Modularity in Ontologies: Incremental Classification, Logical Diff and Forgetting

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ESSLLI, 16 August 2013

Plan for today

1. Incremental Classification
2. Logical Difference for Ontology Versioning
3. Forgetting and Uniform Interpolation
4. Conclusion

And now . . .

Incremental Classification: Motivation

- early detection of modelling errors is important for developing and maintaining ontologies
- frequent classification of ontologies required
- issue:
  - long response times from reasoners
- solutions:
  - classification algorithms and optimisation techniques
  - lightweight logics with tractable classification such as $\mathcal{EL}$
- limitation: similarities between versions of ontologies are not taken into account (reasoning is repeated from scratch)

Example: Incremental Classification

Original ontology $\mathcal{O}_1$

$$\text{Cystic\_Fibrosis} \equiv \text{Fibrosis} \sqcap \exists \text{located\_In.Pancreas} \quad (1)$$

$$\text{Genetic\_Fibrosis} \equiv \text{Fibrosis} \sqcap \text{has\_Origin.Genetic\_Origin} \quad (2)$$

$$\text{Pancreatic\_Fibrosis} \equiv \text{Fibrosis} \sqcap \text{Pancreatic\_Disorder} \quad (3)$$

$$\text{Genetic\_Fibrosis} \sqsubseteq \text{Genetic\_Disorder} \quad (4)$$

$$\text{Pancreatic\_Disorder} \equiv \text{Disorder} \sqcap \exists \text{located\_In.Pancreas} \quad (5)$$

- 9 concept names $+ \top + \bot$
- taxonomy: 121 subsumptions

Example: Incremental Classification

Updated ontology $\mathcal{O}_2 = (\mathcal{O}_1 \setminus \mathcal{O}^-) \cup \mathcal{O}^+$

$$\begin{aligned}
\text{Cystic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \exists\text{located_In.Pancreas} && (1)\\
&\quad \sqcap \exists\text{has_Origin.Genetic_Origin} \\
\text{Genetic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{has_Origin.Genetic_Origin} && (2)\\
\text{Pancreatic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{Pancreatic_Disorder} && (3)\\
\text{Genetic_Fibrosis} &\sqsubseteq \text{Genetic_Disorder} && (4)\\
\text{Pancreatic_Disorder} &\equiv \text{Disorder} \sqcap \exists\text{located_In.Pancreas} && (5)
\end{aligned}$$

- $\mathcal{O}^- = \{(1)\}$
- $\mathcal{O}^+ = \{(1) \sqcap \exists\text{has\_Origin.Genetic\_Origin}\}$

Example: Incremental Classification

Updated ontology $\mathcal{O}_2 = (\mathcal{O}_1 \setminus \mathcal{O}^-) \cup \mathcal{O}^+$

$$
\begin{aligned}
\text{Cystic\_Fibrosis} &\equiv \text{Fibrosis} \sqcap \exists \text{located\_In.Pancreas} & (1)\\
&\quad \sqcap \exists \text{has\_Origin.Genetic\_Origin} & \\
\text{Genetic\_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{has\_Origin.Genetic\_Origin} & (2)\\
\text{Pancreatic\_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{Pancreatic\_Disorder} & (3)\\
\text{Genetic\_Fibrosis} &\sqsubseteq \text{Genetic\_Disorder} & (4)\\
\text{Pancreatic\_Disorder} &\equiv \text{Disorder} \sqcap \exists \text{located\_In.Pancreas} & (5)
\end{aligned}
$$

- Which subsumptions have changed?

Incremental Classification

Idea

- small changes in ontologies affect relatively few subsumptions
- avoid recomputing unaffected subsumptions
- identify subsumptions affected by change using modules
- [Cuenca Grau et al., JAR 2010]

Incremental Classification using Modules

Definition

$\mathcal{M} \subseteq \mathcal{O}$ is **a module for axiom α in $\mathcal{O}$** if:

$\mathcal{M} \models \alpha$ iff $\mathcal{O} \models \alpha$.

- locality-based modules for sig(α) of $\mathcal{O}$ have this property

Proposition

Let $\mathcal{O}_1$ and $\mathcal{O}_2$ be ontologies and α an axiom. Let $\mathcal{M}^i_\alpha$ be a module for α in $\mathcal{O}_i$, for $i = 1, 2$.

