Systeme Hoher Sicherheit und Qualität
Universität Bremen WS 2015/2016

## Lecture 13 (25.01.2016)

# Modelchecking with LTL and CTL

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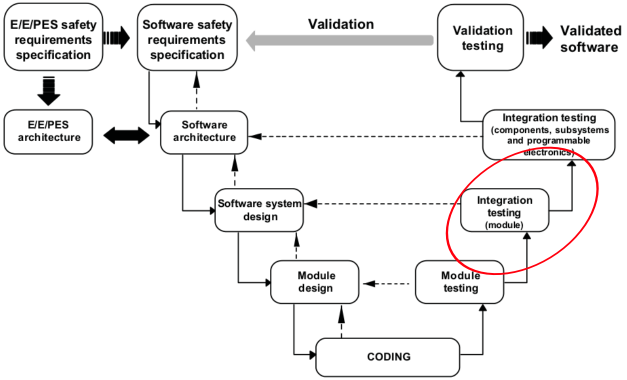
## Organisatorisches

- Evaluation: auf der stud.ip-Seite (unter Lehrevaluation)
- Prüfungen & Fachgespräche:
  - KW 7 (15./16. Februar), oder
  - 02. Februar (letzte Semesterwoche, zum Übungstermin).

## Where are we?

- 01: Concepts of Quality
- 02: Legal Requirements: Norms and Standards
- 03: The Software Development Process
- 04: Hazard Analysis
- 05: High-Level Design with SysML
- 06: Formal Modelling with SysML and OCL
- 07: Detailed Specification with SysML
- 08: Testing
- 09: Program Analysis
- 10: Foundations of Software Verification
- 11: Verification Condition Generation
- 12: Semantics of Programming Languages
- 13: Model-Checking
- 14: Conclusions and Outlook

# Modelchecking in the Development Process


E/E/PES safety requirements specification
Software safety requirements specification
Validation
Validation testing
Validated software
E/E/PES architecture
Software architecture
Integration testing (components, subsystems and programmable electronics)
Software system design
Integration testing (module)
Module design
Module testing
CODING

- Model-checking proves properties of abstractions of the system.
- Thus, it scales also to higher levels of the development process

## Introduction

- In the last lectures, we were verifying program properties with the Floyd-Hoare calculus and related approaches. Program verification was reduced to a deductive problem by translating the program into logic (specifically, state change becomes substitution).
- Model-checking takes a different approach: instead of directly working with the program, we work with an abstraction of the system (a model). Because we build abstractions, this approach is also applicable in the higher verification levels.
- But what are the properties we want to express? How do we express them, and how do we prove them?

## The Model-Checking Problem

**The Basic Question**

Given a model $\mathcal{M}$, and a property $\phi$, we want to know whether

$$\mathcal{M} \models \phi$$

- What is $\mathcal{M}$?
- What is $\phi$?
- How to prove it?

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**The Basic Question**

Given a model $\mathcal{M}$, and a property $\phi$, we want to know whether

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- What is $\mathcal{M}$? Finite state machines
- What is $\phi$? Temporal logic
- How to prove it? Enumerating states — model checking
  - The basic problem: state explosion

## Finite State Machines

### Finite State Machine (FSM)

A FSM is given by $\mathcal{M} = \langle \Sigma, \rightarrow \rangle$ where

- $\Sigma$ is a finite set of states, and
- $\rightarrow \subseteq \Sigma \times \Sigma$ is a transition relation, such that $\rightarrow$ is left-total:

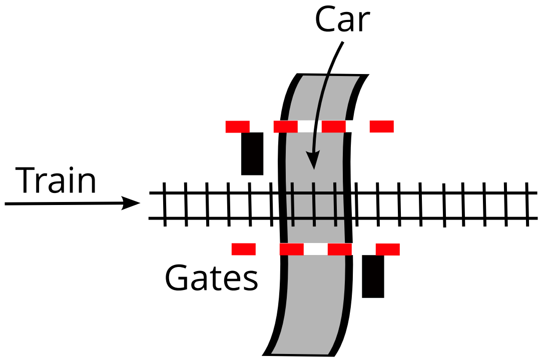
$$\forall s \in \Sigma. \exists s' \in \Sigma. s \rightarrow s'$$

- Many variations of this definition exists, e.g. sometimes we have state variables or labelled transitions.
- Note there is no final state, and no input or output (this is the key difference to automata).
- If $\rightarrow$ is a function, the FSM is deterministic, otherwise it is non-deterministic.

