Prof. Dr. Hans-Jörg Kreowski **May 2007**

Theoretical Aspects of Logistics

Modelling of Processes (Part 3)

This note lists a few interesting properties of processes. To introduce them formally, let $proc = (\mathcal{C}, \rightarrow, \mathcal{I}, \mathcal{T})$ be a process. Moreover, a size function $size\colon \mathcal{C} \rightarrow \mathbb{N}$ is assumed which can be chosen properly. For example, the size of a sequence may be its length, the size of a set its number of elements, the size of a natural number the number itself (or the length of its decimal representation).

1. A process is potentially *nondeterministic* meaning that there may be steps $x \rightarrow y$ and $x \rightarrow z$ with $y \neq z$.

2. A process is *deterministic* if $y = z$ for each two steps $x \rightarrow y$ and $x \rightarrow z$.

   Note that this includes the case that there may be no y_0 with $x_0 \rightarrow y_0$ for some x_0.

3. For each $x \in \mathcal{C}$, one may consider the set of *direct neighbours* $dn(x) = \{y \mid x \rightarrow y\}$.

4. The number of direct neighbours of x is called the *branching degree* of x and denoted by $bd(x)$, i.e. $bd(x) = \#dn(x)$.

5. A process is *polynomially branching* if there are constants c_{size} and n such that $bd(x) \leq c_{size} \cdot (size(x))^n$.

6. A process has potentially infinite runs meaning that there may be an infinite sequence $(x_i)_{i \in \mathbb{N}}$ with $x_i \rightarrow x_{i+1}$ for all $i \in \mathbb{N}$.

   Note that a process may also have infinitely many runs of finite length.

7. A process is *terminating* if there is no infinite run.

8. A process has the *steps number bound* $b\colon \mathcal{C} \rightarrow \mathbb{N}$ if $x \xrightarrow{k} y$ implies $k \leq b(x)$ for all $x \in \mathcal{C}$.

9. A process is *polynomial* if there are constants c and n such that $x \xrightarrow{k} y$ implies $k \leq c \cdot (size(x))^n$.

   Note that a polynomially bounded number of steps is only significant as a time bound if single steps take only times that are also polynomially bounded.

10. Accordingly, a process is *linear* (quadratic, cubic, etc.) if $n = 1$ $(n = 2, n = 3, etc.)$.

    The linear (quadratic, cubic, etc.) number of steps is a time bound if single steps have a constant time bound.

While the properties above are concerned with the step relation, the following properties are related to the semantic relation.

11. A process *solves the problems* $prob \subseteq \mathcal{C} \times \mathcal{C}$ *correctly* if $prob = SEM(proc)$.

12. A process assigns a set of *results* $RES(x) = \{y \in \mathcal{T} \mid x \xrightarrow{*} y\}$ to each $x \in \mathcal{I}$.

13. A process is potentially *partial* meaning that there may be some x_0 with no results, i.e. $RES(x_0) = \emptyset$.

14. A process is *total* if $RES(x) \neq \emptyset$ for all $x \in \mathcal{I}$.

15. A process is potentially *non-functional* meaning that there may be some $x_0 \in \mathcal{I}$ with more than one result, i.e. $\#RES(x_0) \geq 2$.

16. A process is functional if $\#RES(x) \leq 1$ for all $x \in \mathcal{I}$.

    In other terms, *proc* is functional if $x \xrightarrow{*} y$ and $x \xrightarrow{*} y'$ with $x \in \mathcal{I}$ and $y, y' \in \mathcal{T}$ implies $y = y'$ always.

17. A process is *injective* if $x \xrightarrow{*} \overline{x}$ and $y \xrightarrow{*} \overline{y}$ with $x, y \in \mathcal{I}, \overline{x}, \overline{y} \in \mathcal{T}$ and $x \neq y$ implies $\overline{x} \neq \overline{y}$ always.

18. A process is *surjective* if for each $y \in \mathcal{T}$ there is an $x \in \mathcal{I}$ with $x \xrightarrow{*} y$.