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Theoretical Aspects of Logistics

Modelling of Processes (Part 2)

This is to introduce a precise notion of configurations and steps.

A *configuration* is

(A) a Boolean or a truth value, $TRUE$ or $FALSE$ ($BOOL = \{TRUE, FALSE\}$),
(B) a natural number $n \in I\!N$,
(C) an integer (or real number) $z \in \mathbb{Z}$ ($\mathbb{R}$),
(D) a symbol of an alphabet, an element of an enumeration type, $a \in A$,
(E) a tuple or vector of configurations,
(F) a sequence of configurations,
(G) a set of configurations,
(H) a mapping from a set of configurations into a set of configurations.

This notion may be extended later if something is missing.

The various types of configurations are equipped with basic operations.

(A) $BOOL$ with the Boolean operators $\neg, \wedge, \vee, \Longrightarrow, \Leftrightarrow$,

(B+C) $I\!N, \mathbb{Z}, \mathbb{R}$ with the usual arithmetic operations $+, -, \cdot, \ldots$ and the usual predicates $\langle \leq, =, \neq, \geq \rangle$.

(D) Usually, one assumes an equality predicate $=$ only. If the elements are enumerated in some order, this may be used to induce an order predicate $<$.

(E) Given some k-tuple $(c_1, \ldots, c_k)$, one may pick the i-th component c_i for $i \in [k]$. This defines the i-th projection $pr_i(c_1, \ldots, c_k) = c_i$.

(F) Given some sequence $c_1 \ldots c_n$ of configurations $c_1, \ldots, c_n$ for $n \in I\!N$, one can add configurations to the right $c_1 \ldots c_n c$ or to the left $cc_1 \ldots c_n$. Moreover, one may concatenate two sequences $c_1 \ldots c_m$ and $c'_1 \ldots c'_m$ into $c_1 \ldots c_m\ c'_1 \ldots c'_m$. It is also convenient to assume that the length of a sequence $|c_1 \ldots c_n|$ is given by n.

(G) For sets of configurations, we may assume to have the whole machinery of set-theoretic operations available like union ($\cup$), intersection($\cap$), set difference ($-$). Moreover, given a finite set $X, \#X$ denotes the number of elements of X.

(H) Given two mappings $f\colon A \to B$ and $g\colon B \to C$, there is at least the sequential composition $g \circ f\colon A \to C$ defined by $g \circ f(x) = g(f(x))$ for all $x \in A$.

Without further details, we can also assume that these operations on the various types of configurations have the usual properties like associativity, compativity, distributivity, etc. where it applies.

A *step* $c \to c'$ from a configuration c to a configuration c' is given by some proper combination of the basic operations applied to c yielding c'.

Example:

To illustrate the notion of configurations and steps, the Traveling Salesman Problem is modeled.

TSP

configurations: $(G = (V, E, s, t), dist\colon E \to I\!N, N, trip)$

where G is a directed graph with a set of nodes V, a set of edges E, and two mappings $s\colon E \to V$ and $t\colon E \to V$ assigning a source $s(e)$ and a target $t(e)$ to each edge $e \in E, dist\colon E \to I\!N$ is a mapping, $N \in I\!N$ is an upper bound, and $trip \in V^*$ is a sequence of nodes representing the current travelling tour.

steps: $(G, dist, N, trip) \to (G, dist, N, trip\ v)$ if $v \notin set(trip), dist(trip\ v) \leq N$

$(set\colon V^* \to 2^v$ is defined by $set(\lambda) = \emptyset$ and $set(xu) = \{x\} \cup set(n); dist\colon V^* \to I\!N^\infty$ is defined by $dist(\lambda) = 0, dist(x) = 0$ and $dist(xyu) = dist(xy) + dist(yu)$ with $dist(xy) = min\{dist(e) \mid s(e) = x,\ t(e) = y\}; I\!N^\infty = I\!N \cup \{\infty\}$ with $n + \infty = \infty + n = \infty$ and $min\ \emptyset = \infty)$.

initial: $(G, dist, N, \lambda)$

terminal: $(G, dist, N, trip)$ with $|trip| = \#V$ and $set(trip) = V$