## The Railway Crossing

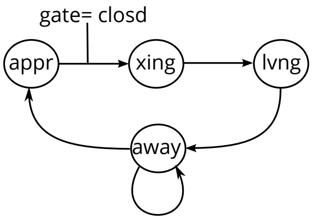
Source: Wikipedia

## The Railway Crossing — Abstraction

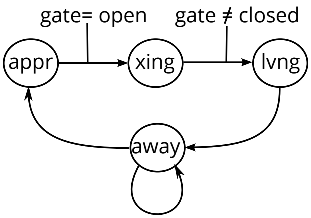


# The Railway Crossing — Model

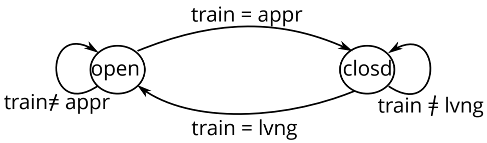
States of the train:



States of the car:



States of the gate:



## The FSM

- The states here are a map from variables *Car*, *Train*, *Gate* to the domains

$$\begin{array}{rcl} \Sigma_{Car} & = & \{appr, xing, lvng, away\} \\ \Sigma_{Train} & = & \{appr, xing, lvng, away\} \\ \Sigma_{Gate} & = & \{open, clsd\} \end{array}$$

  or alternatively, a three-tuple $S \in \Sigma = \Sigma_{Car} \times \Sigma_{Train} \times \Sigma_{Gate}$.

- The transition relation is given by e.g.

$$\begin{array}{l} \langle away, open, away\rangle \to \langle appr, open, away\rangle \\ \langle appr, open, away\rangle \to \langle xing, open, away\rangle \\ \ldots \end{array}$$

## Railway Crossing — Safety Properties

- Now we want to express safety (or security) properties, such as the following:
  - Cars and trains never cross at the same time.
  - The car can always leave the crossing
  - Approaching trains may eventually cross.
  - There are cars crossing the tracks.
- We distinguish safety properties from liveness properties:
  - Safety: something bad never happens.
  - Liveness: something good will (eventually) happen.
- To express these properties, we need to talk about sequences of states in an FSM.

## Linear Temporal Logic (LTL) and Paths

- LTL allows us to talk about paths in a FSM, where a path is a sequence of states connected by the transition relation.
- We first define the syntax of formula,
- then what it means for a path to satisfy the formula, and
- from that we derive the notion of a model for an LTL formula.

**Paths**

Given a FSM $\mathcal{M} = \langle \Sigma, \rightarrow \rangle$, a path in $\mathcal{M}$ is an (infinite) sequence $\langle s_1, s_2, s_3, \ldots \rangle$ such that $s_i \in \Sigma$ and $s_i \rightarrow s_{i+1}$ for all $i$.

- For a path $p = \langle s_1, s_2, s_3, \ldots \rangle$, we write $p_i$ for $s_i$ (selection) and $p^i$ for $\langle s_i, s_{i+1}, \ldots \rangle$ (the suffix starting at $i$).

## Linear Temporal Logic (LTL)

$$
\begin{array}{rcll}
\phi & ::= & \top \mid \bot \mid p & \text{— True, false, atomic} \\
 & \mid & \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \phi_1 \longrightarrow \phi_2 & \text{— Propositional formulae} \\
 & \mid & \mathsf{X}\,\phi & \text{— Next state} \\
 & \mid & \Diamond\phi & \text{— Some Future State} \\
 & \mid & \Box\phi & \text{— All future states (Globally)} \\
 & \mid & \phi_1 \; U \; \phi_2 & \text{— Until}
\end{array}
$$

- Operator precedence: Unary operators; then $U$; then $\wedge$, $\vee$; then $\longrightarrow$.
- An atomic formula $p$ above denotes a state predicate. Note that different FSMs have different states, so the notion of whether an atomic formula is satisfied depends on the FSM in question. A different (but equivalent) approach is to label states with atomic propositions.
- From these, we can define other operators, such as $\phi \; R \; \psi$ (release) or $\phi \; W \; \psi$ (weak until).

## Satifsaction and Models of LTL

Given a path $p$ and an LTL formula $\phi$, the satisfaction relation $p \models \phi$ is defined inductively as follows:

$$\begin{array}{lcl}
p & \models & \mathit{True} \\
p & \not\models & \mathit{False} \\
p & \models & p \text{ iff } p(p_1) \\
p & \models & \neg\phi \text{ iff } p \not\models \phi
\end{array}
\qquad
\begin{array}{lcl}
p & \models & \phi \land \psi \text{ iff } p \models \phi \text{ and } p \models \psi \\
p & \models & \phi \lor \psi \text{ iff } p \models \phi \text{ or } p \models \psi \\
p & \models & \phi \longrightarrow \psi \text{ iff whenever } p \models \phi \text{ then } p \models \psi
\end{array}$$

$$\begin{array}{lcl}
p & \models & \mathrm{X}\,\phi \text{ iff } p^2 \models \phi \\
p & \models & \Box\phi \text{ iff for all } i \text{, we have } p^i \models \phi \\
p & \models & \Diamond\phi \text{ iff there is } i \text{ such that } p^i \models \phi \\
p & \models & \phi\; U\; \psi \text{ iff there is } i\ p^i \models \psi \text{ and for all } j = 1, \ldots, i-1,\ \ p^j \models \phi
\end{array}$$

