluation & Containment
3. Conclusion and Outlook

Next . . .

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Schedule

We want to show:

Lemma 3

Satisfiability in UNFO$^{\text{reg}}$ is in 2ExpTime.

Previous approach for UNFO

Reduction to μ-calculus [ten Cate & Segoufin, LMCS'13]

✘ Does not transfer to UNFO$^{\text{reg}}$

Our approach

- show tree-like model property: treewidth $\leq$ width(φ)
- represent tree-like models via labelled trees
- characterise satisfaction of C2RPQs in labelled trees via tree-shaped witnesses
- build automata that accept the tree-like models of the input fma.

Normal form

UNFO$^{\text{reg}}$ formulas in **normal form**:

- connected C2RPQs with $= 1$ free variable and normal UNFO$^{\text{reg}}$ formulas $\psi(x)$? as tests
- $\neg\varphi(x) \quad \varphi(x) \vee \psi(x) \quad \exists x\, \varphi(x)$

Lemma 4

Every UNFO$^{\text{reg}}$ sentence can be transformed into an equivalent normal UNFO$^{\text{reg}}$ sentence φ' in exponential time.

- $|\varphi'|$ is exponential in $|\varphi|$
- **Width** of φ' (max. number of variables in a sub-C2RPQ): linear!

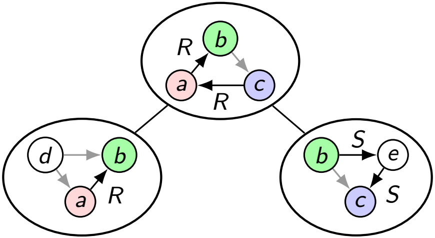
Tree-like structures

Tree-like structure of width m: pair $\mathfrak{T} = (T, \mathrm{bag})$ where

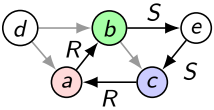
- T is a tree
- $\mathbf{bag}(w)$, $w \in T$: finite structure of domain size $\leq m$ such that for every element a, the set of nodes containing a is connected

Example

$\mathfrak{T}$ of width $m = 3$:



corresponding struc. $\mathfrak{A}_{\mathfrak{T}}$:



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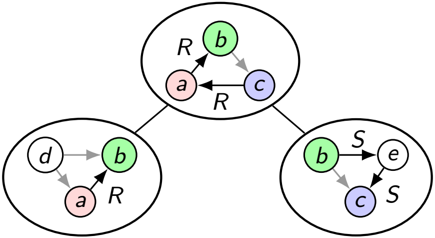

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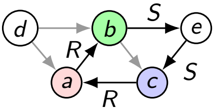

$$\mathfrak{A}_{\mathfrak{T}} \models \exists x \, RS^* R(x, x)$$

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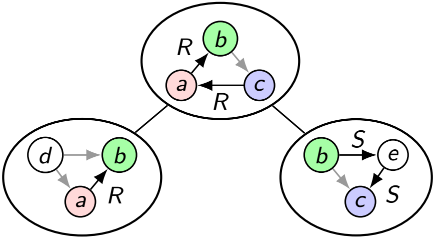

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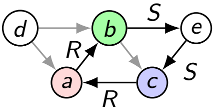

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because

$\mathfrak{A}_{\mathfrak{T}} \models RS^*R(a, a)$

Type-decorated tree-like structures

Fix UNFO$^{\text{reg}}$ formula φ_0.

Replace regular atoms $E(x, y)$ with atoms $\boldsymbol{\mathcal{A}(x, y)}$, $\mathcal{A}$ an **NFA**.

Type-decorated tree-like structure (TDTLS) for φ_0: pair $(\mathfrak{T}, \tau)$ s.t.

- $\mathfrak{T}$ is a tree-like structure
- τ assigns a 1-type $t(x) \subseteq \mathrm{cl}(\varphi_0)$ to each element in $\mathfrak{A}_{\mathfrak{T}}$

Satisfaction for regular atoms $\boldsymbol{(\mathfrak{T}, \tau)} \models \boldsymbol{\mathcal{A}(a, b)}$ is defined via "looking up" tests $\varphi(a)$? directly in $\tau(a)$

Proper TDTLSs

Labelling τ must be "globally" consistent with all $\exists$- and C2RPQ-subformulas of φ_0:

A TDTLS is proper if

(1) $\exists x\, \varphi(x) \in \tau(a)$ are witnessed by $\varphi(x) \in \tau(b)$ for some b

(2) C2RPQs in $\tau(a)$ are witnessed by homomorphisms into $(\mathfrak{T}, \tau)$

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Lemma 5 (Tree-like model property of UNFO$^{\text{reg}}$)

Let φ_0 be a normal UNFO$^{\text{reg}}$ sentence.

φ_0 is satisfiable iff there is a proper TDTLS $(\mathfrak{T}, \tau)$ for φ_0 s.t. $\varphi_0 \in \tau(a)$ for some a.

