Verifikation von C-Programmen
Universität Bremen, WS 2014/15

Lecture 05 (19.11.2013)

Statische Programmanalyse

Christoph Lüth

Today: Static Program Analysis

- Analysis of run-time behavior of programs without executing them (sometimes called static testing)
- Analysis is done for **all** possible runs of a program (i.e. considering all possible inputs)
- Typical tasks
  - Does the variable *x* have a constant value ?
  - Is the value of the variable *x* always positive ?
  - Can the pointer *p* be null at a given program point ?
  - What are the possible values of the variable *y* ?
- These tasks can be used for verification (e.g. is there any possible dereferencing of the null pointer), or for optimisation when compiling.

Usage of Program Analysis

Optimising compilers
- Detection of sub-expressions that are evaluated multiple times
- Detection of unused local variables
- Pipeline optimisations

Program verification
- Search for runtime errors in programs
- Null pointer dereference
- Exceptions which are thrown and not caught
- Over/underflow of integers, rounding errors with floating point numbers
- Runtime estimation (worst-caste executing time, wcet; *AbsInt* tool)

Program Analysis: The Basic Problem

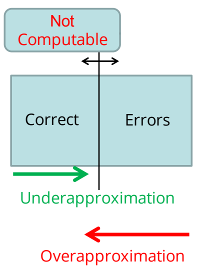
- Basic Problem:

  All interesting program properties are undecidable.

- Given a property P and a program p, we say $p \vDash P$ if a P holds for p. An algorithm (tool) ϕ which decides P is a computable predicate $\phi: p \to Bool$. We say:
  - ϕ is **sound** if whenever $\phi(p)$ then $p \vDash P$.
  - ϕ is **safe** (or **complete**) if whenever $p \vDash P$ then $\phi(p)$.
- From the basic problem it follows that there are no sound and safe tools for interesting properties.
  - In other words, all tools must either under- or overapproximate.

Program Analysis: Approximation

- **Underapproximation** only finds correct programs but may miss out some
  - Useful in optimising compilers
  - Optimisation must respect semantics of program, but may optimise.
- **Overapproximation** finds all errors but may find non-errors (false positives)
  - Useful in verification.
  - Safety analysis must find all errors, but may report some more.
  - Too high rate of false positives may hinder acceptance of tool.



Program Analysis Approach

- Provides approximate answers
  - yes / no / don't know or
  - superset or subset of values
- Uses an abstraction of program's behavior
  - Abstract data values (e.g. sign abstraction)
  - Summarization of information from execution paths e.g. branches of the if-else statement
- Worst-case assumptions about environment's behavior
  - e.g. any value of a method parameter is possible
- Sufficient precision with good performance

Flow Sensitivity

Flow-sensitive analysis
- Considers program's flow of control
- Uses control-flow graph as a representation of the source
- Example: available expressions analysis

Flow-insensitive analysis
- Program is seen as an unordered collection of statements
- Results are valid for any order of statements e.g. *S1* ; *S2* vs. *S2* ; *S1*
- Example: type analysis (inference)

Context Sensitivity

Context-sensitive analysis
- Stack of procedure invocations and return values of method parameters
  then results of analysis of the method *M* depend on the caller of *M*

Context-insensitive analysis
- Produces the same results for all possible invocations of *M* independent of possible callers and parameter values

Intra- vs. Inter-procedural Analysis

Intra-procedural analysis

- Single function is analyzed in isolation
- Maximally pessimistic assumptions about parameter values and results of procedure calls

Inter-procedural analysis

- Whole program is analyzed at once
- Procedure calls are considered

Data-Flow Analysis

Focus on questions related to values of variables and their lifetime

Selected analyses:

- **Available expressions (forward analysis)**
  - Which expressions have been computed already without change of the occurring variables (optimization)?
- **Reaching definitions (forward analysis)**
  - Which assignments contribute to a state in a program point? (verification)
- **Very busy expressions (backward analysis)**
  - Which expressions are executed in a block regardless which path the program takes (verification)?
- **Live variables (backward analysis)**
  - Is the value of a variable in a program point used in a later part of the program (optimization)?

