Prof. Dr. Hans-Jörg Kreowski **June 2007**

Theoretical Aspects of Logistics

The Packing Problem

input: a container with capacity $cap \in \mathbb{N}$, some items numbered by $1, \ldots, m$ with weights $w_1, \ldots, w_m$
solutions: a load $load \subseteq [m]$ with $w(load) = \sum_{i \in load} w_i \leq cap$
output: a maximum load

A load $load \subseteq [m]$ can be represented by a bit string $l_1 \ldots l_m \in \{0,1\}^*$ with $l_i = 1$ if and only if $i \in load$. Accordingly, $w(l_1 \ldots l_m) = \sum_{l_i = 1} w_i$.

A bit string $l'_1 \ldots l'_m$ that differs from $l_1 \ldots l_m$ in a single bit (i.e.$l_{i_0} \neq l'_{i_0}$ for some $i_0 \in [m]$ and $l_i = l'_i$ otherwise) is a *mutation*.

Let $l_1 \ldots l_m$ and $l'_1 \ldots l'_m$ be bit strings and $i, j \in [m]$ with $i \leq j$. Then the bit strings $l_1 \ldots l_{i-1}\ l'_i \ldots l'_j, l_{j+1} \ldots l_m$ and $l'_1 \ldots l'_{i-1}\ l_i \ldots l_j\ l'_{j+1} \ldots l'_m$ are *recombinations* of $l_1 \ldots l_m$ and $l'_1 \ldots l'_m$ wrt. $i \leq j$.

Solving the packing problem by hill climbing

configurations: $\{0,1\}^m = \mathcal{C}_{pack}$ (plus all the invariant stuff)

initial: $\{l_1 \ldots l_m \mid w(l_1 \ldots l_m) \leq cap\} = \mathcal{I}_{pack}$
terminal: $\{\bar{l}_1 \ldots \bar{l}_m \mid w(l_1 \ldots l_m) \leq w(\bar{l}_1 \ldots \bar{l}_m) \leq cap$ for all $l_1 \ldots l_m \in \mathcal{I}_{pack}\} = \mathcal{T}_{pack}$
step: $l_1 \ldots l_m \to l'_1 \ldots l'_m$ provided that $w(l_1 \ldots l_m) < w(l'_1 \ldots l'_m) \leq cap$ and $l'_1 \ldots l'_m$ is a mutation of $l_1 \ldots l_m$ (note that this means $l_{i_0} = 0$ and $l'_{i_0} = 1$)

This hill climbing algorithm terminates always, but finds only local maxima rather than the maxima. Without loss of generality, one may always start with 0^m because other initial configurations can be obtained by hill climbing starting in 0^m.

Example

The capacity is 47, the number of items is 14, and the sequence of weights is 5 5 5 5 5 7 7 7 7 7 7 7 10 10.

A sample load is $\{1, 2, 6, 7, 13, 14\}$ with weight 44. This is a local maximum because no item can be added without the extension of the capacity.

The bit string representation of this load is 1 1 0 0 0 1 1 0 0 0 0 0 1 1. It may be computed by the following hill climbing steps starting in 0^{14} :

$$0^{14} \to 0^5 10^8 \to 010^3 10^8 \to 010^3 110^7 \to 010^3 110^5 10 \to 010^3 110^5 11 \to 110^3 110^5 11.$$

One of the maxima is $0^4 1^7 0^3$.