Systeme Hoher Qualität und Sicherheit

Vorlesung 6 vom 25.11.2013: Detailed Specification, Refinement & Implementation

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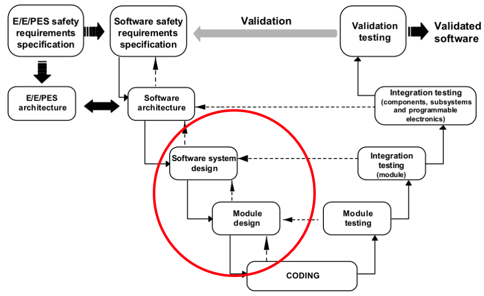
Where are we?

Your Daily Menu

- **Refinement**: from abstract to concrete specification
- **Implementation**: from concrete specification to code
- Running examples: the safe autonomous robot, the birthday book

Design Specification



- At this point, we want to be **relate** implementation to the more abstract specifications in the higher lever, and have a **systematic** way to go from higher to lower levels (**refinement**).

Refinment in the Development Process

- Recall that we have **horizontal** and **vertical** structuring.
- **Refinement** is a **vertical** structure in the development process.
- The simplest form of refinement is **implicational**, where an implementation I implies the abstract requirement A

$$I \Rightarrow A$$

- Recall that refinement typically preserves **safety** requirements, but not **security** — thus, there is a systematic way to construct safe systems, but not so for secure ones.

The Autonomous Robot: Basic Types

- We first declare a datatype for the time:

$[\mathit{Time}]$

- We then declare the robot parameters, and the state of the world — these are the things which do not change.

RobotParam
$\mathit{cont} : \mathit{POLY}$

- Obstacles are just a set of points (instead of polygons)

World
RobotParam
$\mathit{obs} : \mathbb{P}\,\mathit{VEC}$

The Autonomous Robot: Safety Requirements

- The robot's state depends on the time, so we do not have pre/post conditions. It has a position vector, o, which determines the current contour polygon c.

Robot
RobotParam
$c : \mathit{Time} \rightarrow \mathit{POLY}$
$o : \mathit{Time} \rightarrow \mathit{VEC}$
———
$c(t) = \mathit{move}(\mathit{cont}, o(t))$

- Here is the **main safety requirement**: the robot is safe if its current contour never contains any obstacles.

RobotSafe
Robot
———
$\forall\, t . c(t) \cap \mathit{obs} = \emptyset$

The Autonomous Robot: Implementation

- When implementing the autonomous robot, we assume a **control loop** architecture, where a **control function** is called each T ms. It can read the**current** system state, and sets **control variables** which determine the system's behaviour over the next clock cycle.
- The cycle time ("tick") T is part of the robot parameters. We also add the braking accelaration a_{brk}.

RobotParam
$\mathit{cont} : \mathit{POLY}$
$a_{brk} : \mathbb{Z}$
$T : \mathbb{Z}$

World
RobotParam
$\mathit{obs} : \mathbb{P}\,\mathit{VEC}$

The Autonomous Robot: Implementation

- This specifies the **control behaviour** of the robot.
- Velocity is given by the **linear velocity** *vel*, and steering angle ω. This describes the velocity vector v in polar form.
- This does not yet describe how the velocity is controlled.

Robot
$$\begin{array}{|l}
RobotParam \\
vel, \omega : \mathbb{Z} \\
v, o : VEC \\
c : POLY \\
\hline
c = move\,(cont, o) \\
v = cart\,(vel, \omega)
\end{array}$$

- The function *cart* converts a vector in polar form to the cartesian form. A simple specification in Z might be this:

$$\begin{array}{|l}
cart : \mathbb{Z} \times R \to VEC \\
\hline
\forall\, r : \mathbb{Z};\ \omega : R;\ p : VEC \bullet cart(r, \omega) = p \Rightarrow r * r = p.x * p.x + p.y * p.y
\end{array}$$

- Unfortunately, the Mathematical Toolkit does not support trigonometric functions (or real numbers).

The Autonomous Robot: Control

- The velocity is controlled by two **input variables** $a?$ and $d\omega?$, which set the acceleration and change of steering angle for the next cycle. This determines *vel* and ω, and hence v.

RobotMoves
$$\begin{array}{|l}
\Delta Robot \\
\Xi World \\
a? : \mathbb{Z} \\
d\omega? : \mathbb{Z} \\
\hline
vel' = vel + a? * T \\
\omega' = \omega + d\omega? * T \\
o' = add\,(o, v')
\end{array}$$

- This now describes the control loop behaviour of the robot.
- But when is it **safe**?

Moving and Driving Safely

- It is easy to say what it means for the robot to **move safely**: it will not run into any obstacles.

RobotMovesSafely
$$\begin{array}{|l}
RobotMoves \\
\hline
cov\,(c, v') \cap obs = \emptyset
\end{array}$$

- Is that **enough**?
- No, this will give us a **false sense** of safety — it only fails when it is **far too late** to **initiate** braking.
- To ensure safety here we would need:

$RobotMovesSafely \Rightarrow RobotMovesSafely'$

Braking and Safe Braking

- Our safety strategy: we must **always** be able to **brake safely**
- We first need to specify **braking** and **safe braking**. Braking is safe if the braking area is clear of obstacles.

RobotBrakes
$$\begin{array}{|l}
\Delta Robot \\
\Xi World \\
\hline
vel' = vel - a_{brk} * T \\
\omega' = \omega \\
o' = add\,(o, v')
\end{array}$$

RobotBrakesSafely
$$\begin{array}{|l}
RobotBrakes \\
\hline
cov\,(c, brk\,(v, \omega, a_{brk})) \cap obs = \emptyset
\end{array}$$

- **Implementing** the overall strategy: if we can move safely, we do, otherwise we brake.
- **Invariant**: we can always brake safely.

