Lightweight Description Logics & Branching Time: A Troublesome Marriage

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Description logics are inherently atemporal

DLs are …

… **good** at expressing **static domain knowledge**:

$$\text{Diabetes} \equiv \text{MetabolicDisorder} \sqcap \exists\, \text{hasFinding.Pancreas}$$

… **bad** at expressing **temporal knowledge**:

"A patient who has diabetes **now**
may develop certain disorders **in the future**"

$$\rightsquigarrow$$

$$\exists\, \text{hasDisease.Diabetes} \sqsubseteq \exists\, \text{mayDevelop.Glaucoma}$$

Temporal extensions of DLs

Applications: KR and reasoning ...

... over temporal conceptual data models
(EER, UML + temporal constraints)

... in the medical domain

Approach

Extend DLs with point-based temporal operators [Schild 1993]

$\rightsquigarrow$ **Temporal description logics (TDLs)**

Complexity results for satisfiability/subsumption (selection)

- $\mathcal{ALC}$ + LTL operators: ExpTime ... undecidable
- DL-Lite + LTL: NP ... undecidable
- $\mathcal{ALC}$ or $\mathcal{EL}$ + CTL$^{(*)}$: PTime ... 3ExpTime

[Artale et al. 2002/03/12, Baader et al. 2008, Gutiérrez-Basulto et al. 2012]

TDLs: syntax

TDLs are . . . modal description logics

Components: DL of your choice + temporal operators, e.g.:

$\mathsf{E}\Diamond\varphi$	"in some future, eventually φ"
$\mathsf{A}\Box\varphi$	"in all futures, always φ"
$\mathsf{A}\bigcirc\varphi$	"in all futures, next time φ"

Example: $\exists\,\mathsf{hasDisease.Diabetes} \sqsubseteq \mathsf{E}\Diamond\,\exists\,\mathsf{hasDisease.Glaucoma}$

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Design choice #1: Temporal operators from ...

✓ CTL $\rightsquigarrow$ **B-TDLs**

LTL $\rightsquigarrow$ L-TDLs (quite well-understood)

...

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TDLs are . . . modal description logics

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Example: $\exists\,\text{hasDisease.Diabetes} \sqsubseteq \mathsf{E}\Diamond\,\exists\,\text{hasDisease.Glaucoma}$

Design choice #2: Scope of temporal operators

✓ Temporal concepts

Temporal roles

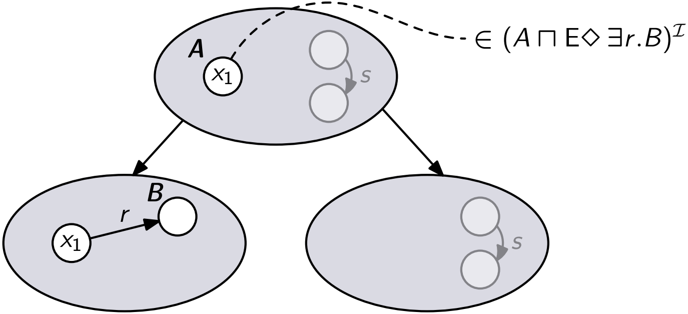
} combination tends to be **hard**

Temporal axioms

B-TDLs: semantics

Temporal dimension: worlds + tree-shaped “future” relation

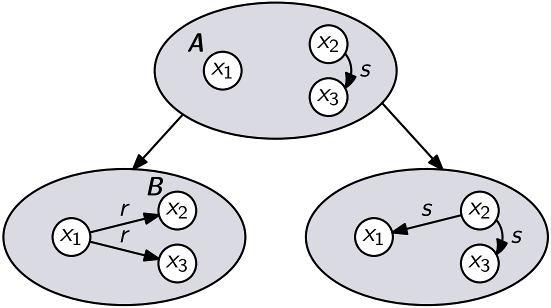
DL dimension: one full DL interpretation per world



