Glossary

Ambiguity [Mehrdeutigkeit]: A feature of natural languages that makes it possible for a single sentence to have two or more meanings. For example, Max is happy or Claire is happy and Carl is happy, can be used to claim that either Max is happy or both Claire and Carl are happy, or it can be used to claim that at least one of Max and Claire is happy and that Carl is happy. Ambiguity can also arise from words that have two meanings, as in the case of puns. FOL does not allow for ambiguity.

Antecedent [Antezedens]: The antecedent of a conditional is its first component clause. In $P \rightarrow Q$, P is the antecedent and Q is the consequent.

Argument [Argument]: The word "argument" is ambiguous in logic.

1. One kind of argument consists of a sequence of statements in which one (the conclusion) is supposed to follow from or be supported by the others (the premises).
2. Another use of "argument" refers to the term(s) taken by a predicate in an atomic wff. In the atomic wff LeftOf(x, a), x and a are the arguments of the binary predicate LeftOf.

Arity [Stelligkeit]: The arity of a predicate indicates the number of arguments (in the second sense of the word) it takes. A predicate with arity of one is called unary. A predicate with an arity of two is called binary. It's possible for a predicate to have any arity, so we can talk about 6-ary or even 113-ary predicates.

Atomic sentences [Atomare Sätze]: Atomic sentences are the most basic sentences of FOL, those formed by a predicate followed by the right number (see arity) of names (or complex terms, if the language contains function symbols). Atomic sentences in FOL correspond to the simplest sentences of English.

Axiom [Axiom]: An axiom is a proposition (or claim) that is accepted as true about some domain and used to establish other truths about that domain.

Boolean connective (Boolean operator) [Boolscher Operator]: The logical connectives conjunction, disjunction, and negation allow us to form complex claims from simpler claims and are known as the Boolean connectives after the logician George Boole. Conjunction corresponds to the English word and, disjunction to or, and negation corresponds to the phrase it is not the case that. (See also Truth-functional connective.)

Bound variable [Gebundene Variable]: A bound variable is an instance of a variable occurring within the scope of a quantifier used with the same variable. For example, in $\forall x\ P(x, y)$ the variable x is bound, but y is "unbound" or "free."

Claim [Behauptung]: Claims are made by people using declarative sentences. Sometimes claims are called propositions.

Completeness [Vollständigkeit]: "Completeness" is an overworked word in logic.

1. A formal system of deduction is said to be complete if, roughly speaking, every valid argument has a proof in the formal system. This sense is discussed in Section 8.3 and elsewhere in the text. (Compare with Soundness.)
2. A set of sentences of FOL is said to be formally complete if for every sentence of the language, either it or its negation can be proven from the set, using the rules of the given formal system. Completeness, in this sense, is discussed in Section 19.8.
3. A set of truth-functional connectives is said to be truth-functionally complete if every truth-functional connective can be defined using only connectives in the given set. Truth-functional completeness is discussed in Section 7.4.

Conclusion [Konklusion]: The conclusion of an argument is the statement that is meant to follow from the other statements, or premises. In most formal systems, the conclusion comes after the premises, but in natural language, things are more subtle.

Conditional [Implikation]: The term "conditional" refers to a wide class of constructions in English including if. . . then. . . , . . . because. . . , . . . unless. . . ., and the like, that express some kind of conditional relationship between the two parts. Only some of these constructions are truth functional and can be represented by means of the material conditional of FOL. (See Material conditional.)

Conditional proof [Beweis durch Annahme]: Conditional proof is the method of proof that allows one to prove a conditional statement $P \rightarrow Q$ by temporarily assuming P and proving Q under this additional assumption.

Conjunct [Konjunktions-Glied]: One of the component sentences in a conjunction. For example, A and B are the conjuncts of $A \wedge B$.

Conjunction [Konjunktion]: The Boolean connective corresponding to the English word and. A conjunction of sentences is true if and only if each conjunct is true.

Conjunctive normal form (CNF) [Konjunktive Normalform]: A sentence is in conjunctive normal form if it is a conjunction of one or more disjunctions of one or more literals.