A Very Simple Programming Language

- In the following, we use a very simple language with
  - Arithmetic operators given by
    $a \ ::= x \mid n \mid a_1 \ op_a \ a_2$
    with x a variable, n a numeral, op_a arith. op. (e.g. +, -, *)
  - Boolean operators given by
    $b \ ::= \text{true} \mid \text{false} \mid \text{not } b \mid b_1 \, op_b \ b_2 \mid a_1 \, op_r \ a_2$
    with op_b boolean operator (e.g. and, or) and op_r a relational operator (e.g. =, <)
  - Statements given by
    $S \ ::=$
    $[x := a]^l \mid [\text{skip}]^l \mid S_1; S_2 \mid \text{if } [b]^l \text{then } S_1 \text{else } S_2 \mid \text{while } [b]^l \text{do } S$
- An Example Program:

```
[x := a+b]1;
[y := a*b]2;
while [y > a+b]3 do ( [a:=a+1]4; [x:= a+b]5 )
```

The Control Flow Graph

- We define some functions on the abstract syntax:
  - The initial label (entry point) $\text{init}: S \ \rightarrow Lab$
  - The final labels (exit points) $\text{final}: S \ \rightarrow \ \mathbb{P}(Lab)$
  - The elementary blocks $\text{block}: S \rightarrow \mathbb{P}(Blocks)$ where an elementary block is
    - an assignment [x:= a],
    - or [skip],
    - or a test [b]
  - The control flow $\text{flow}: S \rightarrow \ \mathbb{P}(Lab \times Lab)$ and reverse control $\text{flow}^{\text{R}}: S \rightarrow \ \mathbb{P}(Lab \times Lab)$.
- The **control flow graph** of a program S is given by
  - elementary blocks $\text{block}(S)$ as nodes, and
  - flow(S) as vertices.

Labels, Blocks, Flows: Definitions

final($[x := a]^l$) = { l }
final($[\text{skip}]^l$) = { l }
final($S_1; S_2$) = final(S_2)
final(if $[b]^l$ then S_1 else S_2) = final(S_1) $\cup$ final(S_2)
final(while $[b]^l$ do S) = { l }

init($[x := a]^l$) = l
init($[\text{skip}]^l$) = l
init($S_1; S_2$) = init(S_1)
init(if $[b]^l$ then S_1 else S_2) = l
init(while $[b]^l$ do S) = l

flow($[x := a]^l$) = $\emptyset$
flow($[\text{skip}]^l$) = $\emptyset$
flow($S_1; S_2$) = flow(S_1) $\cup$ flow(S_2) $\cup$ {(l, init(S_2)) | $l \in$ final(S_1) }
flow(if $[b]^l$ then S_1 else S_2) = flow(S_1) $\cup$ flow(S_2) $\cup$ { (l, init(S_1), (l, init(S_2) }
flow(while $[b]^l$ do S) = flow(S) $\cup$ { (l, init(S) } $\cup$ {(l', l) | $l' \in$ final(S) }

flow^{R}(S) = {(l', l) | (l, l') $\in$ flow(S)}

blocks($[x := a]^l$) = { $[x := a]^l$ }
blocks($[\text{skip}]^l$) = { $[\text{skip}]^l$ }
blocks($S_1; S_2$) = blocks(S_1) $\cup$ blocks(S_2)
blocks(if $[b]^l$ then S_1 else S_2)
  = { $[b]^l$ } $\cup$ blocks(S_1) $\cup$ blocks(S_2)
blocks(while $[b]^l$ do S) = { $[b]^l$ } $\cup$ blocks(S)

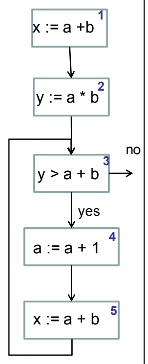
labels(S) = { l | $[B]^l \in$ blocks(S)}
FV(a) = free variables in a
Aexp(S) = nontrivial subexpressions of S

Another Example

P = $[x := a+b]^1$; $[y := a*b]^2$; while $[y > a+b]^3$ do ($[a:=a+1]^4$; $[x:= a+b]^5$)

init(P) = 1
final(P) = {3}
blocks(P) =
  { $[x := a+b]^1$, $[y := a*b]^2$, $[y > a+b]^3$, $[a:=a+1]^4$, $[x:= a+b]^5$ }
flow(P) = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 3)}
flow^{R}(P) = {(2, 1), (3, 2), (4, 3), (5, 4), (3, 5)}
labels(P) = {1, 2, 3, 4, 5}

FV(a + b) = {a, b}

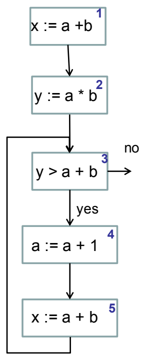


Available Expression Analysis

- The avaiable expression analysis will determine:

For each program point, which expressions must have already been computed, and not later modified, on all paths to this program point.