Semantic design choices

Design choice #3: Relation between DL domains

Constant domains ✓



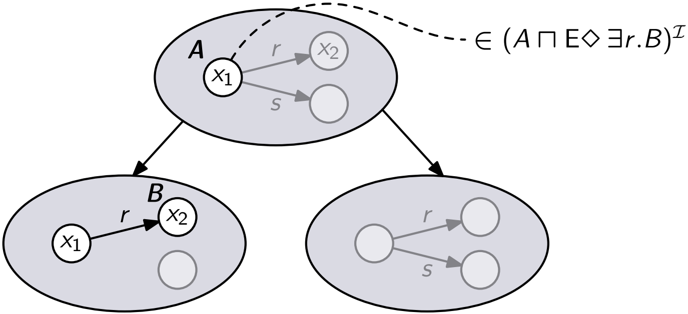
Alternative choices: expanding or decreasing domains

Semantic design choices

Design choice #4: Rigid vs. flexible roles

Rigid role r, flexible role s

We allow both. ✓



Semantic design choices

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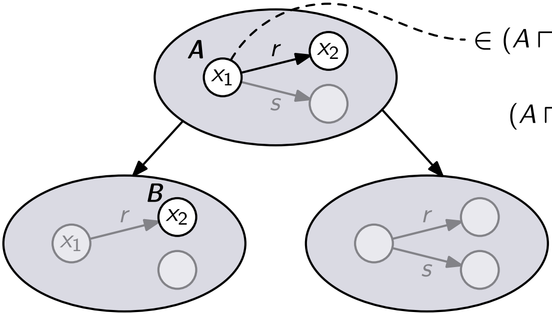

$\in (A \sqcap \mathsf{E}\Diamond\, \exists r.B)^{\mathcal{I}}$

$\equiv$

$(A \sqcap \exists r.\mathsf{E}\Diamond B)^{\mathcal{I}}$

Semantic design choices

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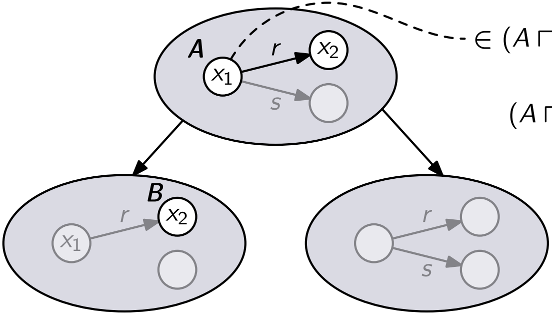

$\in (A \sqcap \mathsf{E}\Diamond\, \exists r.B)^{\mathcal{I}}$

$\equiv$

$(A \sqcap \exists r.\mathsf{E}\Diamond B)^{\mathcal{I}}$

TDLs with rigid roles are usually harder

Semantic design choices

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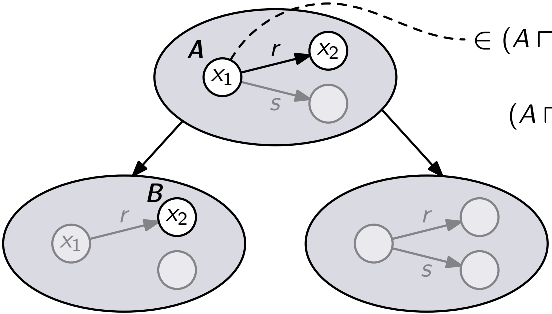

$\in (A \sqcap \mathsf{E}\Diamond\, \exists r.B)^{\mathcal{I}}$

$\equiv$

$(A \sqcap \exists r.\mathsf{E}\Diamond B)^{\mathcal{I}}$

B-TDLs haven't been studied with rigid roles!