- if $\mathcal{O}_1 \models \alpha$ and $\mathcal{M}^1_\alpha \subseteq \mathcal{O}_2$, then $\mathcal{O}_2 \models \alpha$
- if $\mathcal{O}_1 \not\models \alpha$ and $\mathcal{M}^2_\alpha \subseteq \mathcal{O}_1$, then $\mathcal{O}_2 \not\models \alpha$

Algorithm: Incremental Classification using Modules

Algorithm: Step 1

Input: $\mathcal{O}_1$, $\sqsubseteq_1$, $\mathcal{M}^1_A|_{A \in \mathsf{sig}(\mathcal{O}_1)}$, $\mathcal{O}^-$, $\mathcal{O}^+$

Output: $\mathcal{O}_2$, $\sqsubseteq_2$, $\mathcal{M}^2_A|_{A \in \mathsf{sig}(\mathcal{O}_2)}$

$\mathcal{O}_2 := (\mathcal{O}_1 \setminus \mathcal{O}^-) \cup \mathcal{O}^+$

For every $A \in \mathsf{sig}(\mathcal{O}_2) \setminus \mathsf{sig}(\mathcal{O}_1)$

$\quad \mathcal{M}^1_A := \mathcal{M}^1_\top$

$\quad$ for every $(\top, B) \in \sqsubseteq_1$: $\sqsubseteq_1 := \sqsubseteq_1 \cup \{(A, B)\}$

$\quad$ for every $(B, \bot) \in \sqsubseteq_1$: $\sqsubseteq_1 := \sqsubseteq_1 \cup \{(B, A)\}$

Update ontology

Set a module and update classification $\sqsubseteq_1$ for the new concept names

Algorithm: Incremental Classification using Modules

Algorithm: Step 2

$N^- := \emptyset$, $N^+ := \emptyset$

For every $A \in \text{sig}(\mathcal{O}_2)$

For every $\alpha \in \mathcal{O}^-$

if α **not** sig($\mathcal{M}^1_A$)**-local**, then $N^- := N^- \cup \{A\}$

For every $\alpha \in \mathcal{O}^+$

if α **not** sig($\mathcal{M}^1_A$)**-local**, then $N^+ := N^+ \cup \{A\}$

Determine concept names whose modules are affected by the change (using locality check for axioms)

Algorithm: Incremental Classification using Modules

Algorithm: Step 3

For every $A \in \mathrm{sig}(\mathcal{O}_2)$

If $A \in N^- \cup N^+$, then $\mathcal{M}_A^2 := \mathrm{extract\_module}(\{A\}, \mathcal{O}_2)$;

else $\mathcal{M}_A^2 := \mathcal{M}_A^1$

Determine the module for every concept name (recomputing affected modules)

Algorithm: Incremental Classification using Modules

Algorithm: Step 4

For every $A \in \mathsf{sig}(\mathcal{O}_2)$
  For every $B \in \mathsf{sig}(\mathcal{O}_2) \cup \{\bot\}$
    if $(A \in N^-$ and $A \sqsubseteq_1 B)$ or
      $(A \in N^+$ and $A \not\sqsubseteq_1 B)$
    then $\sqsubseteq_2 := \sqsubseteq_2 \cup \{(A, B)\}$ if $\mathcal{M}^2_A \models A \sqsubseteq B$
    else $\sqsubseteq_2 := \sqsubseteq_2 \cup \{(A, B)\}$ if $(A, B) \in \sqsubseteq_1$

Determine new classification $\sqsubseteq_2$ (possibly using a reasoner – any reasoner)

Example: Incremental Classification

Updated ontology $\mathcal{O}_2 = (\mathcal{O}_1 \setminus \mathcal{O}^-) \cup \mathcal{O}^+$

$$\begin{aligned}
\text{Cystic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \exists \text{located_In.Pancreas} \\
&\quad \sqcap \ \exists \text{has_Origin.Genetic_Origin} && (6) \\
\text{Genetic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{has_Origin.Genetic_Origin} && (7) \\
\text{Pancreatic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{Pancreatic_Disorder} && (8) \\
\text{Genetic_Fibrosis} &\sqsubseteq \text{Genetic_Disorder} && (9) \\
\text{Pancreatic_Disorder} &\equiv \text{Disorder} \sqcap \exists \text{located_In.Pancreas} && (10)
\end{aligned}$$