Connective [Junktor]: An operator for making new statements out of simpler statements. Typical examples are conjunction, negation, and the conditional.

Consequent [Konsequens]: The consequent of a conditional is its second component clause. In $P \rightarrow Q$, Q is the antecedent and P is the consequent.

Context sensitivity [Kontextabhängigkeit]: A predicate, name, or sentence is context sensitive when its interpretation depends on our perspective on the world. For example, in Tarski's World, the predicate Larger is not context sensitive since it is a determinate matter whether one block is larger than another, regardless of our perspective on the world, whereas the predicate LeftOf depends on our perspective on the blocks world. In English many words are context sensitive, including words like I, here, now, friend , home, and so forth.

Counterexample [Gegenbeispiel]: A counterexample to an argument is a possible situation in which all the premises of the argument are true but the conclusion is false. Finding even a single counterexample is sufficient to show that an argument is not logically valid.

Contradiction [Widerspruch] ($\perp$): Something that cannot possibly be true in any set of circumstances, for example, a statement and its negation. The symbol $\perp$ represents contradiction.

Corollary [Korollar]: A corollary is a result which follows with little effort from an earlier theorem. (See Theorem.)

Deductive system [Deduktionssytem]: A deductive system is a collection of rules and a specification of the ways they can be use to construct formal proofs. The system F defined in the text is an example of a deductive system, though there are many others.

Determinate property [Bestimmte Eigenschaft]: A property is determinate if for any object there is a definite fact of the matter whether or not the object has that property. In first-order logic we assume that we are working with determinate properties.

Determiner [Bestimmungswort]: Determiners are words such as every, some , most, etc., which combine with common nouns to form quantified noun phrases like every dog, some horses, and most pigs.

Disjunct [Disjunktions-Glied]: One of the component sentences in a disjunction. For example, A and B are the disjuncts of $A \vee B$.

Disjunction [Disjunktion]: The basic Boolean connective corresponding to the English word or. A disjunction is true if at least one of the disjuncts is true. (See also Inclusive disjunction and Exclusive disjunction.)

Disjunctive normal form (DNF) [Disjunktive Normalform]: A sentence is in disjunctive normal form if it is a disjunction of one or more conjunctions of one or more literals.

Domain of discourse [Individuen-Bereich]: When we use a sentence to make a claim, we always implicitly presupposesome domain of discourse. In FOL this becomes important in understanding quantification, since there must be a set of objects under consideration when evaluating claims involving quantifiers. For example, the truth-value of the claim "Every student received a passing grade" depends on our domain of discourse. The truth-values may differ depending on whether our domain of discourse contains all the students in the world, in the university, or just in one particular class.

Domain of quantification [Quantifizierungs-Bereich]: See Domain of discourse.

Empty set [Leere Menge]: The unique set with no elements, often denoted by ∅.

Equivalence classes [Äquivalenzklassen]: An equivalence class is the set of all things equivalent to a chosenobject with respect to a particular equivalence relation. More specifically, given an equivalence relation R on a set S, we can define an equivalence class for any $x \in D$ as follows:
$\{y \in D \mid \langle x, y\rangle \in R\}$

Equivalence relation [Äquivalenzrelation]: An equivalence relation is a binary relation that is reflexive, symmetric, and transitive.

Exclusive disjunction [Exklusiv-Disjunktion]: This is the use of or in English that means exactly one of the two disjuncts is true, but not both. For example, when a waiter says "You may have soup or you may have salad," the disjunction is usually meant exclusively. Exclusive disjunctions can be expressed in FOL, but the basic disjunction of FOL is inclusive, not exclusive.

Existential quantifier (∃) [Existenzquantor]: In FOL , the existential quantifier is expressed by the symbol ∃ and is used to make claims asserting the existence of some object in the domain of discourse. In English, we express existentially quantified claims with the use of words like something, at least one thing, a, etc.

First-order consequence [Folgerung erster Stufe]: A sentence S is a first-order consequence of some premises if S follows from the premises simply in virtue of the meanings of the truth-functional connectives, identity, and the quantifiers.