The Safe Robot: Implementation

- We drive **safe** if we **will** be able to brake safely.

RobotDrivesSafely
$$\begin{array}{|l}
\Delta Robot \\
\Xi World \\
\hline
(cov\,(c, v') \cup cov\,(move\,(c, v'), brk\,(v', \omega', a_{brk}))) \cap obs = \emptyset \\
vel' = vel + a? * T \\
\omega' = \omega + d\omega? * T \\
o' = add\,(o, v')
\end{array}$$

- The safe robot implements the safety strategy:

$RobotSafeImpl = RobotDrivesSafely \vee RobotBrakes$

Showing Safety

- We need to **show:**

$RobotSafeImpl \Rightarrow RobotMovesSafely$
$RobotSafeImpl \Rightarrow RobotMovesSafely'$

- The first holds directly.
- The second holds because of the following:

$RobotSafeImpl \Rightarrow RobotBrakesSafely'$
$RobotBrakesSafely \Rightarrow RobotMovesSafely$
$RobotBrakesSafely' \Rightarrow RobotMovesSafely'$

Missing Pieces

- We start off at the origin (or anywhere else), and with velocity 0.
- We need to specify that initially we are **clear of obstacles**.

InitRobot
$$\begin{array}{|l}
Robot \\
\hline
o = (0, 0) \\
vel = 0 \\
\omega = 0 \\
cont \cap obs = \emptyset
\end{array}$$

Summing Up

- The first, abstract, safety specification was *RobotSafe*.
- We implemented this via a second, more concrete specification *RobotSafeImpl*.
- Showing refinement required several lemmas.
- The general safety argument:
  - Safety holds for the initial position: $InitRobot \Rightarrow RobotMovesSafely$
  - Safety is preserved: $RobotSafeImpl \Rightarrow RobotMovesSafely \wedge RobotMovesSafely'$
  - Thus, safety holds always (proof by **induction**).

From Specification to Implementation

- How would we **implement** the birthday book?
- We need a **data structure** to keep track of names and dates.
- And we need to **link** this data structure with the **specification**.
- There are two ways out of this:
  - Either, the specification language also models datatypes (**wide-spectrum language**).
  - Or there is fixed mapping from the specification language to a programming language.

Implementing Arrays

- In Z, arrays can be represented as functions from $\mathbb{N}_1$. Thus, if we want to keep names and dates in arrays (linked by the index), we take

  $names : \mathbb{N}_1 \to NAME$
  $dates : \mathbb{N}_1 \to DATE$

- To look up $names[i]$, we just apply the function: $names(i)$.
- To assignment $names[i] := v$, we change the function with the **pointwise update operator** $\oplus$:

  $names' = names \oplus \{i \mapsto v\}$.

Implementing the Birthday Book

- We need a variable *hwm* which indicates how many date/name pairs are known.
- The axiom makes sure that each name is associated to exactly one birthday.

BirthdayBookImpl

$names : \mathbb{N}_1 \to NAME$
$dates : \mathbb{N}_1 \to DATE$
$hwm : \mathbb{N}$

$\forall i, j : 1 \mathinner{..} hwm \bullet$
$\quad i \neq j \Rightarrow names(i) \neq names(j)$

Linking Specification and Implementation

- We need to **link** specification and implementation.
- This is done in an abstraction or linking schema:

Abs

BirthdayBook
BirthdayBookImpl

$known = \{ i : 1 \mathinner{..} hwm \bullet names(i) \}$

$\forall i : 1 \mathinner{..} hwm \bullet$
$\quad birthday(names(i)) = dates(i)$

- This specificies how *known* and *birthday* are reflected by the implementing arrays.

Operation: Adding a birthday

- Adding a birthday changes the **concrete state**:

AddBirthdayImpl

$\Delta BirthdayBookImpl$
$name? : NAME$
$date? : DATE$

$\forall i : 1 \mathinner{..} hwm \bullet name? \neq names(i)$

$hwm' = hwm + 1$
$names' = names \oplus \{hwm' \mapsto name?\}$
$dates' = dates \oplus \{hwm' \mapsto date?\}$

- We need to show that the pre- and post-states of *AddBirthday* and *AddBirthdayImpl* are related via *Abs*.

Showing Correctness of the Implementation

- Assume a state where the precondition of the specification holds, find the corresponding state of the implementation via *Abs*, and show that this state satisfies the precondition.
- Similarly, assume a pair of states where the invariant of *AddBirthdayBook* holds, find the corresponding states of the implementation via *Abs*, and show that they satisfy the invariant.

Operation: Finding a birthday

- We specify that the found day corresponds to the name via an index *i*.

FindBirthdayImp

$\Xi BirthdayBookImpl$
$name? : NAME$
$date! : DATE$

$\exists i : 1 \mathinner{..} hwm \bullet$
$\quad name? = names(i) \wedge date! = dates(i)$

- Note that we are still some way off a concrete implementation — we do not say how we **find** the index *i*.
- To formally show that an iterative loop from 1 to *hdw* always returns the right *i*, we need the **Hoare calculus** (later in these lectures); presently, we argue **informally**.

Summary

- We have seen how we **refine** abstract specifications to more **concrete** ones.
- To **implement** specifications, we need to relate the specification language to a programming language
  - In Z, there are some types which correspond to well-known datatypes, such as finite maps $\mathbb{N}_1 \to T$ and arrays of T.
- We have now reached the **bottom** of the V-model. Next week, we will climb our way up on the right-hand side, starting with **testing**.