- Algorithm Step 1:
  - skip since no new symbols were added

Example: Incremental Classification

Updated ontology $\mathcal{O}_2 = (\mathcal{O}_1 \setminus \mathcal{O}^-) \cup \mathcal{O}^+$

$$\begin{aligned}
\text{Cystic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \exists\text{located_In.Pancreas} && (6)\\
&\sqcap\ \exists\text{has_Origin.Genetic_Origin} &&\\
\text{Genetic_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{has_Origin.Genetic_Origin} && (7)\\
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\text{Genetic_Fibrosis} &\sqsubseteq \text{Genetic_Disorder} && (9)\\
\text{Pancreatic_Disorder} &\equiv \text{Disorder} \sqcap \exists\text{located_In.Pancreas} && (10)
\end{aligned}$$

- Algorithm Step 2:
  - $N^- = \{\text{Cystic\_Fibrosis}, \text{Pancreatic\_Fibrosis}\}$
  - $N^+ = \{\text{Cystic\_Fibrosis}\}$

Example: Incremental Classification

Updated ontology $\mathcal{O}_2 = (\mathcal{O}_1 \setminus \mathcal{O}^-) \cup \mathcal{O}^+$

$$\begin{aligned} \text{Cystic\_Fibrosis} &\equiv \text{Fibrosis} \sqcap \exists \text{located\_In.Pancreas} \\ &\sqcap \ \exists \text{has\_Origin.Genetic\_Origin} && (6) \\ \text{Genetic\_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{has\_Origin.Genetic\_Origin} && (7) \\ \text{Pancreatic\_Fibrosis} &\equiv \text{Fibrosis} \sqcap \text{Pancreatic\_Disorder} && (8) \\ \text{Genetic\_Fibrosis} &\sqsubseteq \text{Genetic\_Disorder} && (9) \\ \text{Pancreatic\_Disorder} &\equiv \text{Disorder} \sqcap \exists \text{located\_In.Pancreas} && (10) \end{aligned}$$

- Algorithm Step 3:
  - compute modules for Cystic_Fibrosis and Pancreatic_Fibrosis

Example: Incremental Classification

Updated ontology $\mathcal{O}_2 = (\mathcal{O}_1 \setminus \mathcal{O}^-) \cup \mathcal{O}^+$

$$
\begin{aligned}
\text{Cystic\_Fibrosis} &\equiv \text{Fibrosis} \sqcap \exists \text{located\_In.Pancreas} && (6)\\
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\text{Pancreatic\_Disorder} &\equiv \text{Disorder} \sqcap \exists \text{located\_In.Pancreas} && (10)
\end{aligned}
$$

- Algorithm Step 4:
  - compute whether $\mathcal{O}_2 \models \text{Pancreatic\_Fibrosis} \sqsubseteq B$
  - compute whether $\mathcal{O}_2 \models \text{Cystic\_Fibrosis} \sqsubseteq B$
  - perform only about 13 subsumption tests

And now . . .

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2. Logical Difference for Ontology Versioning
3. Forgetting and Uniform Interpolation
4. Conclusion

Logical Difference: Motivation

Task

- given two versions $\mathcal{T}_1$ and $\mathcal{T}_2$ of an ontology and a signature Σ, compute "the difference" between $\mathcal{T}_1$ and $\mathcal{T}_2$ observable in Σ in a query language $\mathcal{QL}$.

Syntactical difference

- Many tools compute the syntactical difference between versions of texts and program code.
- But many syntactic differences do not affect the semantics of ontologies!
- Example:
  - $\mathcal{T}_1 = \{A \sqsubseteq B_1 \sqcap B_2\}$, $\mathcal{T}_2 = \{A \sqsubseteq B_1, A \sqsubseteq B_2\}$
    $\Sigma = \{A, B_1, B_2\}$
  - Then $\mathcal{T}_1 \neq \mathcal{T}_2$, but $\mathcal{T}_1 \equiv_\Sigma^{SO} \mathcal{T}_2$.

