Monadic second order logic on strings and trees

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A closed formula of monadic second order logic (MSO) can be used to define a graph language (i.e., a set of graphs): all graphs that satisfy the formula. If the graphs are strings (trees), it is well known that the MSO definable languages are exactly the regular languages (the regular tree languages, respectively).

Graph transductions, i.e., translations from one graph into another, can be defined by MSO formulas in the following way. If g is a source graph, then the nodes of the target graph h are defined by means of MSO formulas with one free variable to be interpreted in g. The edges of h are defined by MSO formulas with two free variables, defining binary relations on the nodes of g (and then restricting them to the nodes of h).

If the source graphs are trees, the transduction can be viewed as a context-free graph grammar generating its range (analogous to the fact that a context-free string grammar generates the set of yields of its derivation trees). If the source and target graphs are trees, the transductions can be computed by macro tree transducers which are a formal model for denotational semantics closely related to attribute grammars. If the source and target graphs are strings, the transductions can be computed by 2-way deterministic finite-state transducers.