**Models of LTL formulae**

A FSM $\mathcal{M}$ satisfies an LTL formula $\phi$, $\mathcal{M} \models \phi$, iff every path $p$ in $\mathcal{M}$ satisfies $\phi$.

## The Railway Crossing

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- Approaching trains may eventually cross:

$$\Box(train = appr \longrightarrow \Diamond(train = xing))$$

- There are cars crossing the tracks:

$\Diamond(car = xing)$ means something else!

  - Can not express this in LTL!

## Computational Tree Logic (CTL)

- LTL does not allow us the quantify over paths, e.g. assert the existance of a path satisfying a particular property.
- To a limited degree, we can solve this problem by negation: instead of asserting a property $\phi$, we check wether $\neg\phi$ is satisfied; if that is not the case, $\phi$ holds. But this does not work for mixtures of universal and existential quantifiers.
- Computational Tree Logic (CTL) is an extension of LTL which allows this by adding universal and existential quantifiers to the modal operators.
- The name comes from considering paths in the computational tree obtained by unwinding the FSM.

## CTL Formulae

| | | | |
|---|---|---|---|
| $\phi$ ::= | $\top \mid \bot \mid p$ | | — True, false, atomic |
| $\mid$ | $\neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \phi_1 \longrightarrow \phi_2$ | | — Propositional formulae |
| $\mid$ | $\mathsf{AX}\,\phi \mid \mathsf{EX}\,\phi$ | | — All or some next state |
| $\mid$ | $\mathsf{AF}\,\phi \mid \mathsf{EF}\,\phi$ | | — All or some future states |
| $\mid$ | $\mathsf{AG}\,\phi \mid \mathsf{EG}\,\phi$ | | — All or some global future |
| $\mid$ | $\mathsf{A}[\phi_1\ U\ \phi_2] \mid \mathsf{E}[\phi_1\ U\ \phi_2]$ | | — Until all or some |

## Satifsfaction

- Note that CTL formulae can be considered to be a LTL formulae with a 'modality' ($A$ or $E$) added on top of each temporal operator.
- Generally speaking, the $A$ modality says the temporal operator holds for all paths, and the $E$ modality says the temporal operator only holds for all least one path.
  - Of course, that strictly speaking is not true, because the arguments of the temporal operators are in turn CTL forumulae, so we need recursion.
- This all explains why we do not define a satisfaction for a single path $p$, but satisfaction with respect to a specific state in an FSM.

## Satisfaction for CTL

Given an FSM $\mathcal{M} = \langle \Sigma, \rightarrow \rangle$, $s \in \Sigma$ and a CTL formula $\phi$, then $\mathcal{M}, s \models \phi$ is defined inductively as follows:

$$\begin{array}{lcl}
\mathcal{M}, s & \models & \mathit{True} \\
\mathcal{M}, s & \not\models & \mathit{False} \\
\mathcal{M}, s & \models & p \text{ iff } p(s) \\
\mathcal{M}, s & \models & \phi \wedge \psi \text{ iff } \mathcal{M}, s \models \phi \text{ and } \mathcal{M}, s \models \psi \\
\mathcal{M}, s & \models & \phi \vee \psi \text{ iff } \mathcal{M}, s \models \phi \text{ or } \mathcal{M}, s \models \psi \\
\mathcal{M}, s & \models & \phi \longrightarrow \psi \text{ iff whenever } \mathcal{M}, s \models \phi \text{ then } \mathcal{M}, s \models \psi \\
\ldots & &
\end{array}$$