Logical Difference: Motivation

Structural difference

- extends syntactic diff by taking into account structural meta-information of distinct versions of ontologies
- regards ontologies as structured objects (e.g., taxonomy, set of RDF triplets, set of axioms)
- changes are structural operations (e.g., adding/deleting/extending/renaming classes)
- but:
  - syntax dependent and no formal semantics
  - tailored to applications of ontologies based on taxonomy
  - ontology based data access not captured

Logical Difference

$\mathcal{T}_1$ and $\mathcal{T}_2$ ontologies, $\mathcal{QL}$ a query language, Σ a signature.
The logical difference between $\mathcal{T}_1$ and $\mathcal{T}_2$ wrt. $(\mathcal{QL},\Sigma)$ is defined as

$$\mathsf{Diff}^{\mathcal{QL}}_{\Sigma}(\mathcal{T}_1,\mathcal{T}_2) \cup \mathsf{Diff}^{\mathcal{QL}}_{\Sigma}(\mathcal{T}_2,\mathcal{T}_1),$$

where

- $\mathsf{Diff}^{\mathcal{QL}}_{\Sigma}(\mathcal{T}_1,\mathcal{T}_2) = \{\varphi \in \mathcal{QL} \mid \mathcal{T}_1 \models \varphi, \mathcal{T}_2 \not\models \varphi, \mathsf{sig}(\varphi) \in \Sigma\}$.
- $\mathsf{Diff}^{\mathcal{QL}}_{\Sigma}(\mathcal{T}_2,\mathcal{T}_1) = \{\varphi \in \mathcal{QL} \mid \mathcal{T}_2 \models \varphi, \mathcal{T}_1 \not\models \varphi, \mathsf{sig}(\varphi) \in \Sigma\}$.

Observation: $\mathsf{Diff}^{\mathcal{QL}}_{\Sigma}(\mathcal{T}_1,\mathcal{T}_2) \cup \mathsf{Diff}^{\mathcal{QL}}_{\Sigma}(\mathcal{T}_2,\mathcal{T}_1) = \emptyset$ iff $\mathcal{T}_1 \equiv^{\mathcal{QL}}_{\Sigma} \mathcal{T}_2$.

Problem: How to present $\mathsf{Diff}^{\mathcal{QL}}_{\Sigma}(\mathcal{T}_1,\mathcal{T}_2)$ if it is non-empty?

Σ-difference for $\mathcal{EL}$-terminologies

Take query language $\mathcal{QL}_{\mathcal{EL}}$ consisting of $C \sqsubseteq D$, where C, D are $\mathcal{EL}$-concepts. We also denote $\mathcal{QL}_{\mathcal{EL}}$ simply as $\mathcal{EL}$.
Set

$$\mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2) = \mathsf{Diff}_\Sigma^{\mathcal{EL}}(\mathcal{T}_1, \mathcal{T}_2).$$

Example of 'large' smallest elements in $\mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$:

- $\mathcal{T}_2 = \emptyset$;
- $\mathcal{T}_1 = \{A' \sqsubseteq B_0, A \equiv B_n\} \cup \{B_{i+1} \equiv \exists r.B_i \sqcap \exists s.B_i \mid i < n\}$;
- $\Sigma = \{A', A, r, s\}$.

For the minimal $C \sqsubseteq A \in \mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ we have $|C| = 2^n$.

Σ-difference for $\mathcal{EL}$-terminologies

Theorem ("Primitive Witnesses Theorem")

If $(C \sqsubseteq D) \in \mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ then either

- $(A \sqsubseteq D_0) \in \mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ or
- $(C_0 \sqsubseteq A) \in \mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$,

where A is a concept name and
A, C_0 — subconcepts of C;
D_0, A — subconcepts of D, resp.

In propositional $\mathcal{EL}$: if $C \sqsubseteq A_1 \sqcap A_2 \in \mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$, then

- $C \sqsubseteq A_1 \in \mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ or
- $C \sqsubseteq A_2 \in \mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$.

Compact representation of $\mathsf{Diff}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$

Let

- $\mathsf{diffL}_\Sigma(\mathcal{T}_1, \mathcal{T}_2) =$
$$\left\{ A \in \Sigma \;\middle|\; \begin{array}{l} \text{there is a } \Sigma\text{-concept } C \text{ in } \mathcal{EL} \text{ s.t.} \\ \mathcal{T}_1 \models A \sqsubseteq C \text{ and } \mathcal{T}_2 \not\models A \sqsubseteq C \end{array} \right\}$$
- $\mathsf{diffR}_\Sigma(\mathcal{T}_1, \mathcal{T}_2) =$
$$\left\{ A \in \Sigma \;\middle|\; \begin{array}{l} \text{there is a } \Sigma\text{-concept } C \text{ in } \mathcal{EL} \text{ s.t.} \\ \mathcal{T}_1 \models C \sqsubseteq A \text{ and } \mathcal{T}_2 \not\models C \sqsubseteq A \end{array} \right\}$$

$\mathsf{diffL}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ and $\mathsf{diffR}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ provide a list of concept names in Σ about which $\mathcal{T}_1$ "says more" than $\mathcal{T}_2$.

