

## Begleitunterlagen zur Vorlesung vom 25.11.2014

Die Funktion map:

```
map f [] = []
map f (x:xs) = f x : map f xs
```

In unserer Kernsprache:

$$\mathbf{let\ map = \lambda f\ ys.\ case\ ys\ of\ [] \rightarrow [];\ (x : xs) \rightarrow map\ f\ y\ in\ map}$$

Ableitung mit

$$C_0 \stackrel{def}{=} \{ (: :: \forall \alpha. \alpha \rightarrow [\alpha] \rightarrow [\alpha], [] :: \forall \alpha. \alpha \}$$

- |     |  |                               |
|-----|--|-------------------------------|
| 1.  | $C_0 \vdash (\mathbf{let\ map = \lambda f\ ys.\ case\ ys\ of\ [] \rightarrow [];\ (x : xs) \rightarrow map\ f\ y\ in\ map}) :: \forall \alpha \beta. (\alpha \rightarrow \beta) \rightarrow [\alpha] \rightarrow [\beta]$  | <i>Gen</i> [2]                |
| 2.  | $C_0 \vdash (\mathbf{let\ map = \lambda f\ ys.\ case\ ys\ of\ [] \rightarrow [];\ (x : xs) \rightarrow map\ f\ y\ in\ map}) :: (\alpha \rightarrow \beta) \rightarrow [\alpha] \rightarrow [\beta]$  | <i>LetRec</i> [3, 24]         |
| 3.  | $C_1 \stackrel{def}{=} \{ map :: (\alpha \rightarrow \beta) \rightarrow [\alpha] \rightarrow [\beta] \}, C_0$<br>$C_1 \vdash \lambda f\ ys.\ \mathbf{case\ ys\ of\ [] \rightarrow [];\ (x : xs) \rightarrow f\ x : map\ f\ xs} :: (\alpha \rightarrow \beta) \rightarrow [\alpha] \rightarrow [\beta]$ | <i>Abs, Abs</i>               |
| 4.  | $C_2 \stackrel{def}{=} \{ f :: \alpha \rightarrow \beta, ys :: [\alpha] \}, C_1$<br>$C_2 \vdash \mathbf{case\ ys\ of\ [] \rightarrow [];\ (x : xs) \rightarrow f\ x : map\ f\ xs} :: [\beta]$  | <i>Cases</i> [5, 6, 7, 8, 13] |
| 5.  | $C_2 \vdash ys :: [\alpha]$  | <i>Var</i>                    |
| 6.  | $C_2 \vdash [] :: [\alpha]$  | <i>Var, Spec</i>              |
| 7.  | $C_2 \vdash [] :: [\beta]$   | <i>Var, Spec</i>              |
| 8.  | $C_3 \stackrel{def}{=} \{ x :: \alpha, xs :: [\alpha] \}, C_2 \vdash x : xs :: [\alpha]$   | <i>App</i> [9, 12]            |
| 9.  | $C_3 \vdash (x : ) :: [\alpha] \rightarrow [\alpha]$   | <i>App</i> [10, 11]           |
| 10. | $C_3 \vdash (:) :: \alpha \rightarrow [\alpha] \rightarrow [\alpha]$   | <i>Var, Spec</i>              |
| 11. | $C_3 \vdash x :: \alpha$   | <i>Var</i>                    |
| 12. | $C_3 \vdash xs :: [\alpha]$  | <i>Var</i>                    |
| 13. | $C_3 \vdash f\ x : map\ f\ xs :: [\beta]$  | <i>App</i> [14, 19]           |
| 14. | $C_3 \vdash (f\ x : ) :: [\beta] \rightarrow [\beta]$  | <i>App</i> [15, 16]           |
| 15. | $C_3 \vdash (:) :: \beta \rightarrow [\beta] \rightarrow [\beta]$  | <i>Var, Spec</i>              |
| 16. | $C_3 \vdash f\ x :: \beta$   | <i>App</i> [17, 18]           |
| 17. | $C_3 \vdash f :: \alpha \rightarrow \beta$   | <i>Var</i>                    |
| 18. | $C_3 \vdash x :: \alpha$   | <i>Var</i>                    |
| 19. | $C_3 \vdash map\ f\ xs :: [\beta]$   | <i>App</i> [20, 23]           |
| 20. | $C_3 \vdash map\ f :: [\alpha] \rightarrow [\beta]$  | <i>App</i> [21, 22]           |
| 21. | $C_3 \vdash map :: (\alpha \rightarrow \beta) \rightarrow [\alpha] \rightarrow [\beta]$  | <i>Var</i>                    |
| 22. | $C_3 \vdash f :: \alpha \rightarrow \beta$   | <i>Var</i>                    |
| 23. | $C_3 \vdash xs :: [\alpha]$  | <i>Var</i>                    |
| 24. | $C_1 \vdash map :: (\alpha \rightarrow \beta) \rightarrow [\alpha] \rightarrow [\beta]$  | <i>Var</i>                    |