The Complexity of Temporal Description Logics with Rigid Roles and Restricted TBoxes

In Quest of Saving a Troublesome Marriage

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And now . . .

Description logics are inherently atemporal

DLs are ...

... **good** at expressing **static domain knowledge**:

$$\mathsf{Diabetes} \equiv \mathsf{MetabolicDisorder} \sqcap \exists\, \mathsf{hasFinding.Pancreas}$$

... **bad** at expressing **temporal knowledge**:

'A patient who has diabetes **now**
may develop certain disorders **in the future**'

$$\rightsquigarrow$$

$$\exists\, \mathsf{hasDisease.Diabetes} \sqsubseteq \exists\, \underset{\sim\sim\sim\sim\sim\sim}{\mathsf{mayDevelop}}.\mathsf{Glaucoma}$$

Temporal extensions of DLs

Applications: knowledge representation and reasoning

. . . over temporal conceptual data models
(EER, UML + temporal constraints)

. . . in the medical domain
(e.g., SNOMED CT with temporal knowledge)

Approach

Extend DLs with point-based temporal operators [Schild 1993]

$\rightsquigarrow$ **Temporal description logics (TDLs)**

TDLs: existing work

Several TDLs have been studied, under various **design choices**

$\mathcal{ALC}$ + LTL operators
DL-Lite + LTL
$\mathcal{ALC}$ + CTL$^{(*)}$
$\mathcal{EL}$ + CTL
DL-Lite + CTL

Complexity results from PTime to undecidable

[Artale et al. 2007/14, Baader et al. 2008, Gutiérrez-Basulto et al. 2012/14]

TDLs: syntax

TDLs are . . . modal description logics

Components: DL of your choice + temporal operators, e.g.:

$\mathsf{E}\Diamond\varphi$ ‘in some future, eventually φ’

$\mathsf{A}\Box\varphi$ ‘in all futures, always φ’

$\mathsf{A}\bigcirc\varphi$ ‘in all futures, next time φ’

Example: $\exists\,\mathsf{hasDisease.Diabetes} \sqsubseteq \mathsf{E}\Diamond\,\exists\,\mathsf{hasDisease.Glaucoma}$

$\leadsto$

‘A patient who has diabetes now
may develop certain disorders in the future’

Syntactic design choices

Example: $\exists\,\mathsf{hasDisease}.\mathsf{Diabetes} \sqsubseteq \mathsf{E}\Diamond\,\exists\,\mathsf{hasDisease}.\mathsf{Glaucoma}$

Design choice #1: Temporal operators from ...

✓ CTL

LTL

(or ATL, μ-calculus, ...)

Syntactic design choices

Example: $\exists\,\mathsf{hasDisease.Diabetes} \sqsubseteq \mathsf{E}\Diamond\,\exists\,\mathsf{hasDisease.Glaucoma}$

Design choice #1: Temporal operators from ...

✓ CTL

Design choice #2: Scope of temporal operators

✓ Temporal concepts
Temporal roles
Temporal axioms

} combination tends to be **hard** (Temporal concepts + Temporal roles)

Syntactic design choices

Example: $\exists\,\mathsf{hasDisease.Diabetes} \sqsubseteq \mathsf{E}\Diamond\,\exists\,\mathsf{hasDisease.Glaucoma}$

Design choice #1: Temporal operators from ...

✓ CTL

Design choice #2: Scope of temporal operators

✓ Temporal concepts

Design choice #3: Strength of axioms

General TBoxes (GCIs)

✓ Acyclic terminologies **(NEW)**

✓ No axioms

Syntactic design choices

Example: $\exists\,\mathsf{hasDisease}.\mathsf{Diabetes} \sqsubseteq \mathsf{E}\Diamond\,\exists\,\mathsf{hasDisease}.\mathsf{Glaucoma}$