Σ-difference between $\mathcal{EL}$-terminologies

Theorem

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be $\mathcal{EL}$-terminologies and Σ a signature. Then

- $\mathsf{diffL}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ and
- $\mathsf{diffR}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$

can be computed in polynomial time. In particular, Σ-inseparability wrt. $\mathcal{EL}$ is tractable.

Tools

CEX

- implementation of tractable algorithm computing $\mathrm{DiffL}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ and $\mathrm{DiffR}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ for acyclic $\mathcal{EL}$-terminologies [Konev, Walther, Wolter, 2008]
- `http://www.csc.liv.ac.uk/~konev/software/`

OWLDiff

- CEX-diff for $\mathcal{EL}$-terminologies [Kremen, Smid, Kouba, 2011, to appear]
- plugins for Protégé and NeON toolkit
- `http://krizik.felk.cvut.cz/km/owldiff`

Tools

CEX2

- extends CEX to $\mathcal{ELH}^r$ (i.e. $\mathcal{EL}$ with role inclusion axioms and domain and range restrictions) without loosing tractability [Konev, Ludwig, Walther, Wolter, 2012]
- `http://www.csc.liv.ac.uk/~michel/software/cex2/`

LogDiffViz

- Protégé plugin that calls CEX2 and visualises ontology versions and the differences as a hierarchical structure
- `http://www.csc.liv.ac.uk/~cs8wg/LogDiffViz/`

CEX applied to SNOMED CT

Task: Compute the logical difference of two versions of SNOMED CT

- two versions:
  - SNOMED CT 2005 (SM-05):
    - 379 691 axioms
    - 09 February 2005
  - SNOMED CT 2006 (SM-06):
    - 389 472 axioms
    - 30 December 2006
- $\Sigma \subseteq \mathrm{sig}(\text{SM-05}) \cap \mathrm{sig}(\text{SM-06})$ randomly selected
- compute average (of time/memory/diff-size) over 20 samples for every signature size
- hardware: Intel Core 2 CPU at 2.13 GHz and 3 GB of RAM

SM-05 vs SM-06

Size of Σ	CEX: diff(SM-05,SM-06)			
	Time (Sec.)	Memory (MByte)	$\lvert diffL_\Sigma \rvert$	$\lvert diffR_\Sigma \rvert$
100	513.1	1 393.7	0.10	0.10
1 000	512.4	1 394.6	2.35	2.15
10 000	517.7	1 424.3	155.35	125.35
100 000	559.8	1 473.2	11 795.90	4 108.6

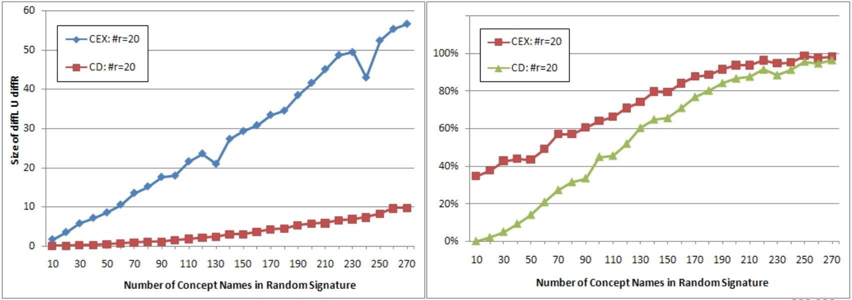
- Note: role box ignored

Comparison on the Joint Signature

- diff(SM-05,SM-06) on
  $\Sigma = \text{sig}(\text{SM-05}) \cap \text{sig}(\text{SM-06})$
  - 689 seconds
  - $|\text{diffL}_\Sigma| + |\text{diffR}_\Sigma| = 162010$
  - Class hierarchy comparison misses 32475 of them

Comparing with classification

- Combined $\mathrm{diffL}_\Sigma(\emptyset, M)$ and $\mathrm{diffR}_\Sigma(\emptyset, M)$
  - M is a subset of SM-05 containing $\sim 140,000$ axioms
  - Σ — randomly selected from M (incl. 20 role names)
  - avg. over 500 samples for each signature size
- Difference in class hierarchy



CEX on MEX

Instead of computing $\text{diffL}_\Sigma(\mathcal{T}_1, \mathcal{T}_2) \cup \text{diffR}_\Sigma(\mathcal{T}_1, \mathcal{T}_2)$ directly,

- first extract minimal Σ-modules $\mathcal{T}_1'$ and $\mathcal{T}_2'$ from $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively,
- then compute $\text{diffL}_\Sigma(\mathcal{T}_1', \mathcal{T}_2') \cup \text{diffR}_\Sigma(\mathcal{T}_1', \mathcal{T}_2')$.

