Which Kind of Module Should I Extract?

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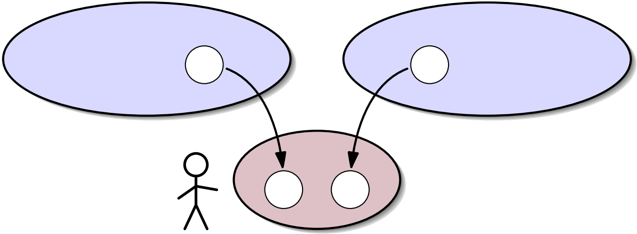
DL, 28 July 2009

And now . . .

Why module extraction?

Reuse external ontologies: borrow knowledge about certain terms



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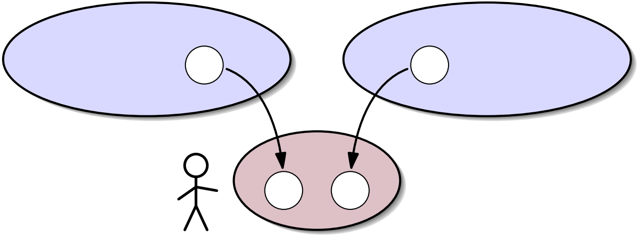

- Provides access to well-established knowledge
- Doesn't require expertise in external disciplines

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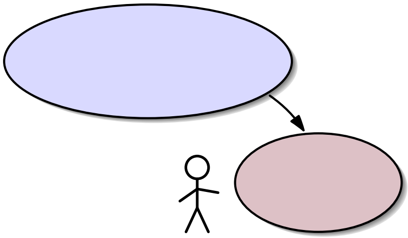

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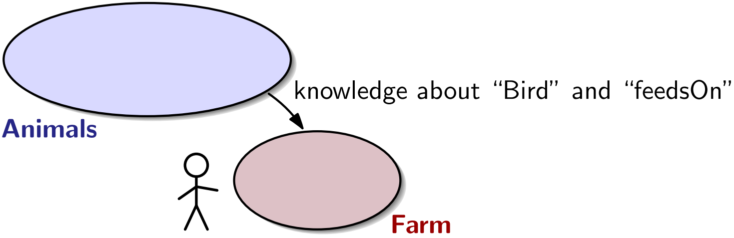

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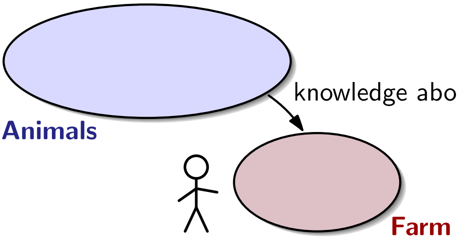

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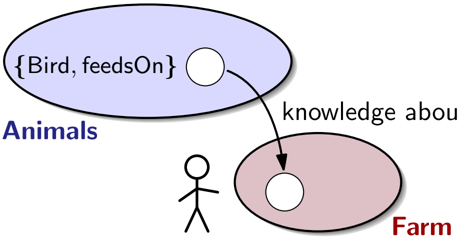

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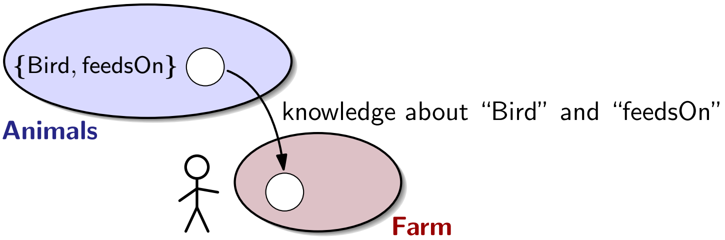

Coverage Import *everything* relevant for the chosen terms.

Economy Import *only* what's relevant for them.
Compute that module quickly.