S :



Available Expression Analysis

gen($[x := a]^l$) = { a' $\in$ Aexp(a) | x $\notin$ FV(a') }
gen($[\text{skip}]^l$) = $\emptyset$
gen($[b]^l$) = Aexp(b)

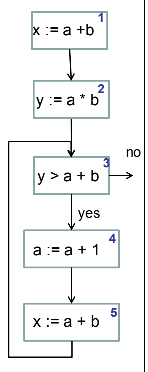
kill($[x := a]^l$) = { a' $\in$ Aexp(S) | x $\in$ FV(a') }
kill($[\text{skip}]^l$) = $\emptyset$
kill($[b]^l$) = $\emptyset$

$AE_{in}(\ l\ ) = \ \emptyset$, if $l \in$ init(S) and
$AE_{in}(\ l\ ) = \ \bigcap \{AE_{out}\ (\ l'\ ) \mid (l', l) \in \text{flow}(S)\ \}$, otherwise
$AE_{out}\ (\ l\ ) = (\ AE_{in}(\ l\ ) \setminus \text{kill}(B^l)\ ) \cup \text{gen}(B^l\ )$ where $B^l \in$ blocks(S)

l	kill(l)	gen(l)
1		
2		
3		
4		
5		

l	AE_{in}	AE_{out}
1		
2		
3		
4		
5		

S :



Available Expression Analysis

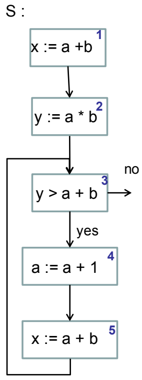
$gen([x := a]^l) = \{ a' \in Aexp(a) \mid x \notin FV(a') \}$
$gen([skip]^l) = \emptyset$
$gen([b]^l) = Aexp(b)$

$kill([x := a]^l) = \{ a' \in Aexp(S) \mid x \in FV(a') \}$
$kill([skip]^l) = \emptyset$
$kill([b]^l) = \emptyset$

$AE_{in}(l) = \emptyset$, if $l \in init(S)$ and
$AE_{in}(l) = \bigcap \{AE_{out}(l') \mid (l', l) \in flow(S)\}$, otherwise
$AE_{out}(l) = (AE_{in}(l) \setminus kill(B^l)) \cup gen(B^l)$ where $B^l \in blocks(S)$

l	kill(*l*)	gen(*l*)
1	∅	{a+b}
2	∅	{a*b}
3	∅	{a+b}
4	{a+b, a*b, a+1}	∅
5	∅	{a+b}

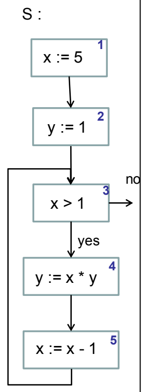
l	AE_{in}	AE_{out}
1	∅	{a+b}
2	{a+b}	{a+b, a*b}
3	{a+b}	{a+b}
4	{a+b}	∅
5	∅	{a+b}



Reaching Definitions Analysis

- Reaching definitions (assignment) analysis determines if:

An assignment of the form $[x := a]^l$ may reach a certain program point k if there is an execution of the program where x was last assigned a value at l when the program point k is reached



Reaching Definitions Analysis

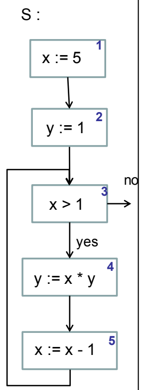
$gen([x := a]^l) = \{ (x, l) \}$
$gen([skip]^l) = \emptyset$
$gen([b]^l) = \emptyset$

$kill([skip]^l) = \emptyset$
$kill([b]^l) = \emptyset$
$kill([x := a]^l) = \{ (x, ?) \} \cup \{ (x, k) \mid B^k$ is an assignment to x in S $\}$

l	kill(B^l)	gen(B^l)
1	{(x,?), (x,1),(x,5)}	{(x, 1)}
2	{(y,?), (y,2),(y,4)}	{(y, 2)}
3	∅	∅
4	{(y,?), (y,2),(y,4)}	{(y, 4)}
5	{(x,?), (x,1),(x,5)}	{(x, 5)}