Branching-time TDLs: a marriage proposal

We study: CTL (fragments) $\times$ $\mathcal{ALC}$, $\mathcal{EL}$, DL-Lite$_{\mathrm{bool}}$

with

- Global TBoxes
- Temporal operators on concepts only
- Rigid roles
- Constant domains

(Un-)decidability and complexity of satisfiability and subsumption

Main motivation:

- B-TDLs with rigid roles: **new**
- **Hope** for happy marriages in contrast to L-TDLs:
  LTL $\times$ $\mathcal{EL}$ is undecidable (non-convex) [Artale et al. 2007]

Up and down between despair and hope

1. Undecidability of CTL $\times \mathcal{ALC}$
2. Lightweight DLs to the rescue
3. Undecidability of non-convex CTL $\times \mathcal{EL}$ fragments
4. Convex fragments of CTL $\times \mathcal{EL}$
5. Lower bounds for convex fragments
6. Decidability for fragments of CTL $\times$ DL-Lite$_{\text{bool}}$
7. Outlook

Despair . . .

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A prototype of a failed marriage

Theorem (bad news, but expected)

Satisfiab. for $\mathrm{CTL}(\mathsf{E}\Diamond, \mathsf{A}\Box) \times \mathcal{ALC}$ with 1 rigid role is undecidable.

Proof sketch.

- Use results for transitive product modal logics [Gabelaia et al.'05]
- Encode transitivity in TBox — Technique by [Tobies 2001]

Implications on a range of product MLs
(global consequence, one transitive component)

Hope . . .

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Saving the marriage by having children

Resort: study **"lightweight" fragments**

- $\mathsf{CTL}(\cdot) \times \mathcal{EL}$
- $\mathsf{CTL}(\cdot) \times \text{DL-Lite}_{\text{bool}}$

Observation: $\mathsf{CTL}(\cdot) \times \mathcal{EL}$ syntax has no disjunction

$$C ::= A \mid C \sqcap C \mid \exists r.C \mid \mathsf{E}\Diamond C \mid \mathsf{A}\Diamond C \mid \mathsf{E}\Box C \mid \ldots$$

Still, some temporal operators can express disjunction, e.g.:

$$\mathsf{E}\Diamond A \quad \sqsubseteq \quad A \sqcup \mathsf{E}\bigcirc\mathsf{E}\Diamond A$$

$\leadsto$ $\mathsf{CTL}(\mathsf{E}\bigcirc, \mathsf{E}\Diamond) \times \mathcal{EL}$ and others are **non-convex**

Despair . . .

More failed marriage proposals

Theorem (bad news, again expected)

Subsumption is undecidable for

- $\mathrm{CTL}(\mathrm{E}\bigcirc, \mathrm{E}\Diamond) \times \mathcal{EL}$
- $\mathrm{CTL}(\mathrm{E}\Diamond, \mathrm{A}\Diamond) \times \mathcal{EL}$
- $\mathrm{CTL}(\mathrm{E}\Diamond, \mathrm{E}\Box) \times \mathcal{EL}$
- $\mathrm{CTL}(\mathrm{E}\mathcal{U}) \times \mathcal{EL}$

Proof sketch.

Use non-convexity witnesses to embed $\mathrm{CTL}(\mathrm{E}\Diamond, \mathrm{A}\Box) \times \mathcal{ALC}$ into $\mathrm{CTL}(\cdot) \times \mathcal{EL}$

(Technique by Artale et al. for $\mathrm{LTL} \times \mathcal{EL}$) [Artale et al. 2007]

Hope . . .

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Candidates for a successful marriage

Consider operators $\mathsf{E}\bigcirc$ $\mathsf{E}\Diamond$ $\mathsf{E}\Diamond,\mathsf{A}\Box$

Theorem (good news)

The following B-TDLs are convex.

$\mathsf{CTL}(\mathsf{E}\bigcirc) \times \mathcal{EL}$ $\mathsf{CTL}(\mathsf{E}\Diamond) \times \mathcal{EL}$ $\mathsf{CTL}(\mathsf{E}\Diamond, \mathsf{A}\Box) \times \mathcal{EL}$

Proof sketch.