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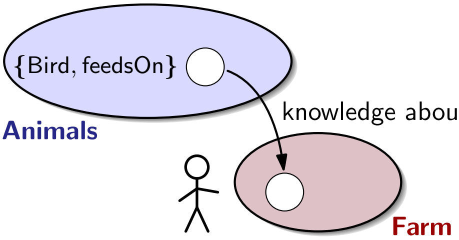

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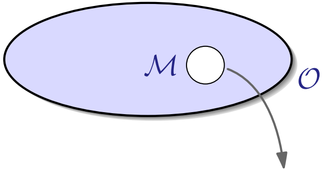
Modules that provide coverage

Input Ontology $\mathcal{O}$ — a set of axioms
Signature Σ (set of concept and role names from $\mathcal{O}$)

Output **a Σ-module $\mathcal{M}$ of $\mathcal{O}$:**

- $\mathcal{M} \subseteq \mathcal{O}$
- $\mathcal{M}$ and $\mathcal{O}$ have the same Σ-entailments:

  For all axioms α using only terms from Σ,
  $\mathcal{O} \models \alpha$ iff $\mathcal{M} \models \alpha$



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conservativity-based modules — locality-based modules

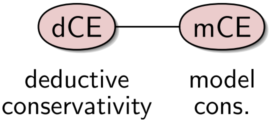
Relevant module types

dCE

deductive conservativity

intractable . . . undecidable

Relevant module types



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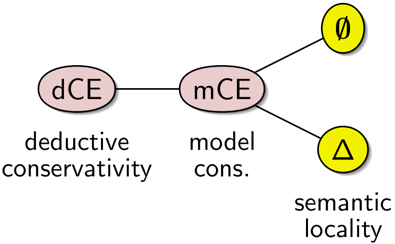

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(pink) intractable . . . undecidable

(yellow) as difficult as reasoning

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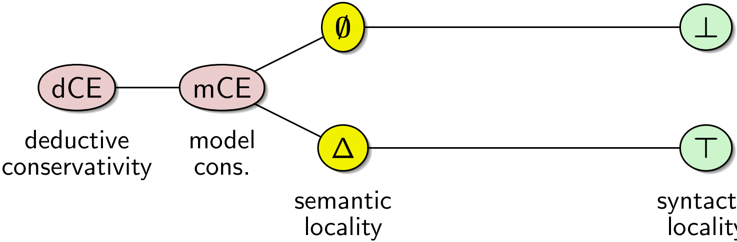

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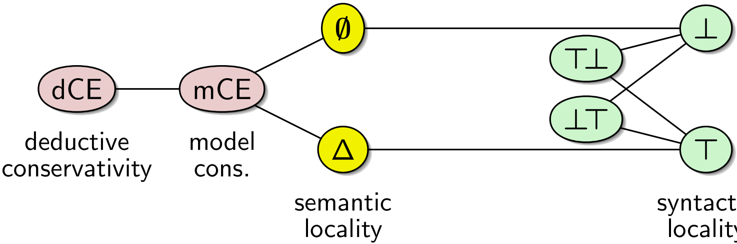

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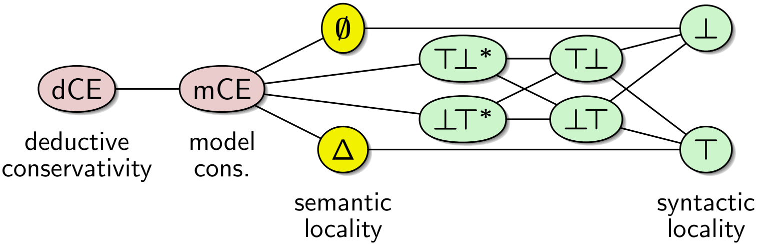

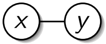

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Goals

- General framework for comparing module notions that provide coverage
- Identify relevant properties
- Application to conservativity-based and locality-based modules

And now . . .

