Systeme Hoher Qualität und Sicherheit
Vorlesung 10 vom 06.01.2014: Verification Condition Generation

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**Frohes Neues Jahr!**



## Where are we?

- Lecture 1: Concepts of Quality
- Lecture 2: Concepts of Safety and Security, Norms and Standards
- Lecture 3: Quality of the Software Development Process
- Lecture 4: Requirements Analysis
- Lecture 5: High-Level Design & Formal Modelling
- Lecture 6: Detailed Specification, Refinement & Implementation
- Lecture 7: Testing
- Lecture 8: Program Analysis
- Lecture 9: Verification with Floyd-Hoare Logic
- **Lecture 10: Verification Condition Generation**
- Lecture 11: Model-Checking with LTL and CTL
- Lecture 12: NuSMV and Spin
- Lecture 13: Conclusions



## Introduction

- In the last lecture, we learned about the **Floyd-Hoare calculus.**
- It allowed us to **state** and **prove** correctness assertions about programs, written as $\{P\}\, c\, \{Q\}$.
- The **problem** is that proofs of $\vdash \{P\}\, c\, \{Q\}$ are **exceedingly** tedious, and hence not viable in practice.
- We are looking for a calculus which reduces the size (and tediousness) of Floyd-Hoare proofs.
- The starting point is the **relative completeness** of the Floyd-Hoare calculus.



## Completeness of the Floyd-Hoare Calculus

**Relative Completeness**

If $\models \{P\}\, c\, \{Q\}$, then $\vdash \{P\}\, c\, \{Q\}$ except for the weakening conditions.

- To show this, one constructs a so-called **weakest precondition**.

**Weakest Precondition**

Given a program $c$ and an assertion $P$, the weakest precondition is an assertion $W$ which

1. is a valid precondition: $\models \{W\}\, c\, \{P\}$
2. and is the weakest such: if $\models \{Q\}\, c\, \{P\}$, then $W \longrightarrow Q$.

- Question: is the weakest precondition **unique?**
  Only up to logical equivalence: if $W_1$ and $W_2$ are weakest preconditions, then $W_1 \longleftrightarrow W_2$.



## Constructing the Weakest Precondition

- Consider the following simple program and its verification:

$\{X = x \wedge Y = y\}$
$\longleftrightarrow$
$\{Y = y \wedge X = x\}$
Z:= Y;
$\{Z = y \wedge X = x\}$
Y:= X;
$\{Z = y \wedge Y = x\}$
X:= Z;
$\{X = y \wedge Y = x\}$

- The idea is to construct the weakest precondition **inductively**.



## Constructing the Weakest Precondition

- There are four straightforward cases:

$$\begin{aligned}
\mathrm{wp}(\mathbf{skip}, P) &\stackrel{def}{=} P\\
\mathrm{wp}(X := e, P) &\stackrel{def}{=} P[e/X]\\
\mathrm{wp}(c_0; c_1, P) &\stackrel{def}{=} \mathrm{wp}(c_0, \mathrm{wp}(c_1, P))\\
\mathrm{wp}(\mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, P) &\stackrel{def}{=} (b \wedge \mathrm{wp}(c_0, P)) \vee (\neg b \wedge \mathrm{wp}(c_1, P))
\end{aligned}$$

- The complicated one is iteration. This is not surprising, because iteration gives us computational power (and makes our language Turing-complete). It can be given recursively:

  $$\mathrm{wp}(\mathbf{while}\ b\ \mathbf{do}\ c, P) \stackrel{def}{=} (\neg b \wedge P) \vee (b \wedge \mathrm{wp}(c, \mathrm{wp}(\mathbf{while}\ b\ \mathbf{do}\, c, P)))$$

  A closed formula can be given using Turing's $\beta$-predicate, but it is unwieldy to write down.
- Hence, $\mathrm{wp}(c, P)$ is not an effective way to **prove** correctness.