$RD_{in}(l) = \{ (x, ?) \mid x \in FV(S)\}$, if $l \in init(S)$ and
$RD_{in}(l) = \bigcup \{RD_{out}(l') \mid (l', l) \in flow(S)\}$, otherwise
$RD_{out}(l) = (RD_{in}(l) \setminus kill(B^l)) \cup gen(B^l)$ where $B^l \in blocks(S)$

l	RD_{in}	RD_{out}
1		
2		
3		
4		
5		



Reaching Definitions Analysis

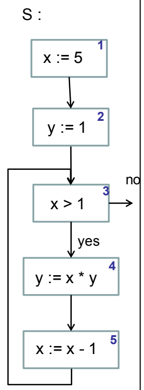
$gen([x := a]^l) = \{ (x, l) \}$
$gen([skip]^l) = \emptyset$
$gen([b]^l) = \emptyset$

$kill([skip]^l) = \emptyset$
$kill([b]^l) = \emptyset$
$kill([x := a]^l) = \{ (x, ?) \} \cup \{ (x, k) \mid B^k$ is an assignment to x in S $\}$

l	kill(B^l)	gen(B^l)
1	{(x,?), (x,1),(x,5)}	{(x, 1)}
2	{(y,?), (y,2),(y,4)}	{(y, 2)}
3	∅	∅
4	{(y,?), (y,2),(y,4)}	{(y, 4)}
5	{(x,?), (x,1),(x,5)}	{(x, 5)}

$RD_{in}(l) = \{ (x, ?) \mid x \in FV(S)\}$, if $l \in init(S)$ and
$RD_{in}(l) = \bigcup \{RD_{out}(l') \mid (l', l) \in flow(S)\}$, otherwise
$RD_{out}(l) = (RD_{in}(l) \setminus kill(B^l)) \cup gen(B^l)$ where $B^l \in blocks(S)$

l	RD_{in}	RD_{out}
1	{(x,?), (y,?)}	{(x,1), (y,?)}
2	{(x,1), (y,?)}	{(x,1), (y,2)}
3	{(x,1), (x,5), (y,2), (y,4)}	{(x,1), (x,5), (y,2), (y,4)}
4	{(x,1), (x,5), (y,2), (y,4)}	{(x,1), (x,5),(y,4)}
5	{(x,1), (x,5),(y,4)}	{(x,5),(y,4)}

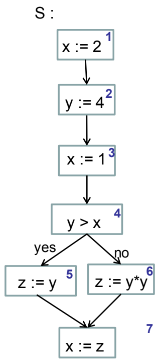


Live Variables Analysis

- A variable x is **live** at some program point (label l) if there exists if there exists a path from l to an exit point that does not change the variable.
- Live Variables Analysis determines:

For each program point, which variables *may* be live at the exit from that point.

- Application: dead code elemination.



Live Variables Analysis

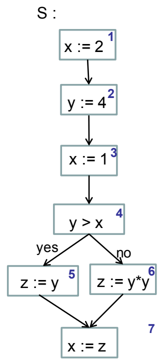
$gen([x := a]^l) = FV(a)$
$gen([skip]^l) = \emptyset$
$gen([b]^l) = FV(b)$

$kill([x := a]^l) = \{x\}$
$kill([skip]^l) = \emptyset$
$kill([b]^l) = \emptyset$

$LV_{out}(l) = \emptyset$, if $l \in final(S)$ and
$LV_{out}(l) = \bigcup \{LV_{in}(l') \mid (l', l) \in flow^R(S)\}$, otherwise
$LV_{in}(l) = (LV_{out}(l) \setminus kill(B^l)) \cup gen(B^l)$ where $B^l \in blocks(S)$

l	kill(*l*)	gen(*l*)
1		
2		
3		
4		
5		
6		
7		

l	LV_{in}	LV_{out}
1		
2		
3		
4		
5		
6		
7		



Live Variables Analysis

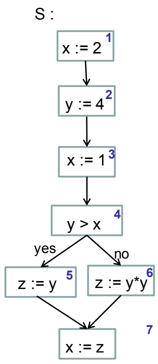
$gen([x := a]^l) = FV(a)$
$gen([skip]^l) = \emptyset$
$gen([b]^l) = FV(b)$

$kill([x := a]^l) = \{x\}$
$kill([skip]^l) = \emptyset$
$kill([b]^l) = \emptyset$

$LV_{out}(l) = \emptyset$, if $l \in final(S)$ and
$LV_{out}(l) = \bigcup \{LV_{in}(l') \mid (l', l) \in flow^R(S)\}$, otherwise
$LV_{in}(l) = (LV_{out}(l) \setminus kill(B^l)) \cup gen(B^l)$ where $B^l \in blocks(S)$

l	kill(*l*)	gen(*l*)
1	{x}	∅
2	{y}	∅
3	{x}	∅
4	∅	{x, y}
5	{z}	{y}
6	{z}	{y}
7	{x}	{z}

l	LV_{in}	LV_{out}
1	∅	∅
2	∅	{y}
3	{y}	{x, y}
4	{x, y}	{y}
5	{y}	{z}
6	{y}	{z}
7	{z}	∅