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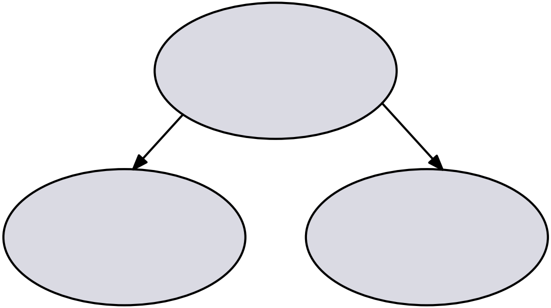
Branching-time TDLs: semantics

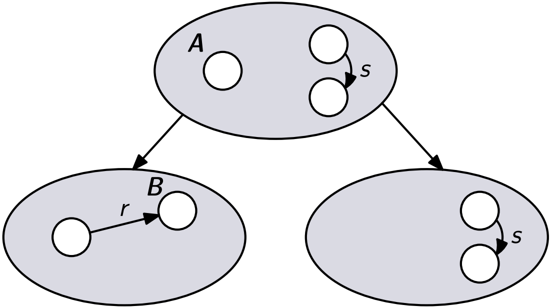
Temporal dimension: worlds + tree-shaped ‘future’ relation



Branching-time TDLs: semantics

Temporal dimension: worlds + tree-shaped ‘future’ relation

DL dimension: one full DL interpretation per world



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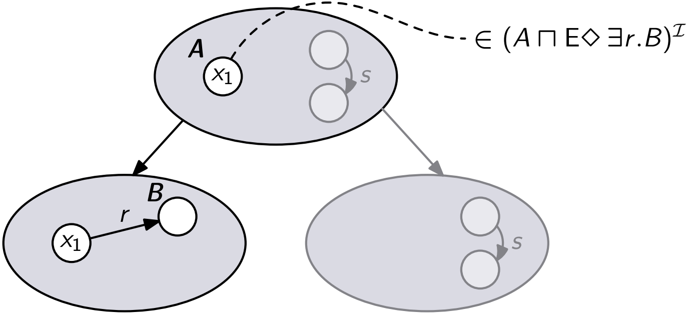
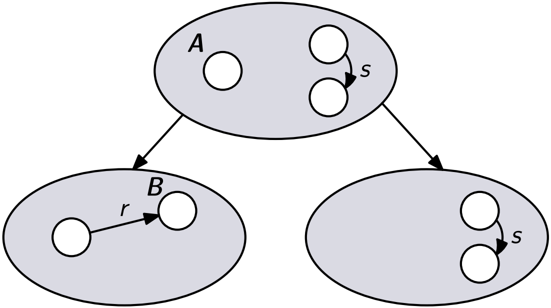


Semantic design choices

Design choice #4: Relation between DL domains

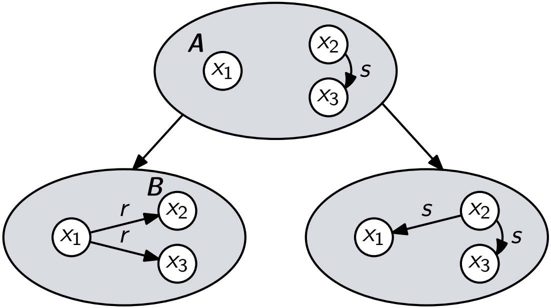
Varying domains



Semantic design choices

Design choice #4: Relation between DL domains

Constant domains ✓



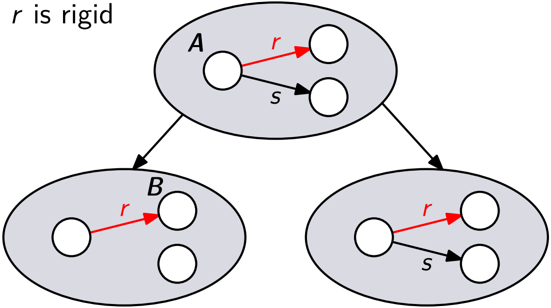
Alternative choices: expanding or decreasing domains