## Verfication Conditions: Annotated Programs

- **Idea**: invariants specified in the program by **annotations**.
- Arithmetic and Boolean Expressions (**AExp**, **BExp**) remain as they are.
- Annotated Statements (**ACom**)

$$\begin{aligned}c ::= &\ \mathbf{skip} \mid \mathbf{Loc} := \mathbf{AExp} \mid \mathbf{assert}\ P \mid \mathbf{if}\ b\ \mathbf{then}\ c_1\ \mathbf{else}\ c_2\\ &\mid \mathbf{while}\ b\ \mathbf{inv}\ I\ \mathbf{do}\ c \mid c_1; c_2 \mid \{c\}\end{aligned}$$

## Calculuation Verification Conditions

- For an annotated statement $c \in \mathbf{ACom}$ and an assertion $P$ (the postcondition), we calculuate a **set** of verification conditions $\mathrm{vc}(c, P)$ and a precondition $\mathrm{pre}(c, P)$.
- The precondition is an auxiliary definition — it is mainly needed to compute the verification conditions.
- If we can prove the verification conditions, then $\mathrm{pre}(c, P)$ is a proper precondition, i.e. $\models \{\mathrm{pre}(c, P)\}\, c\, \{P\}$.



## Calculating Verification Conditions

$$
\begin{aligned}
\mathrm{pre}(\mathbf{skip}, P) &\stackrel{def}{=} P\\
\mathrm{pre}(X := e, P) &\stackrel{def}{=} P[e/X]\\
\mathrm{pre}(c_0; c_1, P) &\stackrel{def}{=} \mathrm{pre}(c_0, \mathrm{pre}(c_1, P))\\
\mathrm{pre}(\mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, P) &\stackrel{def}{=} (b \wedge \mathrm{pre}(c_0, P)) \vee (\neg b \wedge \mathrm{pre}(c_1, P))\\
\mathrm{pre}(\mathbf{assert}\ Q, P) &\stackrel{def}{=} Q\\
\mathrm{pre}(\mathbf{while}\ b\ \mathbf{inv}\ I\ \mathbf{do}\ c, P) &\stackrel{def}{=} I
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{vc}(\mathbf{skip}, P) &\stackrel{def}{=} \emptyset\\
\mathrm{vc}(X := e, P) &\stackrel{def}{=} \emptyset\\
\mathrm{vc}(c_0; c_1, P) &\stackrel{def}{=} \mathrm{vc}(c_0, \mathrm{pre}(c_1, P)) \cup \mathrm{vc}(c_1, P)\\
\mathrm{vc}(\mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, P) &\stackrel{def}{=} \emptyset\\
\mathrm{vc}(\mathbf{assert}\ Q, P) &\stackrel{def}{=} \{Q \longrightarrow P\}\\
\mathrm{vc}(\mathbf{while}\ b\ \mathbf{inv}\ I\ \mathbf{do}\ c, P) &\stackrel{def}{=} \mathrm{vc}(c, I) \cup \{I \wedge b \longrightarrow \mathrm{pre}(c, I)\}\\
&\qquad \cup \{I \wedge \neg b \longrightarrow P\}
\end{aligned}
$$



## Correctness of the VC Calculus

**Correctness of the VC Calculus**

For an annotated program $c$ and an assertion $P$, let $\mathrm{vc}(c, P) = \{P_1, \ldots, P_n\}$. If $P_1 \wedge \ldots \wedge P_n$, then $\models \{\mathrm{pre}(c, P)\}\, c\, \{P\}$.

- Proof: By induction on $c$.



## Example: Faculty

Let *Fac* be the annotated faculty program:

```
{0 ≤ N}
P := 1;
C := 1;
while C ≤ N inv {P = (C − 1)! ∧ C − 1 ≤ N} do {
  P := P × C;
  C := C + 1
}
{P = N!}
```

$\mathrm{vc}(Fac) =$

$\{\ 0 \leq N \longrightarrow 1 = 0! \wedge 0 \leq N,$

$P = (C-1)! \wedge C-1 \leq N \wedge C \leq N \longrightarrow P \times C = C! \wedge C \leq N,$