Size of Σ	CEX: diff(SM-05,SM-06)				CEX: diff(Mod'05,Mod'06)	
	Time (Sec.)	Memory (MByte)	$\|\text{diffL}_\Sigma\|$	$\|\text{diffR}_\Sigma\|$	Time (Sec.)	Memory (MByte)
100	513.1	1 393.7	0.0	0.0	3.66	116.5
1 000	512.4	1 394.6	2.5	2.5	4.46	122.5
10 000	517.7	1 424.3	183.2	122.0	22.29	126.5
100 000	559.8	1 473.2	11 322.1	4 108.5	189.98	615.8
379741	790.0	1999.3	191714	684.1	1850.7	237044

And now . . .

Forgetting Vocabulary: Motivation

Forgetting vocabulary is eliminating that vocabulary from the ontology (involving a reformulation of the ontology).

Use-cases

- re-use: instead of whole ontology, use a potentially much smaller ontology resulting from forgetting
- predicate hiding: concealing confidential information in ontologies
- ontology summary: succinct presentation of what ontology states about non-forgotten vocabulary

The dual notion of forgetting is uniform interpolation.

Uniform Interpolation

Let $\mathcal{T}$ be a $\mathcal{EL}$-TBox and Σ a signature. A TBox $\mathcal{T}'$ is called a uniform interpolant of $\mathcal{T}$ wrt. Σ if the following holds:

- $\mathsf{sig}(\mathcal{T}') \subseteq \Sigma$;
- $\mathcal{T} \equiv_{\Sigma}^{\mathcal{EL}} \mathcal{T}'$.

Theorem

Let $\mathcal{T}_1', \mathcal{T}_2'$ be uniform interpolants of $\mathcal{T}_1$ and $\mathcal{T}_2$ wrt. Σ. The following conditions are equivalent:

- $\mathcal{T}_1 \equiv_{\Sigma}^{\mathcal{EL}} \mathcal{T}_2$;
- $\mathcal{T}_1'$ and $\mathcal{T}_2'$ are logically equivalent.

$\mathcal{EL}$-terminologies

Theorem

There exist an $\mathcal{EL}$-terminology $\mathcal{T}$ and Σ such that there does not exist an uniform interpolant of $\mathcal{T}$ wrt. Σ.

Proof. Let

$$\mathcal{T} = \{A \sqsubseteq B, B \sqsubseteq \exists r.B\}, \quad \Sigma = \{A, r\}.$$

An infinite axiomatisation of the uniform interpolant is given by

$$\{A \sqsubseteq \underbrace{\exists r. \cdots \exists r}_{n}.\top \mid n \geq 1\}.$$

A finite $\mathcal{T}_\Sigma$ does not exist (even in first-order logic).

Acyclic $\mathcal{EL}$-terminologies

Theorem

For acyclic $\mathcal{EL}$-terminologies, uniform interpolants always exist. In the worst case, exponentially many axioms are required.

Proof of second part. Let

$$\mathcal{T} = \{A \equiv B_1 \sqcap \cdots \sqcap B_n\} \cup \{A_{ij} \sqsubseteq B_i \mid 1 \leq i, j \leq n\}.$$

and

$$\Sigma = \{A\} \cup \{A_{ij} \mid 1 \leq i, j \leq n\}.$$

Then

$$\mathcal{T}_\Sigma = \{A_{1j_1} \sqcap \cdots \sqcap A_{n,j_n} \sqsubseteq A \mid 1 \leq j_1, \ldots, j_n \leq n\}$$

is a minimal uniform interpolant. Note that $|\mathcal{T}_\Sigma| = n^n$.