Semantic design choices

Design choice #4: Relation between DL domains ✓ constant

Design choice #5: Permission of rigid roles ✓ yes

here: r is rigid



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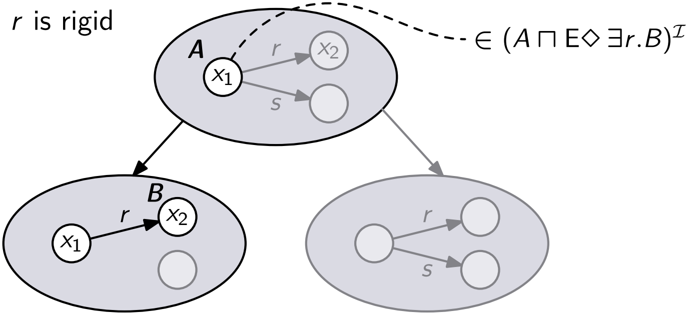

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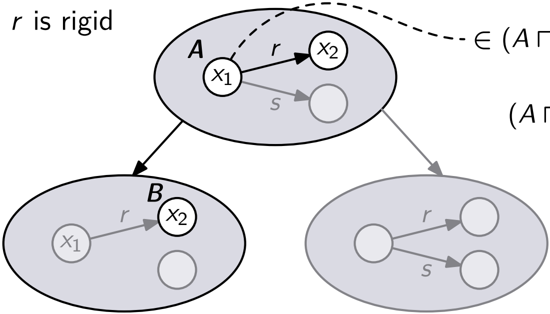

$$\in (A \sqcap \mathsf{E}\Diamond \exists r.B)^{\mathcal{I}}$$
$$\equiv$$
$$(A \sqcap \exists r.\mathsf{E}\Diamond B)^{\mathcal{I}}$$

Semantic design choices

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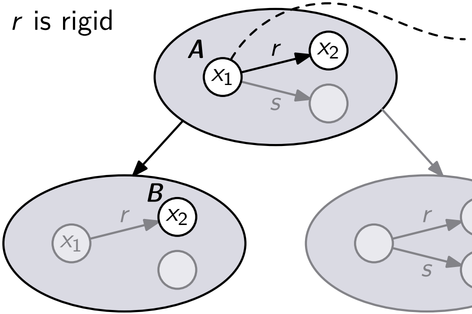


$\in (A \sqcap \mathsf{E}\Diamond\, \exists r.B)^{\mathcal{I}}$

$\equiv$

$(A \sqcap \exists r.\mathsf{E}\Diamond B)^{\mathcal{I}}$

TDLs with rigid roles are usually harder

Branching-time TDLs: a marriage proposal

We study **CTL (fragments) $\times$ $\mathcal{ALC}$, $\mathcal{EL}$** with

- Temporal operators on concepts only
- Acyclic TBoxes
- Constant domains
- Rigid roles

Decidability and complexity
of satisfiability and subsumption

Main motivation

- $\mathcal{EL}$-based TDLs with rigid roles are hard $\leadsto$ **acyclic TBoxes?**
- TDLs based on certain CTL fragments are convex

A troublesome marriage?

With general TBoxes, even very 'small' combinations don't work

CTL(E$\bigcirc$) $\times$ $\mathcal{EL}$ allows concepts of the form

$$C ::= A \mid C \sqcap C \mid \exists r.C \mid \mathsf{E}\bigcirc C$$

Positive, existential, convex – but:

Big, sad theorem ☹

With general TBoxes,

- CTL(E$\bigcirc$) $\times$ $\mathcal{EL}$ is undecidable
- CTL(E$\Diamond$) $\times$ $\mathcal{EL}$ is nonelementary [Gutiérrez-Basulto et al. 2014]

Do acyclic TBoxes permit decidable/elementary/tractable TDLs?

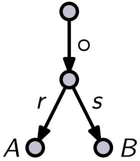
And now . . .