First-order structure [Struktur erster Stufe]: A first-order structure is a mathematical model of the circumstances that determine the truth values of the sentences of a given first-order language. It is analogous to a truth assignment for propositional logic but must also model the domain of quantification and the objects to which the predicates apply.

First-order validity [Gültiger Satz erster Stufe]: A sentence S is a first-order validity if S is a logical truth simply in virtue of the meanings of the truth-functional connectives, identity, and the quantifiers. This is the analog, in first-order logic, of the notion of a tautology in propositional logic.

Formal proof [Formaler Beweis]: See Proof.

Free variable [Freie Variable]: A free variable is an instance of a variable that is not bound. (See Bound variable.)

Generalized quantifier [Verallgemeinerter Quantor]: Generalized quantifiers refer to quantified expressions beyond the simple uses of ∀ (everything) and ∃ (something); expressions like Most students, Few teachers, and Exactly three blocks.

Inclusive disjunction [Inklusiv-Disjunktion]: This is the use of or in which the compound sentence is true as long as at least one of the disjuncts is true. It is this sense of or that is expressed by FOL's disjunction. Compare Exclusive disjunction.

Indirect proof [Indirekter Beweis]: See Proof by contradiction.

Individual constant [Individuenkonstante]: Individual constants, or names, are those symbols of FOL that stand for objects or individuals. In FOL is it assumed that each individual constant of the language names one and only one object.

Inductive definition [Induktive Definition]: Inductive definitions allow us to define certain types of sets that cannot be defined explicitly in first-order logic. Examples of inductively defined sets include the set of wffs, the set of formal proofs, and the set of natural numbers. Inductive definitions consist of a base clause specifying the basic elements of the defined set, one or more inductive clauses specifying how additional elements are generated from existing elements, and a final clause, which tells us that all the elements are either basic or in the set because of (possibly repeated) application of the inductive clauses.

Inductive proof [Induktiver Beweis]: Inductive proofs are used to establish claims about inductively defined sets. Given such a set, to prove that some property holds of every element of that set we need a basis step, which shows that the property holds of the basic elements, and an inductive step, which shows that if the property holds of some elements, then it holds of any elements generated from them by the inductive clauses. See Inductive definition.

Infix notation [Infix-Notation]: In infix notation, the predicate or function symbol appears between its two arguments. For example, a < b and a = b use infix notation. Compare with Prefix notation.

Informal proof [Informaler Beweis]: See Proof.

Intersection (∩) [Durchschnitt]: The operation on sets a and b that returns the set a ∩ b whose members are those objects common to both a and b.

Lemma [Lemma]: A lemma is a claim that is proven, like a theorem, but whose primary importance is for proving other claims. Lemmas are of less intrinsic interest than theorems. (See Theorem.)

Literal [Literal]: A literal is a sentence that is either an atomic sentence or the negation of an atomic sentence.

Logical consequence [Logische Folgerung]: A sentence S is a logical consequence of a set of premises if it is impossible for the premises all to be true while the conclusion S is false.

Logical equivalence [Logische Äquivalenz]: Two sentences are logically equivalent if they have the same truth values in all possible circumstances.

Logical necessity [Logische Notwendigkeit]: See Logical truth.

Logical possibility [Logische Möglichkeit]: We say that a sentence or claim is logically possible if there is no logical reason it cannot be true, i.e., if there is a possible circumstance in which it is true.

Logical truth [Logische Wahrheit]: A logical truth is a sentence that is a logical consequence of any set of premises. That is, no matter what the premises may be, it is impossible for the conclusion to be false. This is also called a logical necessity

Logical validity [Logische Gültigkeit]: An argument is logically valid if the conclusion is a logical consequence of the premises.

Material conditional [Materiale Implikation]: A truth-functional version of the conditional if. . . then. . . . The material conditional P → Q is false if P is true and Q is false, but otherwise is true. (See Conditional.)

Modus ponens [Modus ponens]: The Latin name for the rule that allows us to infer Q from P and P → Q. Also known as → Elimination.