$P = (C-1)! \wedge C-1 \leq N \wedge \neg(C \leq N) \longrightarrow P = N!\ \}$



## The Framing Problem

- One problem with the simple definition from above is that we need to specify which variables stay the same (**framing problem**).
  - Essentially, when going into a loop we use lose all information of the current precondition, as it is replaced by the loop invariant.
  - This does not occur in the faculty example, as all program variables are changed.
- Instead of having to write this down every time, it is more useful to modify the logic, such that we specify which variables are **modified**, and assume the rest stays untouched.
- Sketch of definition: We say $\models \{P, X\}\, c\, \{Q\}$ is a Hoare-Triple with **modification set** $X$ if for all states $\sigma$ which satisfy $P$ if $c$ terminates in a state $\sigma'$, then $\sigma'$ satisfies $Q$, and if $\sigma(x) \neq \sigma'(x)$ then $x \in X$.

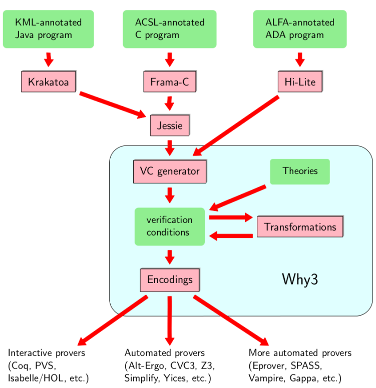


## Verification Condition Generation Tools

- The Why3 toolset (`http://why3.lri.fr`)
  - The Why3 verification condition generator
  - Plug-ins for different provers
  - Front-ends for different languages: C (Frama-C), Java (Krakatoa)
- The Boogie VCG (`http://research.microsoft.com/en-us/projects/boogie/`)
- The VCC Tool (built on top of Boogie)
  - Verification of C programs
  - Used in German Verisoft XT project to verify Microsoft Hyper-V hypervisor

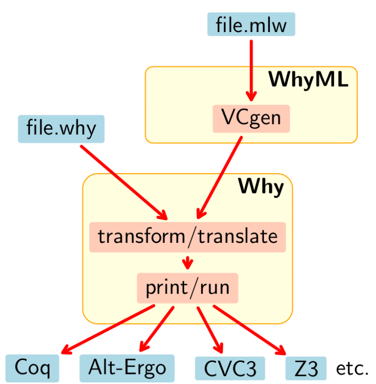


## Why3 Overview: Toolset






## Why3 Overview: VCG



## Why3 Example: Faculty (in WhyML)

```
let fac(n: int): int
  requires { n >= 0 }
  ensures  { result = fact(n) } =
  let p = ref 0 in
  let c = ref 0 in
  p := 1;
  c := 1;
  while !c <= n do
    invariant { !p= fact(!c-1) /\ !c-1 <= n }
    variant { n- !c }
    p:= !p* !c;
    c:= !c+ 1
    done;
 !p
```



## Why3 Example: Generated VC for Faculty

```
goal WP_parameter_fac :
 forall n:int.
  n >= 0 ->
   (forall p:int.
     p = 1 ->
      (forall c:int.
        c = 1 ->
         (p = fact (c - 1) /\ (c - 1) <= n) /\
          (forall c1:int, p1:int.
            p1 = fact (c1 - 1) /\ (c1 - 1) <= n ->
             (if c1 <= n then forall p2:int.
                               p2 = (p1 * c1) ->
                                (forall c2:int.
                                  c2 = (c1 + 1) ->
                                   (p2 = fact (c2 - 1) /\
                                    (c2 - 1) <= n) /\
                                    0 <= (n - c1) /\
                                    (n - c2) < (n - c1))
             else p1 = fact n))))
```



## Summary

- Starting from the **relative completeness** of the Floyd-Hoare calculus, we devised a **Verification Condition Generation** calculus which makes program verification viable.
- Verification Condition Generation reduces an **annotated** program to a set of logical properties.
- We need to annotate **preconditions**, **postconditions** and **invariants**.
- Tools which support this sort of reasoning include **Why3** and **Boogie**. They come with front-ends for **real programming languages**, such as C, Java, C#, and Ada.
- To scale to real-world programs, we need to deal with **framing**, **modularity** (each function/method needs to be verified independently), and **machine arithmetic** (integer word arithmetic and floating-points).