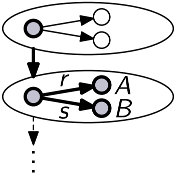
Warming up: subsumption *without* TBoxes

To decide $\models C \sqsubseteq D$, we can

- Construct a canonical model for C

e.g., $\mathsf{E}\bigcirc(\exists r.A \sqcap \exists s.B)$





- Stop the construction after depth $|C| + |D|$
- Check whether D is satisfied at the root

Theorem ☺

Subsumption with empty TBoxes is in polynomial time for

- $\mathrm{CTL}(\mathsf{E}\bigcirc) \times \mathcal{EL}$
- $\mathrm{CTL}(\mathsf{E}\Diamond) \times \mathcal{EL}$

Combining E$\bigcirc$ and E$\Diamond$

CTL(E$\bigcirc$, E$\Diamond$) $\times\, \mathcal{EL}$ is non-convex: $\models\ \mathsf{E}\Diamond A \ \sqsubseteq\ A \sqcup \mathsf{E}\bigcirc\mathsf{E}\Diamond A$

Still, reuse the previous technique to decide $\models C \sqsubseteq D$:

- Replace every E$\Diamond$ in C with some E$\bigcirc$-sequence:

$$C = \ldots \mathsf{E}\Diamond \ldots \qquad \rightsquigarrow \qquad C' = \ldots \underbrace{\mathsf{E}\bigcirc \cdots \mathsf{E}\bigcirc}_{k} \ldots$$

- Suffices to *guess* $k \leqslant |D|$ (technique by Haase & Lutz)

Theorem ☺

Subsumption with empty TBoxes is coNP-complete for CTL(E$\bigcirc$, E$\Diamond$) $\times\, \mathcal{EL}$.

Extend the good news to $\mathcal{ALC}$?

Replacing the lightweight component with $\mathcal{ALC}$ yields:

Theorem ☹

Subsumption with empty TBoxes is decidable but nonelementary for CTL(S) $\times$ $\mathcal{ALC}$ whenever S contains E$\bigcirc$ or E$\Diamond$.

Lower bound

CTL(E$\bigcirc$) $\times$ $\mathcal{ALC}$ and CTL(E$\Diamond$) $\times$ $\mathcal{ALC}$ are nonelementary:

Transfer from product modal logics K $\times$ K, S4 $\times$ K [Göller et al. 2015]

Upper bound

CTL(full) $\times$ $\mathcal{ALC}$ is decidable:

Quasimodel technique [Wolter & Zakharyaschev 1998]

+ reduction to monadic 2nd-order logic over trees [Gabbay et al. 2003]

Summary for the empty TBox

		empty TBox
E$\bigcirc$	$\times\ \mathcal{EL}$	in PTIME
E$\Diamond$	$\times\ \mathcal{EL}$	in PTIME
E$\bigcirc$,E$\Diamond$	$\times\ \mathcal{EL}$	coNP-complete
E$\bigcirc$,...	$\times\ \mathcal{ALC}$	decidable but
E$\Diamond$,...	$\times\ \mathcal{ALC}$	nonelementary

The 'bigger picture' for acyclic TBoxes

Via unfolding, we easily get:

		empty TBox	acyclic TBoxes
$\mathsf{E}\bigcirc$	$\times\,\mathcal{EL}$	in PTIME	in EXPTIME
$\mathsf{E}\Diamond$	$\times\,\mathcal{EL}$	in PTIME	in EXPTIME
$\mathsf{E}\bigcirc,\mathsf{E}\Diamond$	$\times\,\mathcal{EL}$	coNP-complete	in coNEXPTIME
$\mathsf{E}\bigcirc,\ldots$ $\mathsf{E}\Diamond,\ldots$	$\times\,\mathcal{ALC}$ $\times\,\mathcal{ALC}$	decidable but nonelementary	decidable but nonelementary

The ‘bigger picture’ for acyclic TBoxes

But we can do better:

		empty TBox	acyclic TBoxes
E$\bigcirc$	$\times\,\mathcal{EL}$	in PTime	in ExpTime $\rightsquigarrow$ **in PTime**
E$\Diamond$	$\times\,\mathcal{EL}$	in PTime	in ExpTime $\rightsquigarrow$ **in PTime**
E$\bigcirc$,E$\Diamond$	$\times\,\mathcal{EL}$	coNP-complete	in coNExpTime
E$\Diamond$,A$\Box$	$\times\,\mathcal{EL}$	in PSpace	**PSpace-complete**
E$\bigcirc$,...	$\times\,\mathcal{ALC}$	decidable but	decidable but
E$\Diamond$,...	$\times\,\mathcal{ALC}$	nonelementary	nonelementary

E◇ with acyclic TBoxes

Theorem ☺

CTL(E◇) × $\mathcal{EL}$ with acyclic TBoxes is in PTime.