The following are preserved under direct products of models

- FO-translation of $\mathsf{CTL}(\cdot) \times \mathcal{EL}$-TBoxes
- FO-axiomatization of rigid roles

Despair . . .

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Big, sad theorem

Theorem

Subsumption for ...

1. $\mathrm{CTL}(\mathsf{E}\bigcirc) \times \mathcal{EL}$ is undecidable.
2. $\mathrm{CTL}(\mathsf{E}\Diamond) \times \mathcal{EL}$ is inherently nonelementary. (Upper bound?)

$\rightsquigarrow$ **Failed marriage** despite all efforts (positive exist. fragment, convexity)

Proof sketch.

1. For undecidability of $\mathrm{CTL}(\mathsf{E}\bigcirc) \times \mathcal{EL}$:
   reduce from halting problem of 2-counter automata [Minsky '67]
   (Refers to *direct* temporal successors)
2. For nonelementary lower bound of $\mathrm{CTL}(\mathsf{E}\Diamond) \times \mathcal{EL}$:
   encode k-exponential counters, [Stockmeyer, '74]
   reduce from word problem for k-ExpSpace Turing machines

Encoding 2-counter automata

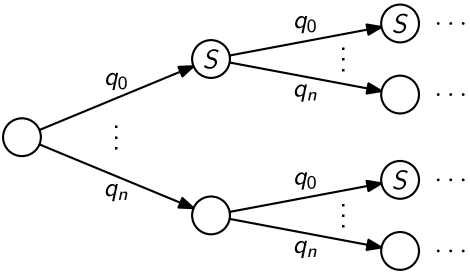
- States $q_0, \ldots, q_n$
- Counters c_1, c_2 (values $\in \mathbb{N}$)
- Instructions *(deterministic)*

$$q_i \rightarrow \texttt{inc}(c_j); q_k \quad \text{or}$$
$$q_i \rightarrow \texttt{if} \;\; c_j = 0 \;\; \texttt{then} \;\; q_k \;\; \texttt{else} \;\; \texttt{dec}(c_j); q_\ell$$

- Configurations $\langle q_i, c_1, c_2 \rangle$
- Halting problem: can M reach q_n from $\langle q_0, 0, 0 \rangle$?

Encoding 2-counter automata

1. **Generate all sequences of states in $\mathcal{EL}$**



Computations start at S and run backwards

2. **Check if one sequence is halting in the root**

Encode counter values along temporal dimension (in unary)

Use E$\bigcirc$ to increment and decrement

Hope . . .

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Theorem

Satisfiability for ...

1. CTL $\times$ DL-Lite$_{bool}$ with *only* rigid roles and
   CTL(E$\mathcal{U}$, E$\Box$) $\times$ DL-Lite$_{bool}$ is ExpTime-complete.
2. CTL(E$\Diamond$) $\times$ DL-Lite$_{bool}$ is PSpace-complete.

(same complexity as the participating CTL fragments) [Meier et al. 2009]

Technique used

[Artale et al. 2012] for LTL $\times$ DL-Lite$_{bool}$

1. Encode TBox and rigidity in 1-var. first-order TL
2. Eliminate $\exists$ quantifiers (using temporal unraveling – **new!**)
3. Instantiate $\forall$ quantifiers with all constants

$\rightsquigarrow$ Poly-time reduction to the participating CTL fragment

Some more hope . . .

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Some more hope ...

For $\mathcal{EL}$

Further taming seems fit
– We're working on acyclic/cyclic terminologies

For DL-Lite$_{\text{bool}}$

- Further restrictions: e.g., DL-Lite$_{\text{core}}$ etc.
- More general result using automata-theoretic techniques

Ambitious ...

- Expanding domains?
- Are there successful marriages with temporal roles?

Thank you.