Names [Namen]: See Individual constants.

Necessary condition [Notwendige Bedingung]: A necessary condition for a statement S is a condition that must hold in order for S to obtain. For example, if you must pass the final to pass the course, then your passing the final is a necessary condition for your passing the course. Compare with Sufficient condition.

Negation normal form (NNF) [Negations-Normalform]: A sentence of FOL is in negation normal form if all occurrences of negation apply directly to atomic sentences. For example, (¬A ∧ ¬B) is in NNF whereas ¬(A ∨ B) is not in NNF.

Numerical quantifier [Zahlquantor]: Numerical quantifiers are those quantifiers used to express numerical claims, for example, at least two, exactly one, no more than five, etc.

Predicate [Prädikat]: Predicates are used to express properties of objects or relations between objects. Larger and Cube are examples of predicates in the blocks language.

Prefix notation [Präfix-Notation]: In prefix notation, the predicate or relation symbol precedes the terms denoting objects in the relation. Larger(a, b) is in prefix notation. Compare with Infix notation.

Premise [Prämisse]: A premise of an argument is one of the statements meant to support (lead us to accept) the conclusion of the argument.

Prenex normal form [Pränexe Normalform]: A wff of FOL is in prenex normal form if it contains no quantifiers, or all the quantifiers are "out in front."

Proof [Beweis]: A proof is a step-by-step demonstration that one statement (the conclusion) follows logically from some others (the premises). A formal proof is a proof given in a formal system of deduction; an informal proof is generally given in English, without the benefit of a formal system.

Proof by cases [Beweis durch Fallunterscheidung]: A proof by cases consists in proving some statement S from a disjunction by proving S from each disjunct.

Proof by contradiction [Beweis durch Widerspruch]: To prove ¬S by contradiction, we assume S and prove a contradiction. In other words, we assume the negation of what we wish to prove and show that this assumption leads to a contradiction.

Proof by induction [Beweis durch Induktion]: See Inductive proof.

Proof of non-consequence [Beweis einer Nicht-Folgerung]: In a proof of non-consequence, we show that an argument is invalid by finding a counterexample. That is, to show that a sentence S is not a consequence of some given premises, we have to show that it is possible for the premises to be true in some circumstance where S is false.

Proposition [Proposition]: Something that is either true or false. Also called a claim.

Quantifier [Quantor]: In English, a quantified expression is a noun phrase using a determiner such as every, some, three, etc. Quantifiers are the elements of FOL that allow us to express quantified expressions like every cube. There are only two quantifiers in FOL, the universal quantifier (∀) and the existential quantifier (∃). From these two, we can, however, express more complex quantified expressions.

Reductio ad absurdum [“Zurückführung auf das Falsche”]: See Proof by contradiction.

Satisfaction [Erfülltheit]: An object named a satisfies an atomic wff S(x) if and only if S(a) is true, where S(a) is the result of replacing all free occurrences of x in S(x) with the name a . Satisfaction for wffs with more than one free variable is defined similarly, using the notion of a variable assignment.

Scope [Sichtbarkeit]: The scope of a quantifier in a wff is that part of the wff that falls under the "influence" of the quantifier. Parentheses play an important role in determining the scope of quantifiers. For example, in

∀x(P(x) → Q(x)) → S(x)

the scope of the quantifier extends only over P(x) → Q(x). If we were to add another set of parentheses, e.g.,

∀x((P(x) → Q(x)) → S(x))

the scope of the quantifier would extend over the entire sentence.

Sentence [Satz]: In propositional logic, atomic sentences are formed by combining names and predicates. Compound sentences are formed by combining atomic sentences by means of the truth functional connectives. In FOL, the definition is a bit more complicated. A sentence of FOL is a wff with no free variables.

Soundness [Korrektheit]: "Sound" is used in two different senses in logic.

1. An argument is sound if it is both valid and all of its premises are true.
2. A formal system is sound if it allows one to construct only proofs of valid arguments, that is, if no invalid arguments are provable within the system. (Compare with Completeness.)