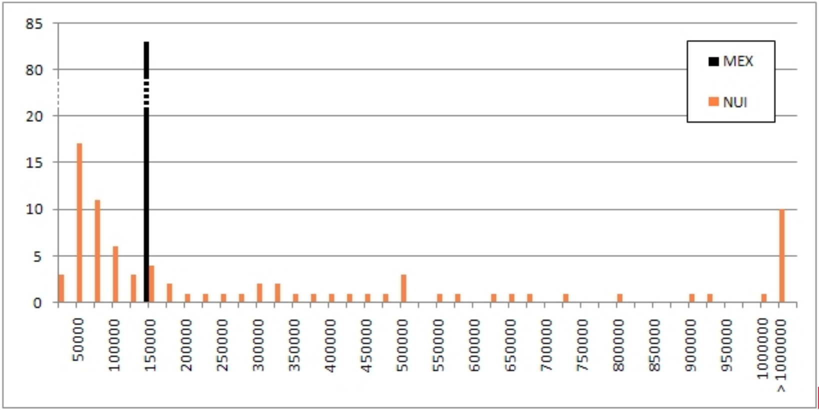
Computing uniform interpolants for SNOMED CT and NCI

- NUI: prototype implementation computing uniform interpolants for acyclic $\mathcal{EL}$-terminologies.
- Σ — randomly selected from sig(SNOMED CT) and sig(NCI), respectively.
- table shows success rate of NUI

$\lvert\Sigma\rvert$	SNOMED CT	$\lvert\Sigma\rvert$	NCI
2 000	100.0%	5 000	97.0%
3 000	92.2%	10 000	81.1%
4 000	67.0%	15 000	72.0%
5 000	60.0%	20 000	59.2%

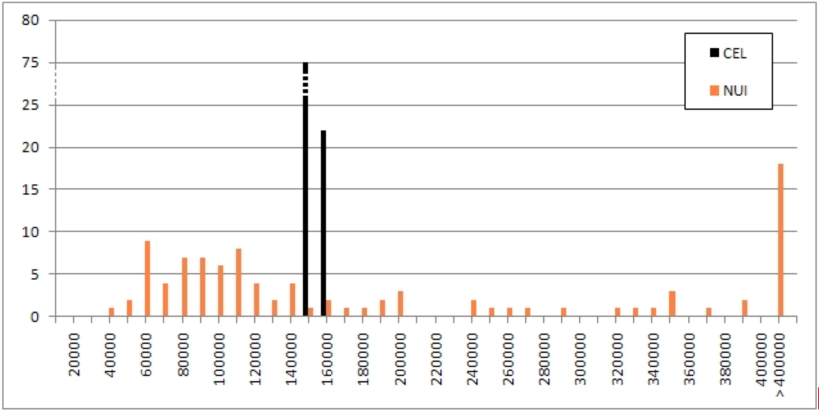
Comparing the size of MEX-modules and Σ-interpolants

- Size distribution of MEX-modules and instance Σ-interpolants of SNOMED CT wrt. signatures containing 3 000 concept names and 20 role names



Comparing the size of ⊤-local modules and Σ-interpolants

- Size distribution of CEL-modules and instance Σ-interpolants of NCI wrt. signatures containing 7 000 concept names and 20 role names



Uniform interpolants beyond $\mathcal{EL}$

Theorem

For $\mathcal{ALC}$-TBoxes, uniform interpolants expressed in FOL do not always exist. [Ghilardi, Lutz, Wolter, 2006]

Theorem

For $\mathcal{ALC}$-TBoxes, deciding the existence of uniform interpolants in ALC is 2ExpTime-complete. If they exist, uniform interpolants are most triple exponential in the size of the original TBox. [Lutz, Wolter, 2011]

And now . . .

Conclusion

What are modules good for (so far)?

- import/reuse of ontologies (locality-based/MEX modules)
- towards understanding the structure of an ontology (atomic decomposition)
- incremental reasoning

Related notions

- Σ-inseparability (foundation of modules)
- logical difference (ontology versioning)
- forgetting (hiding of symbols)

Outlook

Some open problems

- finding appropriate signature for a module (shopping for symbols)
- methodology for collaborative ontology development using modules
- ontology comprehension/visualisation (e.g. using atomic decomposition)
- modular reasoning (improve performance using modules)

So.. that's it.

Thank you for coming!

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Reminder: Workshop on Modular Ontologies (WoMO)
7th International Workshop on Modular Ontologies
September 15, Corunna, Spain
http://www.iaoa.org/womo/2013.html