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Lemma 5 (Tree-like model property of UNFO$^{\text{reg}}$)

Let φ_0 be a normal UNFO$^{\text{reg}}$ sentence **of size n**.

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Lemma 5 (Tree-like model property of UNFO$^{\text{reg}}$)

Let φ_0 be a normal UNFO$^{\text{reg}}$ sentence **of size n and width m**.

φ_0 is satisfiable iff there is a proper TDTLS $(\mathfrak{T}, \tau)$ for φ_0 **of outdegree $O(n^2)$ and width $\leq m$** s.t. $\varphi_0 \in \tau(a)$ for some a.

Automata-based decision procedure

Use 2-way alternating tree automata **(2ATAs)**

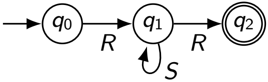
$\mathcal{A}$ accepts input $(\mathfrak{T}, \tau)$ iff $\mathfrak{T}$ proper and $\varphi_0 \in \tau(a)$ for some a

Then: φ_0 satisfiable iff $L(\mathcal{A}) \neq \emptyset$

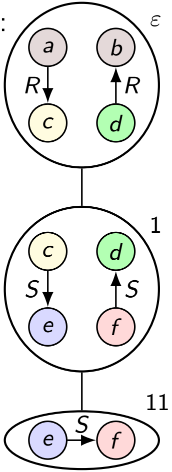
Problem: How can $\mathcal{A}$ check Condition (2) of properness?

(2) C2RPQs in $\tau(a)$ are witnessed by $\underbrace{\text{homomorphisms into } (\mathfrak{T}, \tau)}_{\textbf{need not be local!}}$

Non-locality of regular atoms

Example: C2RPQ $\mathcal{A}(x, y)$ with $\mathcal{A} =$ 

$\mathfrak{T}$:



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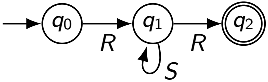
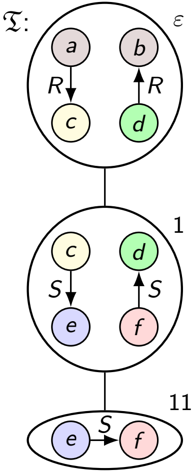
Example: C2RPQ $\mathcal{A}(x, y)$ with $\mathcal{A} =$ 



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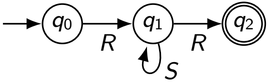

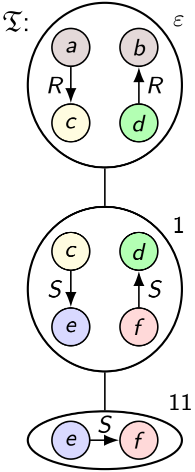

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(1) Subdivide atom $\mathcal{A}(a, b)$

$$\mathcal{A}[q_0, q_1](a, c) \quad \wedge \quad \mathcal{A}[q_1, q_2](d, b) \quad \wedge$$

$$\mathcal{A}[q_1, q_1](c, e) \quad \wedge \quad \mathcal{A}[q_1, q_1](f, d) \quad \wedge$$

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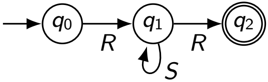
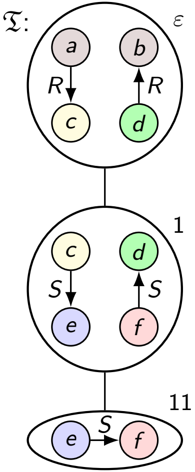
Example: C2RPQ $\mathcal{A}(x, y)$ with $\mathcal{A} =$ 



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Subdivision into $O(|\varphi_0|^2)$ atoms suffices

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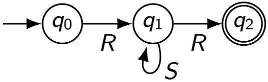
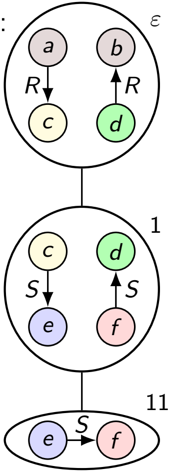
Example: C2RPQ $\mathcal{A}(x,y)$ with $\mathcal{A} =$ 

$\mathfrak{T}$: 

$\mathfrak{A}_{\mathfrak{T}} \models \mathcal{A}(a,b)$; h targets bags $\varepsilon, 1, 11$

(2) Split the resulting set of atoms

$$\begin{array}{c}
\mathcal{A}[q_0,q_1](a,c) \quad \wedge \quad \mathcal{A}[q_1,q_2](d,b) \\
\hline
\mathcal{A}[q_1,q_1](c,e) \quad \wedge \quad \mathcal{A}[q_1,q_1](f,d) \\
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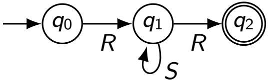

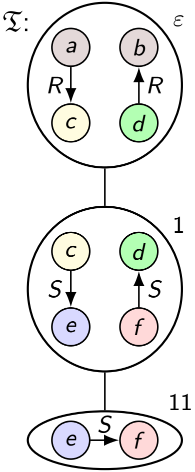

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Resulting witness tree:

$$\Big(\varepsilon,\ \mathcal{A}[q_0,q_1](a,c) \ \wedge\ \mathcal{A}[q_1,q_2](d,b)\Big)$$

$$\Big(1,\ \mathcal{A}[q_1,q_1](c,e) \ \wedge\ \mathcal{A}[q_1,q_1](f,d)\Big)$$

$$\Big(11,\ \mathcal{A}[q_1,q_1](e,f)\Big)$$

Automata-friendly properness condition

Good news:

Lemma 6

Subdivisions and splittings introduce at most exponentially many new sub-C2RPQs.

