# Rich $\omega$-words and monadic second-order arithmetic

**Martin-Luther-Universität Halle-Wittenberg**
**Institut für Informatik**
**Kurt-Mothes-Str. 1**
**D-06120 Halle (Saale)**

Rich $\omega$-words are one-sided infinite strings which have every finite word as a subword (infix).

Regular $\omega$-words are one-sided infinite strings for which the infix-set of a suffix is a regular language. This implies that the set of infinitely often occurring infixes equals the infix-set of a suitably chosen suffix of the regular $\omega$-word.

We show that for a regular $\omega$-language $F$ (a set of predicates definable in monadic second order arithmetic) the following conditions are equivalent:

1. $F$ contains a rich $\omega$-word.
2. $F$ is of second baire category in the Cantor space of $\omega$-words.
3. $F$ is a nonnull set for a class of measures (including the natural Lebesgue measure on Cantor space).
4. $F$ has maximum Hausdorff dimension.

This shows that, although we cannot translate Compton's result on rich Z-words (in the MSO theory of the integers) to MSO arithmetic, a set definable in MSO arithmetic and containing a dijunctive $\omega$-word is large in several respect simultaneously.

Moreover, for we show under the assumption of an exchanging property for 'distinguishing' prefixes that two regular $\omega$-words not necessarily being rich but having the same sets of infixes occurring infinitely often are indistinguishable by MSO formulas or, equivalently, by finite automata.