Intuitions

- $\mathcal{O}_1$ and $\mathcal{O}_2$ are inseparable w.r.t. Σ:
  The knowledge about Σ in $\mathcal{O}_1$ and $\mathcal{O}_2$ can't be distinguished
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- Notation: $\mathcal{O}_1 \equiv_{\Sigma}^{S} \mathcal{O}_2$
- $\equiv_{\Sigma}^{S}$ is an equivalence relation
- Inseparability relation: $S = \{\equiv_{\Sigma}^{S} \mid \Sigma \text{ is a signature}\}$

Different inseparability relations

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- $\mathcal{O}_1 \equiv_{\Sigma}^{\perp} \mathcal{O}_2$ if:

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- $\mathcal{O}_1 \equiv_\Sigma^{\bot} \mathcal{O}_2$ if:
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Analogous definition for

$\equiv_\Sigma^{\emptyset}$ $\equiv_\Sigma^{\Delta}$ $\equiv_\Sigma^{\top}$ $\equiv_\Sigma^{\top\bot}$ $\equiv_\Sigma^{\bot\top}$ $\equiv_\Sigma^{\top\bot^*}$ $\equiv_\Sigma^{\bot\top^*}$

Inseparability relations induce modules

Let S be an inseparability relation, Σ a signature and $\mathcal{M} \subseteq \mathcal{O}$.

$\mathcal{M}$ is called	if	see
an S_Σ-*module of* $\mathcal{O}$	$\mathcal{M} \equiv^S_\Sigma \mathcal{O}$	1

Example: $S = \mathsf{dCE}$, $\Sigma = \{\mathsf{Bird}, \mathsf{feedsOn}\}$, $\mathcal{M}$ contains Grass.

1. $\mathcal{O} \models \mathsf{Bird} \sqsubseteq \exists\,\mathsf{feedsOn}.\top$ iff $\mathcal{M} \models \mathsf{Bird} \sqsubseteq \exists\,\mathsf{feedsOn}.\top$

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a *depleting S_Σ-module of $\mathcal{O}$*	$\emptyset \equiv^S_{\Sigma \cup \mathsf{sig}(\mathcal{M})} \mathcal{O} \setminus \mathcal{M}$	3

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3. $\mathcal{O} \setminus \mathcal{M}$ entails only tautologies w.r.t. $\{\mathsf{Bird}, \mathsf{feedsOn}, \mathsf{Grass}\}$.

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Robustness properties (1)

S is robust under vocabulary restrictions:

If $\mathcal{O}_1 \equiv^S_\Sigma \mathcal{O}_2$ and $\Sigma' \subseteq \Sigma$, then $\mathcal{O}_1 \equiv^S_{\Sigma'} \mathcal{O}_2$.

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Consequences:

If $\mathcal{M}$ is a Σ-module of $\mathcal{O}$ and $\Sigma' \subseteq \Sigma$,
then $\mathcal{M}$ is a Σ'-module of $\mathcal{O}$.

$\leadsto$ On restricting the signature, no new import is necessary.

Robustness properties (2)

- Vocabulary extensions

  If $\mathcal{M}$ is a Σ-module of $\mathcal{O}$ and $(\Sigma' \setminus \Sigma) \cap \mathrm{sig}(\mathcal{O}) = \emptyset$,
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- Joins

  If we have two indistinguishable ontologies,
  it suffices to import one of them.

Overview of properties

Inseparability rel. (IR) / Property	$\equiv_\Sigma^{dCE}$	$\equiv_\Sigma^{mCE}$	$\equiv_\Sigma^{\emptyset}$ $\equiv_\Sigma^{\Delta}$ $\equiv_\Sigma^{\bot}$ $\equiv_\Sigma^{\top}$	$\equiv_\Sigma^{\top\bot}$ $\equiv_\Sigma^{\bot\top}$	$\equiv_\Sigma^{\top\bot*}$ $\equiv_\Sigma^{\bot\top*}$
Modules are induced …					
modules	✓	✓	✓	✗	✓
self-contained modules	✗	✗	✓	✓	✓
depleting modules	✗	✗	✓	✓	✓
IR is robust under …					
vocab. restrictions	✓	✓	✓	✗	✓
vocab. extensions	✗	✓	✓	✗	✓
replacement	✗	✓	✓	✓	✓
joins	✗	✓	✓	✓	✓

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Conclusions

- mCE-based and (most) locality-based modules are very robust.
- dCE-based modules are not robust.
- Locality-based modules can be extracted efficiently.

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