## Satisfaction for CTL (c'ed)

Given an FSM $\mathcal{M} = \langle \Sigma, \rightarrow \rangle$, $s \in \Sigma$ and a CTL formula $\phi$, then $\mathcal{M}, s \models \phi$ is defined inductively as follows:

$$
\begin{array}{lcl}
\ldots & & \\
\mathcal{M}, s & \models & \mathrm{AX}\,\phi \text{ iff for all } s_1 \text{ with } s \rightarrow s_1 \text{, we have } \mathcal{M}, s_1 \models \phi \\
\mathcal{M}, s & \models & \mathrm{EX}\,\phi \text{ iff for some } s_1 \text{ with } s \rightarrow s_1 \text{, we have } \mathcal{M}, s_1 \models \phi \\
\mathcal{M}, s & \models & \mathrm{AG}\,\phi \text{ iff for all paths } p \text{ with } p_1 = s, \\
 & & \qquad \text{we have } \mathcal{M}, p_i \models \phi \text{ for all } i \geq 2 \\
\mathcal{M}, s & \models & \mathrm{EG}\,\phi \text{ iff there is a path } p \text{ with } p_1 = s \text{ and} \\
 & & \qquad \text{we have } \mathcal{M}, p_i \models \phi \text{ for all } i \geq 2 \\
\mathcal{M}, s & \models & \mathrm{AF}\,\phi \text{ iff for all paths } p \text{ with } p_1 = s \\
 & & \qquad \text{we have } \mathcal{M}, p_i \models \phi \text{ for some } i \\
\mathcal{M}, s & \models & \mathrm{EF}\,\phi \text{ iff there is a path } p \text{ with } p_1 = s \text{ and} \\
 & & \qquad \text{we have; } \mathcal{M}, p_i \models \phi \text{ for some } i \\
\mathcal{M}, s & \models & \mathrm{A}[\phi\; U\; \psi] \text{ iff for all paths } p \text{ with } p_1 = s \text{, there is } i \\
 & & \qquad\qquad \text{with } \mathcal{M}, p_i \models \psi \text{ and for all } j < i,\ \mathcal{M}, p_j \models \phi \\
\mathcal{M}, s & \models & \mathrm{E}[\phi\; U\; \psi] \text{ iff there is a path } p \text{ with } p_1 = s \text{ and there is } i \\
 & & \qquad\qquad \text{with } \mathcal{M}, p_i \models \psi \text{ and for all } j < i,\ \mathcal{M}, p_j \models \phi
\end{array}
$$

## Patterns of Specification

- Something bad ($p$) cannot happen: $\mathrm{AG}\,\neg p$
- $p$ occurs infinitly often: $\mathrm{AG}(\mathrm{AF}\,p)$
- $p$ occurs eventually: $\mathrm{AF}\,p$
- In the future, $p$ will hold eventually forever: $\mathrm{AF}\,\mathrm{AG}\,p$
- Whenever $p$ will hold in the future, $q$ will hold eventually: $\mathrm{AG}(p \longrightarrow \mathrm{AF}\,q)$
- In all states, $p$ is always possible: $\mathrm{AG}(\mathrm{EF}\,p)$

## LTL and CTL

- We have seen that CTL is more expressive than LTL, but (surprisingly), there are properties which we can formalise in LTL but not in CTL!
- Example: all paths which have a $p$ along them also have a $q$ along them.
- LTL: $\Diamond p \longrightarrow \Diamond q$
- CTL: Not $\mathrm{AF}\, p \longrightarrow \mathrm{AF}\, q$ (would mean: if all paths have $p$, then all paths have $q$), neither $\mathrm{AG}(p \longrightarrow \mathrm{AF}\, q)$ (which means: if there is a $p$, it will be followed by a $q$).
- The logic $CTL^*$ combines both LTL and CTL (but we will not consider it further here).

## State Explosion and Complexity

- The basic problem of model checking is state explosion.
- Even our small railway crossing has $|\Sigma| = |\Sigma_{Car} \times \Sigma_{Train} \times \Sigma_{Gate}| = |\Sigma_{Car}| \cdot |\Sigma_{Train}| \cdot |\Sigma_{Gate}| = 4 \cdot 4 \cdot 2 = 32$ states. Add one integer variable with $2^{32}$ states, and this gets intractable.
- Theoretically, there is not much hope. The basic problem of deciding wether a particular formula holds is known as the satisfiability problem, and for the temporal logics we have seen, its complexity is as follows:
  - LTL without $U$ is $NP$-complete.
  - LTL is $PSPACE$-complete.
  - CTL is $EXPTIME$-complete.
- The good news is that at least it is decidable. Practically, state abstraction is the key technique. E.g. instead of considering all possible integer values, consider only wether $i$ is zero or larger than zero.

## Summary

- Model-checking allows us to show to show properties of systems by enumerating the system's states, by modelling systems as finite state machines, and expressing properties in temporal logic.
- We considered Linear Temporal Logic (LTL) and Computational Tree Logic (CTL). LTL allows us to express properties of single paths, CTL allows quantifications over all possible paths of an FSM.
- The basic problem: the system state can quickly get huge, and the basic complexity of the problem is horrendous. Use of abstraction and state compression techniques make model-checking bearable.
- Tomorrow: practical experiments with model-checkers (NuSMV and/or Spin)