Sufficient condition [Hinreichende Bedingung]: A sufficient condition for a statement S is a condition that guarantees that S will obtain. For example, if all you need to do to pass the course is pass the final, then your passing the final is a sufficient condition for your passing the course. Compare with Necessary condition.

Tautological consequence [Tautologische Folgerung]: A sentence S is a tautological consequence of some premises if S follows from the premises simply in virtue of the meanings of the truth-functional connectives. We can check for tautological consequence by means of truth tables, since S is a tautological consequence of the premises if and only if every row of their joint truth table that assigns true to each of premise also assigns true to S. All tautological consequences are logical consequences, but not all logical consequences are tautological consequences.

Tautological equivalence [Tautologische Äquivalenz]: Two sentences are tautologically equivalent if they are equivalent simply in virtue of the meanings of the truth-functional connectives.We can check for tautological equivalence by means of truth tables since two sentences Q and S are tautologically equivalent if and only if every row of their joint truth table assigns the same value to the main connectives of Q and S.

Tautology [Tautologie]: A tautology is a sentence that is logically true in virtue of its truth-functional structure. This can be checked using truth tables since S is a tautology if and only if every row of the truth table for S assigns true to the main connective.

Term [Term]: Variables and individual constants are terms of a first-order language, as are the results of combining an n-ary function symbol f with n terms to form a new term.

Theorem [Theorem]: In formal systems, a theorem of is any statement that has been proven from some given set of axioms. Informally, the term "theorem" is usually reserved for conclusions that the author finds particularly interesting or important. (Compare Corollary and Lemma.)

Truth assignment [Wahrheitswert-Belegung]: A function assigning true or false to each atomic sentence of a first-order language. Used to model the informal notion of a world or set of circumstances.

Truth-functional connective [Junktor]: A sentence connective with the property that the truth value of the newly formed sentence is determined solely by the truth value(s) of the constituent sentence(s), nothing more. Examples are the Boolean connectives ($\neg$, $\wedge$, $\vee$) and the material conditional and biconditional ($\rightarrow$, $\leftrightarrow$).

Truth table [Wahrheitstafel]: Truth tables show the way in which the truth value of a sentence built up using truth-functional connectives depends on the truth values of the sentence's components.

Truth value [Wahrheitswert]: The truth value of a statement in some circumstances is true if the statement is true in those circumstances, otherwise its truth value is false. This is an informal notion but also has rigorous counterparts in propositional logic, where circumstances are modeled by truth assignments, and in first-order logic where circumstances are modeled by first-order structures.

Universal quantifier ($\forall$) [Allquantor]: The universal quantifier is used to express universal claims. Its corresponds, roughly, to English expressions such as everything, all things, each thing, etc. (See also Quantifiers.)

Union ($\cup$) [Vereinigung]: The operation on sets a and b that returns the set $a \cup b$ whose members are those objects in either a or b or both.

Validity [Gültigkeit]: "Validity" is used in two ways in logic:

1. Validity as a property of arguments: An argument is valid if the conclusion must be true in any circumstance in which the premises are true. (See also Logical validity and Logical consequence.)
2. Validity as a property of sentences: A first-order sentence is said to be valid if it is logically true simply in virtue of the meanings of its connectives, quantifiers, and identity. (See First-order validity.)

Variable [Variable]: Variables are expressions of FOL that function somewhat like pronouns in English. They are like individual constants in that they may be the arguments of predicates, but unlike constants, they can be bound by quantifiers. Generally letters from the end of the alphabet, x, y, z, etc., are used for variables.

Variable assignment [Variablenbelegung]: A function assigning objects to some or all of the variables of a first-order language. This notion is used in defining truth of sentences in a first-order structure.

Well-formed formula (wff) [Wohlgeformte Formel]: Wffs are the "grammatical" expressions of FOL. They are defined inductively. First, an atomic wff is any n-ary predicate followed by n terms. Complex wffs are constructed using connectives and quantifiers. The rules for constructing complex wffs are found on page 231.

Wffs may have free variables. Sentences of FOL are wffs with no free variables.