$\mathcal{EL}$-style completion algorithm

- Build abstract representation of 'minimal' model for $\mathcal{T}$

  In $\mathcal{EL}$: $$B \in Q(A) \;\Leftrightarrow\; \mathcal{T} \models A \sqsubseteq B$$

- Consider $Q(\cdot)$ relative to worlds $w = AB$

  ensure $$B' \in Q(A, AB) \;\Leftrightarrow\; \mathcal{T} \models A \sqcap \mathsf{E}\Diamond B \sqsubseteq \mathsf{E}\Diamond(B \sqcap B')$$
  $$B \in Q(A, AA) \;\Leftrightarrow\; \mathcal{T} \models A \sqsubseteq B$$

- Complete all $Q(\cdot, \cdot)$ in 3 phases (acyclicity allows separation)
  1. Apply axioms $A \sqsubseteq C$ 'forwards'
  2. Incorporate rigid roles & constant domains
  3. Apply axioms $A \sqsubseteq C$ 'backwards'

E◇ and A□ with acyclic TBoxes

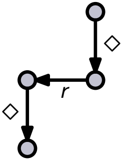
Theorem ☺

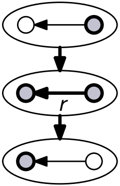
CTL(E◇,A□) $\times$ $\mathcal{EL}$ with acyclic TBoxes is PSPACE-complete.

Lower bound: enforce full binary tree and encode QBF

Upper bound: Resort to a *dynamic* data structure

- Keep a *single trace* in memory at any time





- Complete traces in a tableau-like fashion (cf. K, K4)
- Collect subsumers of *A*: depth-first search through all traces
- Length of traces is limited by a polynomial (acyclicity)

Replacing E$\Diamond$ with E$\bigcirc$

... requires just a few modifications

Theorem ☺

Subsumption with acyclic TBoxes is

- in PTIME for CTL(E$\bigcirc$) $\times \mathcal{EL}$
- PSPACE-complete for CTL(E$\bigcirc$,A$\Box$) $\times \mathcal{EL}$ and CTL(E$\bigcirc$,A$\bigcirc$) $\times \mathcal{EL}$

And now . . .

Conclusion

Main goal achieved!

Fragments of CTL $\times\ \mathcal{EL}$ with elementary (polynomial) complexity

		empty TBox	acyclic TBoxes	general TBoxes
E$\bigcirc$	$\times\ \mathcal{EL}$	in PTIME	in PTIME	undecid.
E$\Diamond$	$\times\ \mathcal{EL}$	in PTIME	in PTIME	nonelem.
E$\bigcirc$,E$\Diamond$	$\times\ \mathcal{EL}$	coNP-complete	in coNEXPTIME	undecid.
E$\Diamond$,A$\Box$	$\times\ \mathcal{EL}$	in PSPACE	PSPACE-complete	undecid.
...	$\times\ \mathcal{ALC}$	decidable but nonelementary		undecid.

$\rightsquigarrow$ Acyclic TBoxes can help design well-behaved $\mathcal{EL}$-based TDLs

Byproduct

Complexity of positive fragments of product MLs: K $\times$ K, S4 $\times$ K

Future work

- More expressive fragments
  e.g., $\mathsf{CTL}(\mathsf{E}\bigcirc, \mathsf{E}\Diamond) \times \mathcal{EL}$ (non-convex) over acyclic TBoxes
- Cyclic TBoxes
- Change the temporal component: LTL, μ-calculus?

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- Cyclic TBoxes
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Ευχαριστώ πολύ!