First Generalized Schema

- $Analyse_{\circ}(l) =$ **EV** , if $l \in$ **E** and
- $Analyse_{\circ}(l) = \sqcup \{ Analyse_{\bullet}(l') \mid (l', l) \in$ **Flow**(S) }, otherwise
- $Analyse_{\bullet}(l) = f_l(Analyse_{\circ}(l))$

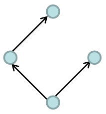
With:

- ⊔ is either ∪ or ∩
- EV is the initial / final analysis information
- Flow is either flow or $flow^R$
- E is either {init(S)} or final(S)
- f_l is the transfer function associated with $B^l \in blocks(S)$

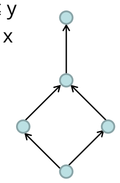
Backward analysis: F = $flow^R$, • = IN, ∘ = OUT
Forward analysis: F = flow, • = OUT, ∘ = IN

Partial Order

- L = (M, ⊑) is a partial order iff
  - Reflexivity: $\forall x \in M.\ x \sqsubseteq x$
  - Transitivity: $\forall x,y,z \in M.\ x \sqsubseteq y \wedge y \sqsubseteq z \Rightarrow x \sqsubseteq z$
  - Anti-symmetry: $\forall x,y \in M.\ x \sqsubseteq y \wedge y \sqsubseteq x \Rightarrow x = y$



- Let L = (M, ⊑) be a partial order, S ⊆ M.
  - y ∈ M is upper bound for S (S ⊑ y) iff $\forall x \in S.\ x \sqsubseteq y$
  - y ∈ M is lower bound for S (y ⊑ S) iff $\forall x \in S.\ y \sqsubseteq x$
  - Least upper bound ⊔X ∈ M of X ⊆ M :
    - $X \sqsubseteq \sqcup X \wedge \forall y \in M : X \sqsubseteq y \Rightarrow \sqcup X \sqsubseteq y$
  - Greatest lower bound ⊓X ∈ M of X ⊆ M:
    - $\sqcap X \sqsubseteq X \wedge \forall y \in M : y \sqsubseteq X \Rightarrow y \sqsubseteq \sqcap X$



Lattice

A lattice ("Verbund") is a partial order L = (M, ⊑) such that

- ⊔X and ⊓X exist for all X ⊆ M
- Unique greatest element $\top = \sqcup M = \sqcap \emptyset$
- Unique least element $\bot = \sqcap M = \sqcup \emptyset$

Transfer Functions

- Transfer functions to propagate information along the execution path
  (i.e. from input to output, or vice versa)

- Let L = (M, ⊑) be a lattice. Set *F* of transfer functions of the form
  $f_l : L \rightarrow L$ with *l* being a label

- Knowledge transfer is monotone
  - $\forall x,y.\ x \sqsubseteq y \Rightarrow f_l(x) \sqsubseteq f_l(y)$

- Space *F* of transfer functions
  - *F* contains all transfer functions f_l
  - *F* contains the identity function id, i.e. $\forall x \in M.\ id(x) = x$
  - *F* is closed under composition, i.e. $\forall f,g \in F.\ (f \circ g) \in F$

The Generalized Analysis

- $Analyse_{\circ}(l) = \bigsqcup \{ Analyse_{\bullet}(l') \mid (l', l) \in Flow(S) \} \sqcup \iota^l_E$
  with $\iota^l_E = EV$ if $l \in E$ and
  $\iota^l_E = \bot$ otherwise
- $Analyse_{\bullet}(l) = f_l(Analyse_{\circ}(l))$

With:

- L property space representing data flow information with (L, ⊔) being a lattice
- Flow is a finite flow (i.e. flow or $flow^R$)
- EV is an extremal value for the extremal labels E (i.e. {init(S)} or final(S))
- transfer functions f_l of a space of transfer functions *F*

Summary

- Static Program Analysis is the analysis of run-time behavior of programs without executing them (sometimes called static testing).
- Approximations of program behaviours by analyzing the program's cfg.
- Analysis include
  - available expressions analysis,
  - reaching definitions,
  - live variables analysis.
- These are instances of a more general framework.
- These techniques are used commercially, e.g.
  - AbsInt aiT (WCET)
  - Astrée Static Analyzer (C program safety)