Condition (2) of properness is equivalent to:

(2′) C2RPQs in $\tau(a)$ are witnessed by witness trees in $(\mathfrak{T}, \tau)$

$\rightsquigarrow$ construct 2ATA with exponentially many states

Next ...

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2. Results for OMQ Evaluation & Containment
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Overview

We want to show:

Lemma 7

OMQ evaluation is in coNP (data complexity).

Observation: $D \cup \mathcal{O} \models q$ iff $\underbrace{D \text{ is unsatisfiable with } \mathcal{O} \wedge \neg q}$

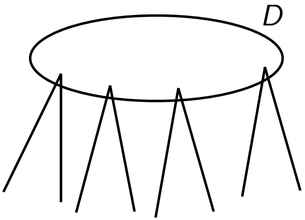
Lemma 8

Satisfiability of databases with UNFO^{reg} sentences is in NP.

Database satisfiability in NP

Fix UNFO$^{\text{reg}}$ sentence φ_0 and database D

Idea: guess & check consistent decoration of dom(D) with 1-types



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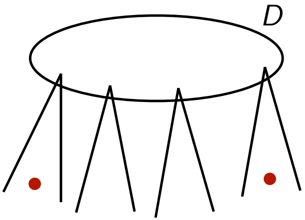

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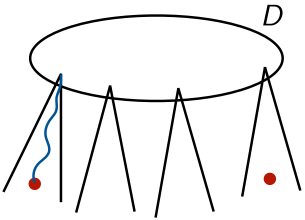

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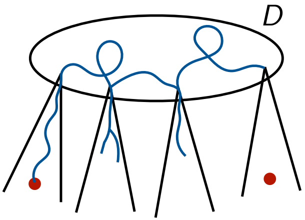

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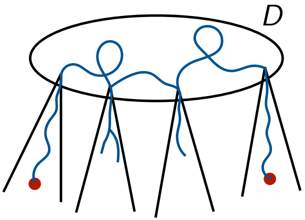

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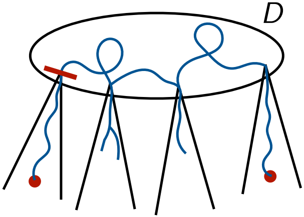

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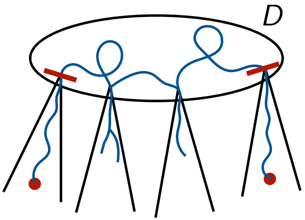

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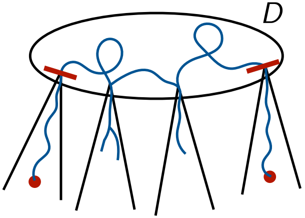

$\rightsquigarrow$ 3-subdivisions of regular atoms $\mathcal{A}(x, y)$ suffice

Database satisfiability in NP

Type decoration τ is **proper** if

- each $\tau(a)$ is satisfiable
- similar conditions as (1) and (2) for proper TDTLSs

Lemma 9

D is satisfiable with φ_0 iff D has a proper type decoration τ with $\varphi_0 \in \tau(a)$ for some $a \in \mathrm{dom}(D)$.

$\rightsquigarrow$ **NP decision procedure:**

- Guess candidate decoration
- Precompute satisfaction of all regular atoms
- Verify properness

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3. Conclusion and Outlook

Conclusion

UNFO$^{\text{reg}}$ is computationally well-behaved:

- SAT and model checking are not harder than in UNFO.
- OMQ evaluation and OMQ containment for (UNFO$^{\text{reg}}$, C2RPQ) are not harder than for ($\mathcal{ALCI}$, CQ).

Outlook

- Finite-model variants of SAT and OMQ evaluation
- Extensions:
  - Base case in regular path expressions with *any* UNFO^{reg} formula $\varphi(x, y)$ instead of $R(x, y)$ $\rightsquigarrow$ contains ICPDL
  - Replace C2RPQs with linear Datalog
  - Add constants, fixed points, role inclusions ... ("DL world")
- OMQ containment, general case: $(\mathcal{O}_1, \mathbf{\Sigma}, q_1) \overset{?}{\subseteq} (\mathcal{O}_2, \mathbf{\Sigma}, q_2)$

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Thank you.